<table>
<tr><th></th><th colspan="3">SHAPES AND DESIGNS Two Dimensional Geometry</th></tr>
<tr><td>Instructional Time and Investigations</td><td>21 days</td><td colspan="2">• Inv. 1: The Family of Polygons (5 Problems)<br>• Inv. 2: Designing Polygons: The Angle Connection (4 Problems)<br>• Inv. 3: Designing Triangles and Quadrilaterals (5 Problems)</td></tr>
<tr><td>Goals</td><td>Properties of Polygons: Understand the properties of polygons that affect their shape.<br>• The shape of a polygon is determined by its sides and angles. Polygons can be sorted into families according to the number and lengths of their sides and the measures of their angles.<br>• Patterns exist among interior and exterior angles in polygons. For example, the sum of the interior angles of a polygon relates to the number of triangles that are formed by drawing diagonals from one vertex.</td><td>Relationships Among Angles: Understand special relationships among angles.<br>• Angles can be classified by their size, their location in relation to each other in a figure or design, and their combined angle measure. Angle classification by location or combined angle measure can help you write equations to find unknown angle measures.</td><td>Constructing Polygons: Understand the properties needed to construct polygons.<br>• Triangles have 3 sides, but not every combination of 3 side lengths will make a triangle.<br>• As with triangles, specific combinations of side lengths and angle measures of a polygon can produce congruent copies of the polygon.<br>• Special properties of polygons, such as angle sum, side-length relationships, and symmetry, make them useful in building, design, and nature.</td></tr>
<tr><td>Common Core Standards</td><td>Common Core Standards for Mathematical Practice<br>MP.1: Make sense of problems and persevere in solving them.<br>MP.2: Reason abstractly and quantitatively.<br>MP.3: Construct viable arguments and critique the reasoning of others.<br>MP.4: Model with mathematics.<br>MP.5: Use appropriate tools strategically.<br>MP.6: Attend to precision.<br>MP.7: Look for and make use of structure.<br>MP.8: Look for and express regularity in repeated reasoning.</td><td colspan="2">Common Core Content Standards<br>7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.<br>7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.<br>Also 7.EE.A.2</td></tr>
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| | SHAPES AND DESIGNS Two Dimensional Geometry | |
|---|---|---|
| | Content Connections to Other Units | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Properties of Polygons:** Understand the properties of polygons that affect their shape. | • Developing mathematical reasoning by analyzing integers and data (*Prime Time*)<br>• Developing shape recognition skills (*Elementary School*)<br>• Finding area and perimeter of 2-D figures (*Covering and Surrounding*)<br>• Developing classification skills through classifying integers (e.g., even, odd, abundant, deficient) (*Prime Time*)<br>• Developing shape recognition skills (*Elementary School*)<br>• Learning important properties of rectangles, triangles, and parallelograms (*Covering and Surrounding*) | • Exploring similarity of 2-D figures (*Stretching and Shrinking*)<br>• Finding surface area and volume of 3-D figures (*Filling and Wrapping; Say It With Symbols*)<br>• Enlarging, shrinking, and distorting 2-D shapes (*Stretching and Shrinking*)<br>• Learning properties of 3-D figures (*Filling and Wrapping*)<br>• Learning and applying the Pythagorean Theorem (*Looking for Pythagoras*)<br>• Enlarging, shrinking, flipping, and translating graphs of functions (*Function Junction*) |
| **Relationships Among Angles:** Understand special relationships among angles. | • *Developing angle recognition skills (Elementary School)*<br>• *Understanding degrees as the unit of angle measure (Elementary School)* | • Enlarging, shrinking, and distorting 2-D shapes (*Stretching and Shrinking*)<br>• Understanding congruence (*Butterflies, Pinwheels, and Wallpaper*) |
| **Constructing Polygons:** Understand the properties needed to construct polygons. | • Understanding area as the exact number of square units needed to cover a 2-D figure (*Covering and Surrounding*)<br>• Exploring how 2-D shapes fit together (*Elementary School*) | • Subdividing figures into similar figures (*Stretching and Shrinking*)<br>• Connecting tessellations to isometries (*Butterflies, Pinwheels, and Wallpaper*)<br>• Connecting symmetry to isometries (*Butterflies, Pinwheels, and Wallpaper; Function Junction*) |

| | ACCENTUATE THE NEGATIVE Integers and Rational Numbers | |
|---|---|---|
| **Instructional Time and Investigations** | 22 days | • Inv. 1: Extending the Number System (4 Problems)<br>• Inv. 2: Adding and Subtracting Rational Numbers (4 Problems)<br>• Inv. 3: Multiplying and Dividing Rational Numbers (4 Problems)<br>• Inv. 4: Properties of Operations (3 Problems) |
| **Goals** | **Rational Numbers:** Develop an understanding that rational numbers consist of positive numbers, negative numbers, and zero.<br>• Rational numbers can be compared, ordered, and located on a number line. They can also be used to indicate a distance or difference between points on a number line. Number lines are useful models for solving problems with rational numbers. | **Operations with Rational Numbers:** Develop understanding of operations with rational numbers and their properties.<br>• Models facilitate understanding the meaning of addition, subtraction, multiplication, and division of positive and negative numbers, and improve understanding of the standard algorithms for these operations. This also helps to identify which operation is helpful to solve a problem.<br>• Mathematical sentences, with or without variables, can model real-world problems. Sometimes rewriting a problem using a different operation can be helpful in finding the solution.<br>• Properties of operations (such as Order of Operations, Commutative Property, and Distributive Property) extend to all rational numbers, and understanding these properties is helpful in solving problems. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.NS.A.1:** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.<br>**7.NS.A.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.<br>**7.NS.A.3:** Solve real-world and mathematical problems involving the four operations with rational numbers.<br>**7.EE.B.3:** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.<br>**7.EE.B.4:** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>**Also 7.NS.A.1a–d, 7.NS.A.2a–d, 7.EE.B.4b** |

| | **ACCENTUATE THE NEGATIVE** Integers and Rational Numbers | |
|---|---|---|
| | **Content Connections to Other Units** | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Rational Numbers:** Develop an understanding that rational numbers consist of positive numbers, negative numbers, and zero. | • Developing understanding of whole numbers and rational numbers (*Prime Time; Comparing Bits and Pieces; Let's Be Rational; Decimal Ops*)<br>• Using models to develop understanding of mathematical concepts (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops; Covering and Surrounding*)<br>• Using a coordinate grid with positive coordinates (*Data About Us; Variables and Patterns*)<br>• Using a number line to develop equivalence and operations of fractions and decimals (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops*) | • Interpreting and applying positive and negative slopes of lines and positive and negative coefficients in equations (*Moving Straight Ahead; Thinking With Mathematical Models; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*)<br>• Developing understanding of square roots and irrational numbers (Looking for Pythagoras)<br>• Understanding relationships between positive and negative coefficients or values for variables (*Moving Straight Ahead; Thinking With Mathematical Models; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*)<br>• Using positive and negative integers to communicate directions in two dimensions (*Stretching and Shrinking; Butterflies, Pinwheels, and Wallpaper*)<br>• Graphing equations and functions on coordinate grids (*Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction; Stretching and Shrinking, Butterflies, Pinwheels, and Wallpaper*)<br>• Locating square roots on the number line (*Looking for Pythagoras*) |
| **Operations with Rational Numbers:** Develop understanding of operations with rational numbers and their properties. | • Understanding and applying arithmetic operations with rational numbers (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops*)<br>• Developing understanding of the Commutative Property and Distributive Property using whole numbers and rational numbers (*Prime Time; Let's Be Rational; Decimal Ops; Variables and Patterns*)<br>• Using the Order of Operations to solve problems in a context (*Prime Time; Covering and Surrounding; Variables and Patterns*) | • Evaluating algebraic expressions involving positive and negative coefficients or values for variables (*Moving Straight Ahead; Data Distributions; Thinking With Mathematical Models; Frogs, Fleas, and Painted Cubes; Say It With Symbols, It's In the System; Function Junction*)<br>• Interpreting isometries in the plane given in symbolic form (*Butterflies, Pinwheels, and Wallpaper*)<br>• Using the properties and Order of Operations to write equivalent expressions and solve equations (*Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*) |

| | STRETCHING AND SHRINKING Understanding Similarity | |
|---|---|---|
| **Instructional Time and Investigations** | 19 days | • Inv. 1: Enlarging and Reducing Shapes (2 Problems)<br>• Inv. 2: Similar Figures (3 Problems)<br>• Inv. 3: Scaling Perimeter and Area (4 Problems)<br>• Inv. 4: Similarity and Ratios (4 Problems) |
| **Goals** | **Similar Figures:** Understand what it means for figures to be similar.<br>• Similar figures have congruent corresponding angles, and corresponding side lengths are in a proportional relationship.<br>• Algebraic rules can be used to stretch or shrink a shape into a new shape that is similar or nonsimilar to the original image. | **Reasoning with Similar Figures:** Develop strategies for using similar figures to solve problems.<br>• The scale factor for two similar figures is established by finding the ratio of a pair of corresponding sides. Scale factor, used with other tools, allows you to make drawings of similar figures and to compare the perimeters and areas of similar figures.<br>• If two figures are similar, then you can use a proportional relationship between corresponding sides to find unknown side lengths. This can be used to solve real-world problems, finding distances and measurements that cannot be measured directly. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.RP.A.2:** Recognize and represent proportional relationships between quantities.<br>**7.RP.A.3:** Use proportional relationships to solve multistep ratio and percent problems.<br>**7.EE.B.3:** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.<br>**7.G.A.1:** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.<br>**7.G.A.2:** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.<br>**Also 7.RP.A.2a–b, 7.EE.B.4, 7.G.B.6** |

| | STRETCHING AND SHRINKING Understanding Similarity | |
|---|---|---|
| | Content Connections to Other Units | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Similar Figures:** Understand what it means for figures to be similar. | • Finding angle measures, lengths, and areas of plane geometric figures (*Covering and Surrounding; Shapes and Designs*)<br>• Developing and applying concepts of vertex, angle, angle measure, side, and side length (*Covering and Surrounding; Shapes and Designs*)<br>• Constructing two–dimensional shapes (*Shapes and Designs*)<br>• Using symbols to communicate operations (*Variables and Patterns; Accentuate the Negative*)<br>• Exploring symmetries of a figure (*Shapes and Designs*) | • Scaling quantities, objects, and shapes up and down (*Comparing and Scaling; Filling and Wrapping; Butterflies, Pinwheels, and Wallpaper*)<br>• Analyzing how two-dimensional shapes are affected by rotations, reflections, translations, and dilations; generating isometric transformations (*Butterflies, Pinwheels, and Wallpaper*)<br>• Finding the equation of a line (*Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models*)<br>• Expressing linear relationships with symbols (*Comparing and Scaling; Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing*)<br>• Determining whether linear expressions are equivalent (*Say It With Symbols*)<br>• Writing directions for isometries in two dimensions (*Butterflies, Pinwheels, and Wallpaper; Function Junction*) |
| **Reasoning with Similar Figures:** Develop strategies for using similar figures to solve problems. | • Using factors and multiples (*Prime Time*)<br>• Measuring two-dimensional figures (*Covering and Surrounding*)<br>• Using ratios in fraction form (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops*)<br>• Using maps (*Variables and Patterns*)<br>• Exploring properties of two-dimensional shapes; finding areas, perimeters, and side lengths of shapes (*Covering and Surrounding; Shapes and Designs*) | • Scaling and comparing figures and quantities (*Comparing and Scaling*)<br>• Using slope to solve problems involving linear relationships (*Moving Straight Ahead; Thinking With Mathematical Models*)<br>• Exploring ratios and proportional relationships (*Comparing and Scaling; Moving Straight Ahead*)<br>• Developing the concept of slope (*Moving Straight Ahead; Thinking With Mathematical Models*) |

| | COMPARING AND SCALING Ratios, Rates, Percents, and Proportions | | |
|---|---|---|---|
| **Instructional Time and Investigations** | 22 days | • Inv. 1: Ways of Comparing: Ratios and Proportions (4 Problems)<br>• Inv. 2: Comparing and Scaling Rates (3 Problems)<br>• Inv. 3: Markups, Markdowns, and Measures: Using Ratios, Percents, and Proportions (3 Problems) | |
| **Goals** | **Ratios, Rates, and Percents:** Understand ratios, rates, and percents.<br>• Ratios make comparisons between two quantities. Rates, unit rates, and percents are all types of ratios.<br>• Knowing the desired ratio between two variables allows you to scale the ratio or find a missing part of a ratio. | **Proportionality:** Understand proportionality in tables, graphs, and equations.<br>• A proportional relationship has particular characteristics when represented in a table, graph or equation. For example, it is a straight line in a graph and can be represented as $y = mx$. | **Reasoning Proportionality:** Develop and use strategies for solving problems that require proportional reasoning.<br>• Being able to change the form of a ratio is a useful problem-solving strategy.<br>• Various strategies can be used to solve problems involving proportions, including scaling, rate tables, percent bars, unit rates, and equivalent ratios. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.RP.A.1:** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.<br>**7.RP.A.2:** Recognize and represent proportional relationships between quantities.<br>**7.RP.A.3:** Use proportional relationships to solve multistep ratio and percent problems.<br>**7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>**Also 7.RP.A.2a–d, 7.NS.A.3, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4a** | |

| | COMPARING AND SCALING Ratios, Rates, Percents, and Proportions | |
|---|---|---|
| | Content Connections to Other Units | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Ratios, Rates, and Percents:** Understand ratios, rates, and percents. | • Exploring and applying rational number concepts (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops; Accentuate the Negative*)<br>• Percent defined as a ratio to 100 and connected to fractions and decimals (*Comparing Bits and Pieces; Let's Be Rational; Decimal Ops*) | • Calculating and applying slope with equations in $y = mx + b$ form (*Moving Straight Ahead; Thinking With Mathematical Models; Say It With Symbols*)<br>• Making comparisons between groups of different sizes (*Samples and Populations; Growing, Growing, Growing*) |
| **Proportionality:** Understand proportionality in tables, graphs, and equations. | • Connecting and comparing rates using ratios, decimals, and percents (*Comparing Bits and Pieces; Let's Be Rational; Stretching and Shrinking*)<br>• Comparing data sets (*Data About Us*)<br>• Representing patterns of change in words, tables, graphs, and equations (*Variables and Patterns*)<br>• Fractions as a part/whole comparison, addition, subtraction, multiplication, and division with fractions (*Comparing Bits and Pieces; Let's Be Rational*) | • Comparing probabilities (*What Do You Expect?*)<br>• Comparing data sets (*Samples and Populations; Thinking With Mathematical Models*)<br>• Finding the equation of a line (*Moving Straight Ahead; Thinking With Mathematical Models*)<br>• Expressing linear relationships with symbols (*Moving Straight Ahead; Thinking With Mathematical Models; Growing, Growing, Growing*)<br>• Expressing and applying probabilities as fractions (*What Do You Expect?*)<br>• Determining if two algebraic expressions are equivalent (*Growing, Growing, Growing; Frogs, Fleas and Painted Cubes; Say It With Symbols; Function Junction*) |
| **Reasoning Proportionality:** Develop and use strategies for solving problems that require proportional reasoning. | • Using percents to make comparisons (*Comparing Bits and Pieces; Decimal Ops*)<br>• Recognizing direct proportionality relationships with a unit rate (*Variables and Patterns*)<br>• Making inferences about quantities (*Data About Us*)<br>• Comparing and subdividing similar figures to determine scale factors (*Stretching and Shrinking*) | • Expressing proportional and nonproportional linear relationships with symbols (*Moving Straight Ahead; Thinking With Mathematical Models*)<br>• Making inferences about quantities and populations based on experimental or theoretical probabilities (*What Do You Expect?*)<br>• Estimating with and comparing large numbers (*Growing, Growing, Growing*)<br>• Developing benchmarks and skills for estimating irrational numbers (*Looking for Pythagoras*) and for estimating populations (*Samples and Populations*)<br>• Scaling up rectangular prisms (*Filling and Wrapping*) |

| | MOVING STRAIGHT AHEAD Linear Relationships | |
|---|---|---|
| **Instructional Time and Investigations** | 25 days | • Inv. 1: Walking Rates (4 Problems)<br>• Inv. 2: Exploring Linear Relationships With Graphs and Tables (4 Problems)<br>• Inv. 3: Solving Equations (5 Problems)<br>• Inv. 4: Exploring Slope: Connecting Rates and Ratios (4 Problems) |
| **Goals** | **Linear Relationships:** Recognize problem situations in which two variables have a linear relationship.<br>• Two variables are in a linear relationship if one variable is changing by a constant amount when the other variable changes by increments of 1 unit.<br>• The rate of change in a linear relationship is represented by the slope of the line representing the relationship.<br>• The equation $y = mx$ is a particular kind of linear relationship in which $x$ and $y$ are proportional to each other. | **Equivalence:** Understand that the equality sign indicates that two expressions are equivalent.<br>• Solutions for linear equations of the form $y = mx + b$ are pairs of values $(x, y)$ which make this equation true. Graphically, solution pairs are points on the graph of the line.<br>• Properties of equality can be used to maintain equivalent expressions on each side of the equation when finding a solution. Determining which equivalent expression to use in solving a problem is important. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.RP.A.2:** Recognize and represent proportional relationships between quantities.<br>**7.EE.A.1:** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.<br>**7.EE.A.2:** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.<br>**7.EE.B.4:** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>**Also 7.RP.A.2a–d, 7.EE.B.3, 7.EE.B4a–b** |

## MOVING STRAIGHT AHEAD Linear Relationships

### Content Connections to Other Units

| Goals of the Unit | Prior Work | Future Work |
|---|---|---|
| **Linear Relationships:** Recognize problem situations in which two variables have a linear relationship. | • Graphing data in the coordinate plane; using symbols to represent relationships between variables (*Variables and Patterns; Accentuate the Negative; Comparing and Scaling*)<br>• Expressing relationships between variables in words, symbols, graphs, and tables (*Variables and Patterns; Covering and Surrounding; Shapes and Designs; Comparing and Scaling*)<br>• Computing and interpreting ratios (*Comparing Bits and Pieces; Decimal Ops; Stretching and Shrinking; Comparing and Scaling*)<br>• Finding rates of change in relationships between two variables (*Variables and Patterns; Comparing and Scaling*)<br>• Understanding positive and negative rational numbers (*Accentuate the Negative*)<br>• Graphing relationships between two variables (*Variables and Patterns; Comparing and Scaling*)<br>• Finding values of the variables in a linear relationship using graphs or tables or numeric reasoning (*Variables and Patterns; Comparing and Scaling*)<br>• Understanding the meaning of parallel and intersecting lines (*Shapes and Designs*) | • Identifying and interpreting patterns of change for exponential ($y = ax$), quadratic ($y = ax^2 + bx + c$), and inverse variation relationships (e.g. $y = k/x$) (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; Function Junction*)<br>• Writing and interpreting equations that represent linear, inverse, exponential, and quadratic relationships (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*)<br>• Analyzing linear models and interpreting slope of lines representing linear relationships (*Thinking With Mathematical Models; Growing, Growing, Growing*)<br>• Finding the slope of a line to determine an equation in $y = mx + b$ form (*Thinking With Mathematical Models; Say It With Symbols; It's In the System*)<br>• Interpreting and constructing graphs of lines; determining the equation of lines (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*)<br>• Graphing step and piecewise-defined functions (*Function Junction*)<br>• Finding values of the variables in more complicated linear equations (*Thinking With Mathematical Models; Say It With Symbols; It's in the System*)<br>• Finding values of the variables for exponential and quadratic relationships using tables, graphs, and symbolic methods (*Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes*)<br>• Solving systems of linear equations; interpreting, graphing, and solving inequalities (*It's In the System*)<br>• Finding and interpreting points of intersection of two or more graphs of relationships from graphs or tables (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes, Say It With Symbols; It's In the System*)<br>• Interpreting parallel and perpendicular lines (*Looking for Pythagoras*)<br>• Analyzing equivalent linear and quadratic expressions (*Frogs, Fleas, and Painted Cubes; Say It With Symbols*)<br>• Finding the solution to a system of linear equations and interpreting and graphing inequalities (*It's In the System*) |
| **Equivalence:** Understand that the equality sign indicates that two expressions are equivalent. | • Understanding inequalities (*Comparing Bits and Pieces; Variables and Patterns; Accentuate the Negative*)<br>• Writing and interpreting equivalent numeric expressions (*Prime Time; Variables and Patterns; Comparing and Scaling*) | • Solving more complicated linear inequalities (*It's In the System*)<br>• Writing and interpreting equivalent linear, exponential and quadratic expressions (*Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System; Function Junction*) |

| | **WHAT DO YOU EXPECT?** Probability and Expected Value | |
|---|---|---|
| **Instructional Time and Investigations** | 26 days | • Inv. 1: A First Look at Chance (4 Problems)<br>• Inv. 2: Experimental and Theoretical Probability (4 Problems)<br>• Inv. 3: Making Decisions With Probability (4 Problems)<br>• Inv. 4: Analyzing Compound Events Using an Area Model (4 Problems)<br>• Inv. 5: Binomial Outcomes (3 Problems) |
| **Goals** | **Experimental and Theoretical Probabilities:** Understand experimental and theoretical probabilities.<br>• The experimental probability of an event can be found by gathering data from experiments or observations, counting the number of times the specified outcome occurred, and comparing that to the number of trials. Long run relative frequencies collected from experiments make good approximations of theoretical probabilities.<br>• Theoretical probability is determined by reasoning about the likelihood of a specific outcome based on all possible outcomes of an event. Lists, tree diagrams, or area models can show all of the possible outcomes and can be used to determine the theoretical probability of a compound event. | **Reasoning with Probability:** Explore and develop probability models by identifying possible outcomes, and analyze probabilities to solve problems<br>• Probabilities are ratios. Probability can be used to predict outcomes in real-world events, to analyze games for fairness, and to determine the long-term average of a game of chance (expected value). |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.SP.C.5:** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.<br>**7.SP.C.6:** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.<br>**7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.<br>**7.SP.C.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.<br>**Also 7.RP.A.2, 7.RP.A.2a, 7.RP.A.3, 7.SP.C.7a–b, 7.SP.C.8a–b** |

| | WHAT DO YOU EXPECT? Probability and Expected Value | |
|---|---|---|
| | Content Connections to Other Units | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Experimental and Theoretical Probabilities:** Understand experimental and theoretical probabilities. | • Gathering, analyzing, and displaying data to show trends (*Data About Us*)<br>• Performing operations with whole numbers; finding factors and multiples (*Prime Time*)<br>• Developing understanding of ratio in fraction, percent, and decimal form (*Comparing Bits and Pieces; Decimal Ops; Comparing and Scaling)*<br>• Working with fractions and ratios (*Comparing Bits and Pieces; Decimal Ops; Accentuate the Negative; Comparing and Scaling*) | • Understanding and describing data distributions, sampling techniques, and using samples to predict population behaviors (*Samples and Populations*)<br>• Using probabilities to make inferences and predictions about populations based on analysis of population samples (*Samples and Populations*)<br>• Analyzing and comparing data (*Samples and Populations*) |
| **Reasoning with Probability:** Explore and develop probability models by identifying possible outcomes, and analyze probabilities to solve problems. | • Analyzing games or situations (*Prime Time*)<br>• Looking for patterns (*Covering and Surrounding; Shapes and Designs*)<br>• Working with ratio and proportion (*Comparing Bits and Pieces; Decimal Ops; Stretching and Shrinking; Comparing and Scaling*)<br>• Using an area model for understanding addition and multiplication of fractions (*Let's Be Rational*)<br>• Using area model to understand the Distributive Property (*Prime Time; Accentuate the Negative*)<br>• Organizing data collected from experiments (*Variables and Patterns; Data About Us; Moving Straight Ahead*)<br>• Using ratio and proportion (*Comparing Bits and Pieces; Decimal Ops; Stretching and Shrinking; Comparing and Scaling*) | • Developing strategies for analyzing complex situations to determine probabilities (*Samples and Populations*)<br>• Designing simulation models using probability concepts to collect and organize data and make predictions about populations (*Samples and Populations*)<br>• Using expected values of favorable and unfavorable outcomes to make inferences and predictions; using expected values to make recommendations or to develop solutions to real-world problems (*Samples and Populations*)<br>• Analyzing data to show trends or the strength of the linear association of between two variables (*Thinking With Mathematical Models*) |

| | FILLING AND WRAPPING Three Dimensional Measurement | | |
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| **Instructional Time and Investigations** | 23 days | • Inv. 1: Building Smart Boxes: Rectangular Prisms (4 Problems)<br>• Inv. 2: Polygonal Prisms (3 Problems)<br>• Inv. 3: Area and Circumference of Circles (4 Problems)<br>• Inv. 4: Cylinders, Cones, and Spheres (5 Problems) | |
| **Goals** | **Surface Areas and Volumes of Polygonal Prisms and Cylinders:** Understand surface areas and volumes of prisms and cylinders and how they are related<br>• Prisms are named for their bases. The name of a prism indicates the number of vertices, edges, and faces the prism has.<br>• Slicing prisms vertically, horizontally, or on a slant can expose different shapes of cross-sections, depending on which of the original edges are intersected.<br>• Comparing, reasoning about, and extending what you know about area and volume leads to an understanding of the formulas for finding the surface area and volume of prisms, cones, and pyramids.<br>• Proportional changes to dimensions of the sides of a prism lead to predictable changes in the surface area and the volume. | **Areas and Circumferences of Circles:** Understand the areas and circumferences of circles and how they are related.<br>• Approximations of the ratio of the circumference of a circle to the circle's diameter lead to exact formulas for the area and circumference of a circle. | **Volumes of Spheres and Cones:** Understand the relationships between the volumes of cylinders and the volumes of cones and spheres.<br>• Comparing, reasoning about, and extending what you know about area of circles and volume of cylinders leads to an understanding of the formulas for finding the volume of cones and spheres. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.NS.A.3:** Solve real-world and mathematical problems involving the four operations with rational numbers.<br>**7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.<br>**7.G.A.3:** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.<br>**7.G.B.4:** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.<br>**7.G.B.6:** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.<br>**Also 7.RP.A.2, 7.EE.A.1, 7.EE.A.2** | |

| | FILLING AND WRAPPING Three Dimensional Measurement | |
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| | Content Connections to Other Units | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **Surface Areas and Volumes of Polygonal Prisms and Cylinders** Understand surface areas and volumes of prisms and cylinders and how they are related. | • Interpreting area as the number of squares that cover a two-dimensional figure (*Covering and Surrounding*)<br>• Interpreting perimeter as the number of linear units that surround a two-dimensional figure; interpreting area as the number of squares that cover a two-dimensional figure (*Covering and Surrounding*)<br>• Comparing areas and perimeters of different two-dimensional figures (*Covering and Surrounding*)<br>• Studying the relationship between perimeter and area in rectangles (*Covering and Surrounding*) | • Finding volumes of cylinders , cones, and spheres (*Say It With Symbols)*<br>• Comparing linear, quadratic, and cubic relationships by analyzing the measurements of a cube (*Frogs, Fleas, and Painted Cubes*)<br>• Developing strategies for finding the distance between two points on a coordinate grid (*Looking for Pythagoras*)<br>• Finding the Pythagorean Theorem and using it to solve problems *(Looking for Pythagoras)*<br>• Algebraically analyzing the relationship between perimeter and area in rectangles *(Frogs, Fleas, and Painted Cubes*) |
| **Areas and Circumferences of Circles:** Understand the areas and circumferences of circles and how they are related. | • Developing strategies and algorithms for finding the perimeter and area of rectangles, triangles, parallelograms, and composite figures (*Covering and Surrounding; Stretching and Shrinking*) | • Finding the equation of a circle (*Looking for Pythagoras*)<br>• Using variables to represent a variety of relationships algebraically (*Thinking With Mathematical Models; Looking for Pythagoras; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols*) |
| **Volumes of Spheres and Cones:** Understand the relationships between the volumes of cylinders and the volumes of cones and spheres. | • Enlarging, shrinking, and distorting two-dimensional figures (*Stretching and Shrinking*)<br>• Scaling quantities up and down using ratios and proportions (*Comparing and Scaling*) | • Describing the relationships among volumes of cylinders, cones, and spheres with algebraic equations (*Say It With Symbols*) |

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| | **SAMPLES AND POPULATIONS** Making Comparisons and Predictions | | | |
| **Instructional Time and Investigations** | 18 days | • Inv. 1: Making Sense of Samples (4 Problems)<br>• Inv. 2: Choosing a Sample From a Population (4 Problems)<br>• Inv. 3: Using Samples to Draw Conclusions (4 Problems) | | |
| **Goals** | **The Process of Statistical Investigation:** Deepen the understanding of the process of investigation and apply this to samples.<br>• A survey allows you to gather data using a sample of a population and use that data to represent the population. Tables and graphs, as well as measures of center and variability enable you to compare data from different samples and draw conclusions about the samples and the populations. | **Analysis of Samples:** Understand that data values in a sample vary and that summary statistics of samples taken from the same population also vary<br>• You can compare two samples with approximately the same measure of variability by using that measure to determine the distance between the centers of the samples. | **Design and Use of Simulations:** Understand that simulations can model real world situations.<br>• Random samples are without bias, and therefore are useful for predicting population characteristics. | **Predictions and Conclusions about Populations:** Understand that summary statistics of a representative sample can be used to gain information about a population.<br>• Probability models allow you to select a random sample from a population. Random samples, even of the same size, vary from each other and from the underlying population. Random samples allow you to make inferences about a population. |
| **Common Core Standards** | **Common Core Standards for Mathematical Practice**<br>**MP.1:** Make sense of problems and persevere in solving them.<br>**MP.2:** Reason abstractly and quantitatively.<br>**MP.3:** Construct viable arguments and critique the reasoning of others.<br>**MP.4:** Model with mathematics.<br>**MP.5:** Use appropriate tools strategically.<br>**MP.6:** Attend to precision.<br>**MP.7:** Look for and make use of structure.<br>**MP.8:** Look for and express regularity in repeated reasoning. | **Common Core Content Standards**<br>**7.SP.A.1:** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.<br>**7.SP.A.2:** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.<br>**7.SP.B.3:** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.<br>**7.SP.B.4:** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.<br>**Also 7.RP.A.2, 7.RP.A.3, 7.NS.A.1, 7.NS.A.1b, 7.SP.C.5, 7.SP.C.7, 7.SP.C.7a** | | |

| | **SAMPLES AND POPULATIONS** Making Comparisons and Predictions | |
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| | **Content Connections to Other Units** | |
| **Goals of the Unit** | **Prior Work** | **Future Work** |
| **The Process of Statistical Investigation:** Deepen the understanding of the process of investigation and apply this to samples. | • Collecting and organizing data in different contexts (*Data About Us; What Do You Expect?*)<br>• Representing data using tables, line plots, dot plots, value or frequency bar graphs, histograms, and box-and-whisker plots (*Data About Us*) | • Continuing to frame exploration of statistical concepts within the process of statistical investigation (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes*)<br>• Interpreting categorical and quantitative data, making inferences and justifying conclusions (*Thinking With Mathematical Models; High School*) |
| **Analysis of Samples:** Understand that data values in a sample vary and that summary statistics of samples taken from the same population also vary. | • For numerical data: Finding measures of center and variability (range, IQR, MAD) and using these measures to make inferences and predictions (*Data About Us*)<br>• Examining the behavior of the mean and median and shapes of distributions (*Data About Us*)<br>• For categorical data: Analyzing frequencies as counts or percents (*Data About Us*)<br>• Understanding units of measure and counts (*Comparing Bits and Pieces; Covering and Surrounding; Variables and Patterns; Data About Us; Shapes and Designs; Accentuate the Negative; Comparing and Scaling; Moving Straight Ahead*) | • Working with graphs, particularly extending to include coordinate graphs representing bivariate data, correlation, and standard deviation (*Thinking With Mathematical Models; High School*)<br>• Interpreting categorical and quantitative data (*Thinking With Mathematical Models; High School*)<br>• Reasoning about quantities and using units to solve problems (*Thinking With Mathematical Models; Looking for Pythagoras; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes; Say It With Symbols; It's In the System*) |
| **Design and Use of Simulations:** Understand that variables can represent unknown values and equations to represent relationships. | • Gathering and organizing data collected from conducting experiments or trials of games (*Moving Straight Ahead; What Do You Expect?*) | • Deciding if a simulation model produces desired results (*High School*) |
| **Predictions and Conclusions about Populations:** Understand that summary statistics of a representative sample can be used to gain information about a population. | • Using counts or percents to report frequencies of data (*Data About Us; Comparing Bits and Pieces; Decimal Ops; Comparing and Scaling*)<br>• Comparing data sets using ratios, proportions, rates, or percents (*Comparing Bits and Pieces; Decimal Ops; Data About Us; Comparing and Scaling; What Do You Expect?*) | • Using linear, inverse, exponential, and quadratic relationships to model data (*Thinking With Mathematical Models; Growing, Growing, Growing; Frogs, Fleas, and Painted Cubes*)<br>• Making inferences and justifying conclusions (*Thinking With Mathematical Models; High School*) |