Accentuate the Negative: Homework Examples from ACE

Investigation 1: #6, 7, 12, 13, 14, 15, 16, 17, 30, 32-35, 52.
Investigation 2: #6, 10, 15, 23.
Investigation3: #7, 26
Investigation 4: #5, 29, 33.

ACE Question	Possible Answer
ACE Investigation 1	
6 - 7. Find each Math Fever's team's score. Write number sentences for each team. Assume that each team starts with 0 points. 6. The Protons answered a 250 point question correctly, a 100 point question correctly, a 200 point question correctly, a 150 point question incorrectly, and a 200 point question incorrectly. 7. The Neutrons answered a 200 point question incorrectly, a 50 point question correctly, a 250 point question correctly, a 150 point question incorrectly, and a 50 point question incorrectly.	6. 250 + 100 + 200 + (-150) + (-200) = 200 7. (-200) + 50 + 250 + (-150) + (-50) = -200 *This game context introduced students to positives and negatives and combining these quantities.*
12 – 17. Copy each pair of numbers in Exercises 12 – 17. Insert "<", " >" or "=" to make true statements. 12. 3 ? 0 13. $^{-}23.4$? 23.4 14. 46 ? $^{-}79$ 15. $^{-}75$? $^{-}90$ 16. $^{-}300$? 100 17. $^{-}1000$? $^{-}999$	12 . 3 is greater than 0, or 3 > 0. 13 . -23.4 is less than 23.4, or -23.4< 23.4 14. 46 > -79 15. -75 > -90 16. -300 < 100 17. -1000 < -999 *Thinking of the number line will help students decide, based on placement on the line, which numbers are lower/less/further left than others.*
30. The greatest one-day temperature change in world records occurred at Browning, Montana, from January 23–24 in 1916. The temperature fell from 44°F to $^{-}56$°F in less than 24 hours.	30. a. Students will probably think of this on a number line model. It takes 44 units to drop from 44 to 0, and then a further 56 units to drop from 0 to -56. This is a total change

a. By how many degrees did the temperature change in that day? **b.** How could you express the calculation of temperature change and the resulting temperature with a number sentence?	(drop) of 100 degrees. If they think of this as a direction as well as a change, they are thinking of the difference from 44 to -56, that is, -56 – 44 = -100, *down* 100 degrees. . b. -56 – 44 = -100.
32 - 35. Find the missing part for each of the situations below: **32.** Start with (B) (B) (B) Add 5 (R) **End with ?** **33 .Start with** (R) (B) (B) Subtract 3 (R) **End with ?** **34.** Start with (R) (R) (R) (R) (R) **?** End with (R) (R) **35.** Start with **?** Subtract 3 (B) End with (R) (R) (R) (R)	32 - 35. In the Chip Board model "B" stands for a black chip with value positive 1 unit, and "R" stands for a red chip with value negative 1 unit. 1B + 1R = 0, 3B + 3R = 0 etc. 32. Start with +3 and add -5. We can think of each pair of "B + R" as 1 + (-1) = 0. Since there are 2 more "R" we end with -2. 33. Start with -1 + 2 and subtract (-3). Since there are not enough "R"'s to subtract 3R we would have to alter the original representation from -1 + 2 to, for example, -3 + 4. Notice that -1 + 2 and -3 + 4 have the same resulting value, so this change does not actually change the value of the result, but it does allow us to take away (-3) or 3R. So the problem becomes: -3 + 4 subtract (-3). The end result is 4. *Note: Alternatively, we could have started with – 1 + 2 and added -3 + 3 to the board, that is 3R + 3B. This focuses on the "3R" which must be subtracted. It also means that, after the "3R" has been subtracted the net result is the addition of 3B to the board. This is the explanation of why subtracting -3 is the same as adding +3.* 34. Start with -5 and do "some operation" so that we end with -2. This could be -5 add 3, or "add three Blacks." Or students might think of this as -5 – (-3), "subtract three Reds." *Note: Here again we see that adding +3 gives the same result as subtracting -3.* 35. Start with some quantity, subtract 3 to end with -4. We must have started with -4 + 3, or 4 Reds and 3 Blacks, so that subtracting 3 Blacks left the 4 Reds.

	Start with -1 subtract 3, or (-4 + 3) subtract 3 = -4.
52 . Find values for A and for B that make the number sentence true. $^{+}A + {}^{-}B = {}^{-}1$	52. There are many possible solutions. We need 2 numbers whose difference is -1. That is, we need A - B = -1. Students will likely think of this in terms of Red and Black chips on a chip board. • If A = 5 and B = 6 then 5 + -6 = -1. That is, 5 Black chips add 6 Reds. • If A = 12 and B = 13 then 12 + -13 = -1. Notice that A does not have to be a positive. If A = -5 and B = -4 then -5 + 4 = -1, or 5 Reds added to 4 Blacks.
ACE Investigation 2	
6. Use your algorithms to find each difference without using a calculator. Show your work. a. $^{+}12 - {}^{+}4$ b. $^{+}4 - {}^{+}12$ c. $^{-}12 - {}^{+}4$ d. $^{-}7 - {}^{+}8$ e. $^{+}45 - {}^{-}40$	6. *Note: An "algorithm" is an efficient and logical procedure. For some students the "algorithm" will involve using a manipulative. For some students the "algorithm" has become a rule that they have observed always works: to subtract an integer we can add the opposite. See notes above for Investigation 1.* a. Students will either think of this as a chip board model ("12 Blacks take away 4 Blacks") or as a number line model ("What is the difference from 4 to 12?" Or "Start at 12 on the line and come down 4 units.") Or they may rewrite this as an addition: +12 – (+4) = +12 + (-4) = 8. b. Students will either think of this as a chip board model ("4 Blacks take away 12 Blacks. We need to represent this as 4 Blacks + (8 Blacks + 8 Reds) take away 12 Blacks") or as a number line model (What is the difference from 12 on the line to 4 on the line? Or "Start at 4 on the line and go down 12 units.") Or they may rewrite this as an addition: +4 – (+12) = +4 +(- 12) = -8. c. -12 – (+4) = -12 + (-4) = -16.

	d. -7 – (+8) = -7 + (-8) = -15. e. +45 – (-40) = +45 + (+40) = 85.
10. Without actually doing any calculations, decide which will give the greater result. Explain your reasoning. **a.** $^{+}5280 + {}^{-}768$ or $^{+}5280 - {}^{-}768$	10. a. Both expressions start with +5280 and one adds a negative and the other subtracts a negative. The first expression results will be less than +5280. If we think in terms of the chip board model then the second computation, “subtracting a negative,” would require a re-representation of the initial +5280 by adding the 768 “positives” and “768” negatives, before taking away the negatives. This ends with a larger result than +5280. So the second expression is greater than the first.
15. Compute each of the following: **a.** $3 + {}^{-}3 + {}^{-}7$ **b.** $3 - {}^{+}3 - {}^{+}7$ **c.** ${}^{-}10 + {}^{-}7 + {}^{-}28$ **d.** ${}^{-}10 - {}^{+}7 - {}^{+}28$ **e.** $7 - {}^{+}8 + {}^{-}5$ **f.** $7 + {}^{-}8 - {}^{+}5$ **g.** ${}^{-}97 + {}^{-}35 - {}^{+}10$ **h.** ${}^{-}97 - {}^{+}35 + {}^{-}10$ **i.** What can you conclude about the relationship between subtracting a positive number and adding a negative number with the same absolute value?	15. a. 3 + (-3) + (-7) = 0 + (-7) = -7. b. 3 – (+3) – (+7) = 0 –(+7) = -7. c. -10 + (-7) + (-28) = (-17) + (-28) = -45 d. -10 – (+7) –(+28) = -10 – (35) = -45 g. h. i. It seems that “add (-3)” gives the same result as “subtract (+3)” or in general “add -A” gives the same result as “subtract +A.”. *Note: this rule generalizes to be “Adding any integer gives the same result as subtracting its opposite, or subtracting any integer gives the same result as adding its opposite.”*
23. Write a related fact for each mathematical sentences to find *n*. What is the value of *n*? **a.** $n - 7 = 10$ **b.** $-\frac{1}{2} + n = -\frac{5}{8}$ **c.** $\frac{2}{3} - n = -\frac{7}{9}$	23. a. n = 10 + 7. So n = 17. b. n = $-\frac{5}{8} - (-\frac{1}{2})$. So n = -1/8. c. n = $\frac{2}{3} - (-\frac{7}{9})$. So n = 13/9. *Note: In elementary school students learned ”fact families” for any addition or subtraction. The idea is that “part A + part B= whole” or “whole – part*

	A = part B" or "whole – part B = part A" are all ways of saying the same relationship.
ACE Investigation 3	
7. You have located fractions such as $-\frac{5}{7}$ on a number line. You have also used fractions to show divisions such as $\frac{^-5}{7} = {^-5} \div 7$, and $\frac{5}{^-7} = 5 \div {^-7}$. Which of the following statements are true? Explain your thinking. a. $\frac{^-1}{2} = \frac{1}{^-2}$ b. $-\frac{1}{2} = \frac{^-1}{^-2}$	7. a. $\frac{-1}{2}$ says "take -1 and divide by 2, or divide into 2 parts." The result is (– half). $\frac{1}{-2}$ says "take 1 and divide by -2." This is hard to think of directly, but we could use fact families to rewrite "1 ÷ (-2) = what?" as "-2 times (what?) = 1." The missing number is (-half) again. This statement is true. b. $-\frac{1}{2}$ means (-half), as above. $\frac{-1}{-2}$ means "-1 ÷ -2." Thinking of how division relates to multiplication we have "-1 ÷ -2 = What?" which can be rewritten as "-1 = -2 times (what)?" The missing number is (+half). This statement is false. *Note: This reasoning leads to the general conclusion that* $\frac{-a}{b}$ *= a/(-b)* $\frac{a}{-b}$ *= -(*$\frac{a}{b}$*) which is NOT equal to* $\frac{-a}{-b}$*.*
26. Write a number sentence to represent each situation. a. The Extraterrestrials had a score of ⁻300, and then they answered four 50 point questions incorrectly. What was their score after missing the four questions? b. The Super Computers answer three 100 point questions incorrectly. They now have 200 points. What was their score before answering the three questions? c. The Bigtown Bears football team are at their own 25 yard line. In the next three	26. a. -300 +4(-50) = -300 + (-200) = -500. b. X + 3(-100) = 200. X +(-300) = 200, or X = 200 – (-300) = 500. c. 25 + 3(-4) = 25 + (-12) = 33. d. 5750(-0.25) = -$1437.50

plays, they lost an average of 4 yards per play. Where did the Bears end up after the three plays? d. When a new convenience store wanted to attract customers, they advertised gasoline at a price $0.25 below their cost. If they sold 5750 gallons on the one-day special, how much did they lose for that day?	
ACE Investigation 4	
5. Rewrite each of these expressions in an equivalent form to show a simpler way to do the arithmetic. Explain how you knew the two results would be equal without actually doing any calculations. a. $(^{-}150 + 270) + 30$ b. $(43 \times 120) + (43 \times {}^{-}20)$ c. $23 + {}^{-}75 + 14 + {}^{-}23 - {}^{-}75$ d. $(0.8 \times {}^{-}23) + (0.8 \times {}^{-}7)$	5. a. Since all the operations are additions we can alter the grouping (Associative Property of addition) and order (Commutative Property). Thus, (-150 + 270) + 30 = -150 + (270 + 30). This has the advantage of putting the positive quantities together and also of creating a “friendly” pair of addends. -150 + (300) = 150. b. There are two expressions added here, and each has a common factor of 43. Thus, we can use the Distributive Property to rewrite this as $43(120 + {}^{-}20) = 43(100) = 4300$. c. This expression has additions and subtractions and can be rewritten in terms of additions only. Thus, $23 + {}^{-}75 + 14 + {}^{-}23 - {}^{-}75$ $= 23 + {}^{-}75 + 14 + {}^{-}23 + {}^{+}75$, and then the order can be changed since addition is commutative, to $23 + {}^{-}23 + {}^{-}75 + {}^{+}75 + 14$. Then, taking advantage of opposites, we have a final result = 14. d. The Distributive Property can be used to factor 0.8 out of both expressions. $(0.8 \times {}^{-}23) + (0.8 \times {}^{-}7)$

	$= 0.8(^-23 + ^-7)$ $= 0.8(-30)$ $=-24$
29. Write a related fact. Use it to find the value of n that makes the sentence true. a. $n - {}^-5 = 35$. **b.** $4 + n = {}^-43$.	29. a. $n - {}^-5 = 35$ can be thought of as "Unknown whole – Part A = Part B," and can be rewritten as n = 35 + (-5) = 30. This makes it easier to find n, since n is now the subject of the sentence. *(This strategy takes advantage of the fact family, "part A + part B = whole" can be rewritten as "whole – Part A = Part B" or "whole – Part B =Part A.")* b. $4 + n = {}^-43$ can be thought of as "part + part = whole." Rewriting, we have n = -43 – 4 = -47.
33. Insert parentheses where needed in each expression to show how to get the following results. **a.** $1 + {}^-3 \times {}^-4 = 8$ **b.** $1 + {}^-3 \times {}^-4 = 13$ **c.** ${}^-6 \div {}^-2 + {}^-4 = 1$ **d.** ${}^-6 \div {}^-2 + {}^-4 = {}^-1$ **d.** ${}^-4 \times 2 - 10 = {}^-18$ **e.** ${}^-4 \times 2 - 10 = 32$	33. This problem requires students to apply the parentheses in such a way that the correct order of operations will give the required result. This order is: operations in Parentheses first, then exponents, then multiplication or division from the left, then addition or subtraction from the left. a. $(1 + {}^-3) \times {}^-4 = (-2)\times(-4) = 8$. b. $1 + ({}^-3 \times {}^-4) = 1 + (12) = 13$. c. $({}^-6 \div {}^-2) + {}^-4 = 3 + {}^-4 = -1$. d. $({}^-4 \times 2) - 10 = (-8) - 10 = -18$. e. $-4(2 - 10) = -4(-8) = 32$.