Arc of Learning for *Connected Mathematics*

What is the Arc of Learning?

The Arc of Learning framework is a resource for curriculum design and use that makes explicit the intentions of the curriculum designers about how students engage in the learning of mathematics over time. The framework focuses on middle grades students' development of conceptual understanding from a carefully sequenced set of mathematics problems. This framework is important to mathematics teachers and researchers because it can inform

- what it means to understand an important mathematical idea,
- how student thinking and learning might unfold within and across mathematics sequences of problems,
- how teachers understand the development of long-term mathematical goals embedded in a coherent sequence of multiday problem-solving tasks, and
- the mathematical, pedagogical, and assessment decisions teachers make when planning or enacting lessons that respond to students' mathematical conceptions.

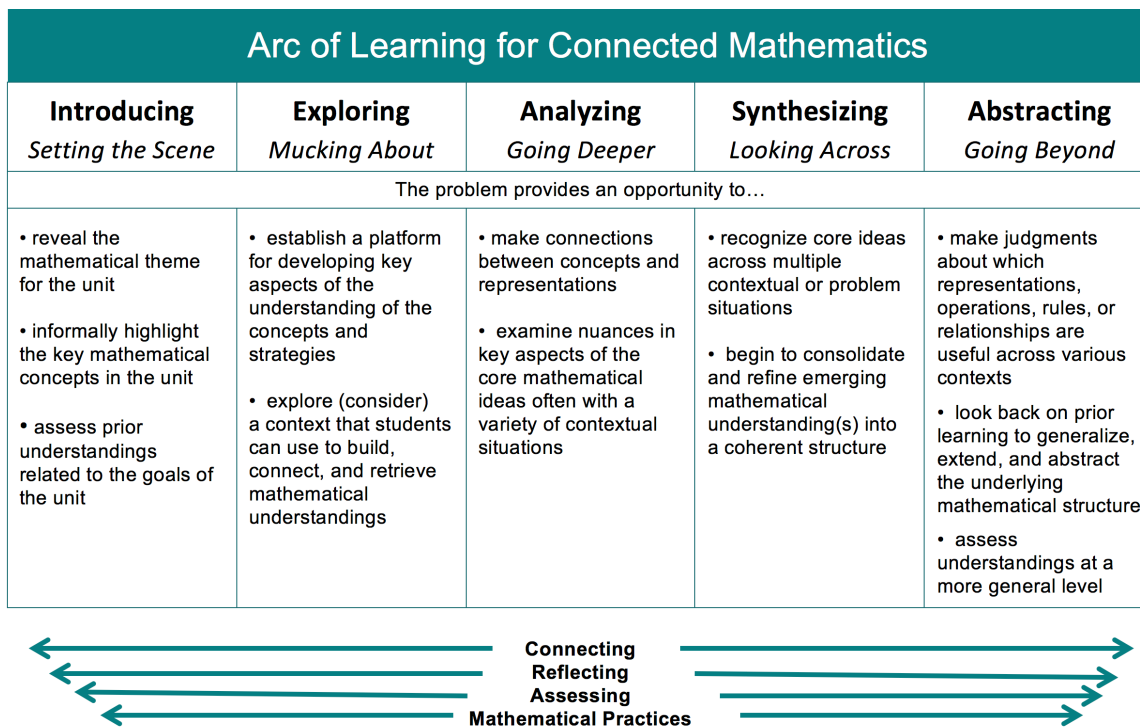**Why does the Arc of Learning matter?**

The Arc of Learning framework provides a tool for characterizing deeply grounded and connected learning that is different than the prevalent focus on student learning as passively watching and imitating isolated skills. The framework can support teachers to understand the various entry points in how students develop mathematics over time. In turn, the teachers can provide more students with more opportunities to powerful mathematics.

What are the five phases of the Arc of Learning framework?

1) **Introduction (Setting the Scene)** – students explore the mathematical theme and informally highlight the key mathematical concepts. The problems provide an opportunity to assess what students bring to the lesson in terms of the goals of the unit.

2) **Exploration (Mucking About)** – students consider and explore a context that students can use to build, connect, and retrieve mathematical understandings. Students establish a point of reference for developing key aspects of concepts and strategies.

3) **Analysis (Going Deeper)** – students engage with a variety of contextual situations and examine nuances in key aspects of the core mathematical ideas. Students make connections between concepts and representations.

4) **Synthesis (Looking Across)** – students consolidate and refine their emerging mathematical understanding(s) into a coherent structure. They recognize core ideas across multiple contextual or problem situations. Students begin to generalize their mathematical understandings.

5) **Abstraction (Going Beyond)** – students make judgments about which representations,

operations, rules, or relationships are useful across various contexts. Students look back on prior learning to generalize, extend, and abstract the underlying mathematical structure. Teachers can assess student thinking at a more sophisticated level.

What does the Arc of Learning framework look like?

Arc of Learning for Connected Mathematics				
Introducing *Setting the Scene*	**Exploring** *Mucking About*	**Analyzing** *Going Deeper*	**Synthesizing** *Looking Across*	**Abstracting** *Going Beyond*
The problem provides an opportunity to…				
• reveal the mathematical theme for the unit • informally highlight the key mathematical concepts in the unit • assess prior understandings related to the goals of the unit	• establish a platform for developing key aspects of the understanding of the concepts and strategies • explore (consider) a context that students can use to build, connect, and retrieve mathematical understandings	• make connections between concepts and representations • examine nuances in key aspects of the core mathematical ideas often with a variety of contextual situations	• recognize core ideas across multiple contextual or problem situations • begin to consolidate and refine emerging mathematical understanding(s) into a coherent structure	• make judgments about which representations, operations, rules, or relationships are useful across various contexts • look back on prior learning to generalize, extend, and abstract the underlying mathematical structure • assess understandings at a more general level

Connecting
Reflecting
Assessing
Mathematical Practices

Are there CMP unit specific Arcs of Learning?

Specific frameworks for the CMP curriculum units have been developed that position problems in context of the learning goal. For example, the figure below shows the development of understanding similarity for the *Stretching and Shrinking* unit, moving from initial ideas for the meaning of similarity from everyday life, to a generalization of understandings of similar shapes using strategies extending beyond contextual situations.

Stretching and Shrinking: Developing Proportional Reasoning in the Context of Similarity (Scale Drawings)					
■ **Proportional Reasoning** ■ **Similar Figures**	**Introduction** *Setting the Scene*	**Exploration** *Mucking About*	**Analysis** *Going Deeper*	**Synthesis** *Looking Across*	**Abstraction** *Going Beyond*
Investigation 1: Enlarging and Reducing Shapes					
1.1 Using Proportional Reasoning to Solve a Mystery	1.1		1.1		

1.2 Stretching a Figure: Comparing Similar Figures	1.2	1.2	1.2		
1.3 Scaling Up and Down	1.2	1.2	1.3		
Mathematical Reflections		MR	MR		
Investigation 2: Similar Figures					
2.1 Drawing Wumps: Making Similar Figures		2.1	2.1 2.1		
2.2 Hats Off to the Wumps: Changing a Figure's Size and Location			2.2 2.2 2.2		
2.3 Mouthing Off and Nosing Around: Scale Factors			2.3 2.3		
Mathematical Reflections			MR MR		
Investigation 3: Scaling Perimeter and Area					
3.1 Using Proportional Reasoning to Rep-tile			3.1	3.1	
3.2 Designing Under Constraints: Scale Factors and Similar Shapes			3.2	3.2 3.2	
3.3 Out of Reach: Finding Lengths With Similar Triangles				3.3 3.3	
Mathematical Reflections				MR MR	
Investigation 4: Similarity (Scale Drawings) and Ratios					
4.1 Equivalent Ratios Within Similar Figures			4.1	4.1	
4.2 Finding Missing Parts: Using Similarity to Find Measurements				4.2 4.2	
4.3 Using Shadows to Find Heights				4.3 4.3	
Mathematical Reflections				MR	

How can I access more information about the Arc of Learning framework?
Please visit the Connected Mathematics Project website at Michigan State University:
https://connectedmath.msu.edu/research/ongoing-research/arc-of-learning-research-project/

Edson, A.J., Phillips, E., Slanger-Grant, Y., & Stewart, J. (2019). The Arc of Learning framework: An ergonomic resource for design and enactment of problem-based curriculum, *International Journal of Educational Research*, *93*, 118-135.

Edson, A.J., Gilbertson, N.J., Gonulates, F., Grant, Y., Nimtz, J.L., Phillips, E., & Satyam, V.R. (2015). Studying the arc of learning in middle school mathematics curriculum materials. In T.G. Bartell, K.N. Bieda, R.T. Putnam, K. Bradfield, & H. Dominguez (Eds.) *Proceedings of the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 104-107). East Lansing, MI: Michigan State University.