Vocabulary: ***What do You Expect?***

<table>
<tr><th>Concept</th><th>Example</th></tr>
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Meaning of Probability:
The term probability is applied to situations that have uncertain outcomes on individual trials but a predictable pattern of outcomes over many trials. Students find experimental and theoretical probabilities in this unit.

(See How Likely Is It? for more detail)

In general, theoretical probability can be expressed as follows:

P(outcome we are interested in) =

$$\frac{\#\text{ of equally likely favorable outcomes}}{\text{total }\#\text{ of equally likely outcomes}}$$
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1. What is the probability of a boy being the youngest child in a family of 4 children?
Assuming that the births of a boy or girl are equally likely events we can assign a probability of 50% to each. We can now proceed by listing all possible families of 4 in an organized way, starting with 4 girls, then 3 girls etc: GGGG, GGGB, GGBG, GBGG, BGGG, GGBB, GBGB. BGGB, GBBG, BGBG, BBGG, GBBB, BGBB, BBGB, BBBG, BBBB. (The order indicates the order of the births. So BBBG and BGBB both represent families with 3 boys, but the order of the births was different.) There are 16 possibilities for 4 children. In 8 of these cases a boy was the youngest child, so theoretically P(boy is youngest) = $\frac{8}{16}$ or 0.5.

Note: You can answer many other questions about families of 4 children from this list.

See How Likely Is It? For other examples.
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Simulations:
Creating a Simulation is a strategy for finding the probability of an event by experimentation, without actually repeating the experiment with the variables in the real situation. In order to create a simulation there has to be an underlying probability that is known, which means that the experiment starts by assuming a theoretical probability. Then some model such as a die or a spinner or a coin or a random number table is coded so that an outcome on this model represents some outcome in the real world, with a matching probability. Thus, instead of repeating a real event many times, which might be impractical, we can repeatedly use the model and record outcomes. A simulation blends a theoretical or given probability with an experimental approach.
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2. What is the probability of a family having to wait through the births of 4 or more children of one gender before a child of the other gender is born?
We could try to set up a list as above, but another way to solve this problem would be to use a coin to represent the birth of a child, and code the coin so that H = girl and T = boy. Then we toss the coin and record each family as it is created, noting how many times a family of 4 or more is created before both genders are present. (We can stop a trial when both genders are present.)
<table>
<tr><th>Trial</th><th>Family</th></tr>
<tr><td>1</td><td>GB</td></tr>
<tr><td>2</td><td>GGB</td></tr>
<tr><td>3</td><td>BG</td></tr>
<tr><td>4</td><td>BBBBBG</td></tr>
<tr><td>5</td><td>GGGB</td></tr>
<tr><td>6</td><td>GB</td></tr>
<tr><td>7</td><td>BBG</td></tr>
<tr><td>8</td><td>BBBG</td></tr>
<tr><td>9</td><td>BBG</td></tr>
<tr><td>10</td><td>BG</td></tr>
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<td>Only trial 4 has at least 4 of one gender before the other gender appears. This is the only trial in which the “wait” was at least 4. So, the experimental probability of having to wait through at least 4 of one gender before the other gender appears is 1/10 or 10%. If we did many more trials we would approach the theoretical probability.</td>
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<td>Models for working with Probabilities

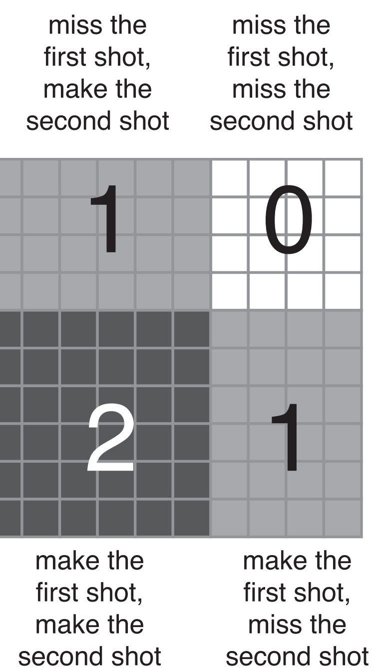
Area:
Area models are useful for finding probabilities in situations involving successive events, such as a basketball player who is allowed to attempt a second free-throw only if the first succeeds. An area model is particularly powerful in situations in which the possible outcomes are not equally likely. A rectangular area is set up; one dimension represents the first event (first toss of a coin, or first free throw, or genetic information from one parent, for example); the other dimension represents the other event. These dimensions are partitioned to represent the underlying probabilities.

List:
An organized list is a useful way to organize all the possibilities when finding the theoretical probability of an event. See How Likely is It? for examples.

Counting Tree: is the equivalent of a list, but the organization of successive events on the tree makes it easy to ensure that all are counted. Counting trees are most useful when all events are equally likely. See How Likely Is It? for examples.

Note: To use a counting-tree approach in a</td>
<td>3. What is the probability that a 60% free-throw shooter will score 0 points (or 1 or 2) in a 2 free throws situation? (In this two-shot situation, the player will attempt a second free-throw whether or not the first free-throw succeeds.)

The first shot has two possible outcomes—making or missing the shot. The probability of making the shot is 60% or 0.6 or $\frac{60}{100}$. The probability of missing the shot is 40% or 0.4 or $\frac{40}{100}$. The grid below is shaded to indicate this.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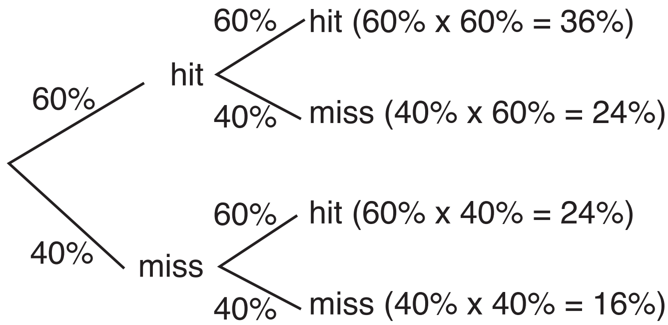
situation where outcomes are not equally likely, each branch of the tree must be weighted by the probability that it will be selected.

4. Below is a counting tree version of the same problem as example 3. This is not part of the student experience in this unit. Students generally use counting trees when all outcomes are equally likely.



Expected Value

The *expected value* is a long-term average of the payoff of each outcome weighted by its probability.

5. *What is the long-term average or expected value for a 60% free-throw shooter in two-shot free-throw situations, as in example 3 above?*

If the player goes to the line 100 times, then he expects:

A score of 0 to occur 16 times for a total of 0 points and

A score of 1 to occur 48 times for a total of 48 points and

A score of 2 to occur 36 times for a total of 72 points.

The total number of points expected in 100 situations is 0 + 48 + 72 = 120 points.

The average number of points expected in 100 trials is 120 points ÷ 100 trials or 1.2 points per trial. We could also arrive at this result by the computation

$$\left(\frac{16}{100}\right)(0) + \left(\frac{48}{100}\right)(1) + \left(\frac{36}{100}\right)(2)$$

	$= 0 + \frac{48}{100} + \frac{72}{100}$ $= 1.2$ which shows each payoff weighted by the probability that it will occur.
Independent and Dependent Events The idea of *independent* and *dependent* events is introduced informally in this unit. Two (or more) events are said to be independent if the outcome of one event has no impact on the probability of the other event. For example, tossing a coin and getting a head does not make it more or less likely that the next toss will also be a head. Thus, two tosses of a coin are independent. However, suppose a teacher puts all the names of her students in a hat to choose 1 boy and 1 girl for a project from her 16 students. On the first pick each student has a $\frac{1}{16}$ chance of being chosen. But if the first choice is a boy, then the probability of any particular boy being chosen next drops to 0 (because the teacher will not choose another boy) while the probability of any particular girl being chosen next becomes 1 out of 8 (8 girls left in the hat).	6. *Suppose you twice draw a marble from a bag containing two red marbles and two blue marbles. If you replace the selected marble after the first draw, what is the probability of a red on the first draw? On the second draw?* The two draws will be independent of each other, because what you draw the first time will not affect what you draw the second time. Therefore, the probability of a red marble is 2 out of 4 each time. 7. *If you do not replace the selected marble, as above, then what is the probability of a red on the first draw? On the second draw?* The second draw will be dependent on the first draw, because the probability of drawing each color the second time depends on the color chosen on the first draw. The probability of a red on the first draw is 2 out of 4. If you draw a red marble the first time and do not replace it, the probability of drawing a red marble the second time is 1 out of 3 rather than 2 out of 4. Yet if you had drawn a blue marble the first time, the probability of drawing red the second time would be greater: 2 out of 3. It is in this sense that the probability of drawing a red on the second draw is a dependent event. We can give two answers to the second question: The probability of a red on the second draw given that a red occurred on the first draw is $\frac{1}{3}$; The probability of a red on the second draw given that a red did NOT occur on the first draw

	is $\frac{2}{3}$.
The Law of Large Numbers (See *How Likely Is It?)*	
Binomial Situations: are situations where there are only two outcomes, and when 1 occurs the other cannot occur (so the probabilities add to 1) and the probability of each event does not change from trial to trial. Though there are only 2 possible outcomes the probability of each outcome does not have to be 50%.	Examples of binomial situations: 8. The free throw situation in examples 3, 4, 5 above can be considered a binomial situation if the two outcomes are Hit and Miss. 9. The situation of a child being born is a binomial situation. There are two possible outcomes: Male and Female. 10. Examples 6 and 7, about drawing marbles, have more than 2 outcomes, if we consider the different colors that are possible (RR, RB, BR, BB). However, if we define the outcomes as Win and Lose (in a marble drawing game) then we have a binomial situation (as long as the marbles are replaced so that the probabilities do not change). 11. A True-False quiz is a binomial situation, either you are right or you are wrong. So can a multiple choice quiz, if we define the outcomes as Right and Wrong.