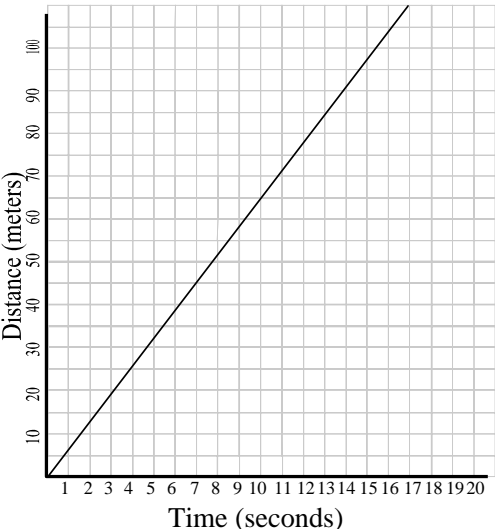# Moving Straight Ahead: Homework Examples from ACE
ACE Investigation 1: #4
ACE Investigation 2: #4
ACE Investigation 3: # 12
## ACE Investigation 4: #15.

ACE Question	Possible Answer
ACE 1	
4. Mike makes the following table of the distances he travels during the first day of the trip. Table below a. Suppose Mike continues riding at this rate. Write an equation for the distance Mike traveled after *t* hours. b. Sketch a graph of the equation. How did you choose the range of values for the time axis? For the distance axis? c. How can you find the distances Mike traveled in 7 hours and in $9\frac{1}{2}$ hours, using the table? The graph? The equation? d. How can you find the numbers of hours it took Mike to travel 100 miles and 237 miles, using the table? The graph? The equation? e. For parts c and d, what are the advantages and disadvantages of using	*Notice the connection with the work students did in Variables and Patterns.* a. Students should see the rate of change in distance is constantly 6.5 miles for each one hour change. $d = 6.5t$ b. Students look at the range of numbers needed for time and for distance in the table, and then decide on a reasonable scale. Since the distance values reach about 40 it is convenient to mark the vertical axis in increments of 5 or 10. The time axis only has to cover times to 6 hours, therefore a unit scale will be appropriate. Students may want to mark the scale on the time axis in half hours.

Table for ACE 1 #4:

Time (hours)	Distance (miles)
0	0
1	6.5
2	13
3	19.5
4	26
5	32.5
6	39

Mike's Cycling Data

Graph with vertical axis "Distance (meters)" marked 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 and horizontal axis "Time (seconds)" marked 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20.

each form of representation—a table, a graph, and an equation—to find the answers?

f. Compare the rate at which Mike rides with the rates at which Jose, Mario, and Melanie ride. Who rides the fastest? How can you determine this from the tables? From the graphs? From the equations? (You need to refer back to question 3 for this part.)

The graph above has a larger range of values for both axes because parts c and d ask for times greater than 6 hours and distances greater than 40 miles.

c. The table can be extended to show 7 and $9\frac{1}{2}$ *hours.* On the graph, the distances at these points may be approximated. In the equation, the values of 7 and $9\frac{1}{2}$ can be substituted for t, which gives the answers of 45.5 and 61.75 mi.

d. The table can be extended by adding increments of 6.5 miles, to show values that are close to 100 mi. and 237 mi. On the graph, the times at these points may be approximated after the graph has been extended. In the equation, the values of 100 mi. and 237 mi can be substituted for d, which gives the approximate answers 15.4 hours and 36.5 hours.

e. Possible answer: If the given value for one variable is already showing, the table or graph would be easy to use to find the corresponding value for the other variable. If the values are far from those shown on the table, graph, or if you need an exact quantity, it is easier to use equations to get the answer.

f. See question 3.

ACE 2	

4. A band decides to sell protein bars to raise money for an upcoming trip. The cost (the amount the band pays for the protein bars) and income the band receives for the protein bars are represented on the graph below.
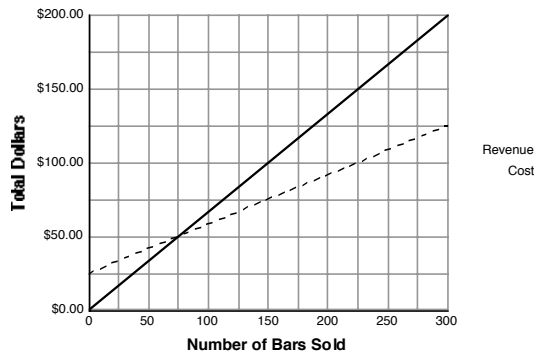
a. How many protein bars must be sold for the band's costs to equal the band's income?

b. What is the income from selling 50 protein bars? 125 bars?

c. If the income is $200, how many protein bars were sold? How much of this income was profit?

Note: This problem is still about constant rate, though this time it is "per protein bar" not "per hour."

4. a. about 75 bars (Note: Students are reading answers from the graph, so some inaccuracy is expected.) One line is a representation of the relationship between cost and number of protein bars. The other represents the relationship between income and number of protein bars. So the intersection point gives the number of protein bars for an equal cost and income.

b. $33.50 because 0.67(50) = 33.5 (Note: the 0.67 was derived from points (0, 0) and points (300, 200) showing that 300 bars cost $200, so each bar would sell for 67¢. The slope of the "income" line is 0.67.) Students could also answer this from the graph. $83.75 because 0.67(125) = 83.75.

c. Using the graph we see that, for an income of $200, the band would have to sell about 300 protein bars. Again using the graph, we see that the cost for 300 bars would be $125, leaving a profit of about $75. (Note: In part b we saw that the income per bar is $0.67. Using the "cost" line we see that it costs $125 for 300 bars, so each bar costs about $0.42. so the profit per bar is $0.25. For 300 bars the profit is 300(0.25) = $75.)

ACE 3	

12. Use properties of equality and numbers to solve each equation for *x*. Check your answers.
 	
a. $7 + 3x = 5x + 13$
 	
b. $3x - 7 = 5x + 13$
 	
c. $7 - 3x = 5x + 13$
 	
d. $3x + 7 = 5x - 13$
	Notice that students already have table and graph ways to solve these equations. For example, to solve $7 + 3x = 5x + 13$ students could graph $y = 7 + 3x$ and $y = 5x + 13$. The intersection point will give the value of x that makes $7 + 3x = 5x + 13$. The goal in MSA Investigation 3 is not to suggest that graphical methods are always inferior, but rather that symbolic methods, using properties of equality, are accurate and efficient.
	
	As they proceed to other units, other equations, such as Exponential (*Growing, Growing*) and Quadratic (*Frogs and Fleas*) will be introduced. For some complex equations *graphical* methods are often more efficient, though perhaps less accurate, than symbol-manipulation methods. In fact, for some complex equations there are *no* symbol-manipulation methods, and graphical solutions are the only solutions possible.
	
	12.
	
	  a. $7 + 3x = 5x + 13$. Subtracting 7 from each side we get
	
	   $3x = 5x + 6$. Subtracting 5x from each side we get
	
	   $-2x = 6$; so
	
	   $x = -3$.
	
	  (Check: Is $7 + 3(-3) = 5(-3) + 13$? Yes.)
	
	  b. $3x - 7 = 5x + 13$. Add 7 to each side.
	
	   $3x = 5x + 20$. Subtract 5x from each side.

-2x = 20. Divide by -2.

 X = -10.

(Check: Is 3(-10) – 7 = 5(-10) + 13? Yes.)

c. 7 – 3x = 5x + 13. Add 3x to both sides.

   7 = 8x + 13. Subtract 13 from both
    sides.

  -6 = 8x. Divide by 8 on both sides.

  -0.75 = x.

(Check: Is 7 – 3(-0.75) = 5(-0.75) + 13?
    Yes.)

d. 3x + 7 = 5x – 13. Subtract 3x from both
    sides.

   7 = 2x – 13. Add 13 to both sides.

  20 = 2x. Divide by 2.

  10 = x.

(Check: Is 3(10) + 7 = 5(10) – 13? Yes.

ACE 4	

15.
 	
In parts a and b, the equations represent linear relationships. Use the given information to find the value of b.
 	
a. The point (1, 5) lies on the line representing $y = b - 3.5x$.
 	
b. The point (0, -2) lies on the line representing $y = 5x - b$.
 	
c. What are the y-intercepts in the linear relationships in a and b? What are the patterns of change for the linear relationships in a and b?
 	
d. Find the x-intercepts for the linear relationships in a and b. (The x-intercept is the point where the graph intersects the x-axis.)
	15.
	  a. If a point lies on a line then it must make the equation of the line true. Thus,
	       $5 = b - 3.5(1)$, so
	       $5 = b - 3.5$, so
	       $b = 8.5$.
	       Thus, the only way that (1, 5) can lie on the line $y = b - 3.5x$ is if the value of b is 8.5.
	  b. $-2 = 5(0) - b$, so $b = 2$.
	  c. The linear relationship in part a is $y = 8.5 - 3.5x$; this shows y decreasing at a rate of -3.5 for each unit change in x, with a y intercept of (0, 8.5).
	       The linear relationship in part b is $y = 5x - 2$; this shows y increasing at a rate of 5 for each unit change in x, with a y intercept of (0, -2).
	  d. Students will probably use what they know about the meaning of "m" and "b" in the linear equation "y = mx + b" to answer part c. But there is no shortcut for finding the x-intercept by inspecting the equation. They must understand that the x-intercept is the point where the line crosses the x-axis, so has coordinates (x, 0).
	       Thus, substituting y = 0 into the equation in part a,
	       y= 8.5 – 3.5x, we have
	       $0 = 8.5 - 3.5x$, so
	       $3.5 x = 8.5$, so
	       $x = 8.5/3.5 = 17/7$. The x-intercept is about (2.4,0).
	       For the linear relationship in part b,
	       y = 5x – 2, we have
	       $0 = 5x - 2$, so
	       $x = 0.4$.
	       Thus the x-intercept is (0.4, 0).