# 6-1: Prime Time

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Factors and Multiples** Understand relationships among factors, multiples, divisors, and products

- Classify numbers as prime, composite, even, odd, or square
- Recognize that factors of a number occur in pairs
- Recognize situations that call for common factors and situations that call for common multiples
- Recognize situations that call for the greatest common factor and situations that call for the least common multiple
- Develop strategies for finding factors and multiples
- Develop strategies for finding the least common multiple and the greatest common factor
- Recognize and use the fact that every whole number can be written in exactly one way as a product of prime numbers
- Use exponential notation to write repeated factors
- Relate the prime factorization of two numbers to the least common multiple and greatest common factor of two numbers
- Solve problems involving factors and multiples

**Equivalent Expressions** Understand why two expressions are equivalent

- Relate the area of a rectangle to the Distributive Property
- Recognize that the Distributive Property relates the multiplicative and additive structures of whole numbers
- Use the properties of operations of numbers, including the Distributive Property, and the Order of Operations convention to write equivalent numerical expressions
- Solve problems involving the Order of Operations and Distributive Property

# 6-1 Prime Time: Focus Questions (FQ) and Mathematical Reflections

| Investigation 1<br>Building on Factors and Multiples | Investigation 2<br>Common Multiples and Common Factors | Investigation 3<br>Factorizations: Searching for Factor Strings | Investigation 4<br>Linking Multiplication and Addition: The Distributive Property |
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| **Problem 1.1**<br>**Playing the Factor Game: Finding Proper Factors**<br>FQ: How can you find all the factors (or divisors) of a number?<br><br>**Problem 1.2**<br>**Playing to Win: Prime and Composite Numbers**<br>FQ: What information about a number can you find by looking at its factors?<br><br>**Problem 1.3**<br>**The Product Game: Finding Multiples**<br>FQ: If you know one factor of a number, how can you find another factor of the number?<br><br>**Problem 1.4**<br>**Rectangles and Factor Pairs**<br>FQ: How do you know when you have found all of the factors of a number? | **Problem 2.1**<br>**Riding Ferris Wheels: Choosing Common Multiples or Common Factors**<br>FQ: How can you decide when finding common multiples is useful in solving a problem?<br><br>**Problem 2.2**<br>**Looking at Cicada Cycles: Choosing Common Multiples or Common Factors**<br>FQ: How can you find the least common multiple of two or more numbers?<br><br>**Problem 2.3**<br>**Bagging Snacks: Choosing Common Multiples or Common Factors**<br>FQ: How can you decide when finding common factors is useful in solving a problem? How can you find the greatest common factor of two numbers? | **Problem 3.1**<br>**The Product Puzzle: Finding Factor Strings**<br>FQ: How can you find the prime factorization of a number?<br><br>**Problem 3.2**<br>**Finding the Longest Factor String**<br>FQ: How many unique prime factorizations of a number are there?<br><br>**Problem 3.3**<br>**Using Prime Factorizations**<br>FQ: How can the prime factorization of a number be used to find the LCM and GCF of two or more numbers?<br><br>**Problem 3.4**<br>**Unraveling the Locker Problem: Putting It All Together**<br>FQ: What characteristics of numbers, such as factors and multiples did you use to answer the questions? What special numbers, such as prime numbers, composite numbers, and square numbers, did you use? | **Problem 4.1**<br>**Reasoning With Even and Odd Numbers**<br>FQ: How do you decide whether a number is even or odd?<br><br>**Problem 4.2**<br>**Using the Distributive Property**<br>FQ: How is the Distributive Property used to create equivalent expressions? How is finding the area of a rectangle related to the Distributive Property?<br><br>**Problem 4.3**<br>**Ordering Operations**<br>FQ: How do you decide the order when you work on number sentences with more than one operation?<br><br>**Problem 4.4**<br>**Choosing and Operation**<br>FQ: How do you decide what operations are needed in a given situation? |
| **Mathematical Reflections**<br>1a. Explain how factors and multiples of a number are related.<br><br>1b. Describe a situation where it is useful to know about factors and multiples.<br><br>1c. Describe strategies for finding factors or multiples of a number.<br><br>2. You can describe a number by both the number of its factors and the kind of its factors. Describe several kinds of numbers that you studied in this Investigation. Give examples. | **Mathematical Reflections**<br>1. How can you decide if finding common multiples or common factors is helpful in solving a problem? Explain.<br><br>2a. Describe how you can find the common factors and the greatest common factor of two numbers.<br>2b. What information does the greatest common factor of two numbers provide in a problem?<br><br>3a. Describe how you can find the common multiples and the least common multiple of two numbers.<br>3b. What information does the least common multiple of two numbers provide in a problem? | **Mathematical Reflections**<br>1a. Why it is helpful to write a number as a product of primes?<br><br>1b. Describe how you can find the prime factorization of a number.<br><br>2a. When it is useful to find the LCM or GCF of two or more numbers to solve a problem?<br><br>2b. Describe a method for finding the LCM of two numbers. Is there another method? Explain.<br><br>2c. Describe a method for finding the GCF of two numbers. Is there another method? Explain. | **Mathematical Reflections**<br>1a. Explain what the Distributive Property means for multiplication, addition, and subtraction. Use the area of a rectangle to illustrate your answer.<br><br>1b. Explain how you can use the Distributive Property to write a number as two equivalent expressions. Give two examples.<br><br>2a. What rules for ordering computations with numbers does the Order of Operations convention provide? Why is it important?<br><br>2b. How do you decide what operation, addition, subtraction, multiplication, or division, is needed to solve a problem? |

# 6-2: Comparing Bits and Pieces

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Fractions as numbers**. Understand fractions and decimals as numbers that can be located on the number line, compared, counted, partitioned and decomposed.

- Expand interpretations of a fraction to include, expressing fractions as a part-whole relationship, as a number, and as an indicated division.
- Reason about the roles of numerator and denominator in each of the interpretations of a fraction.
- Use multiple interpretations of proper fractions, improper fractions and mixed numbers.
- Use decimals to represent fractional quantities, with attention to place value.
- Recognize that the set of positive and negative fractions is called rational numbers and recognize rational numbers as points on the number line.
- Use the number line to reason about rational number relationships.
- Use benchmarks to estimate the size of fractions (and decimals), to compare and order fractions (and decimals).
- Recognize that fractions (both positive and negative) can represent both locations and distances on the number line.
- Recognize that a number and its opposite are equal distances from zero on the number line. The opposite of a is –a and the opposite of –a is a.
- Understand that the *absolute value* of a number is its distance from 0 on the number line and use it to describe real-world quantities.
- Introduce percents as a part-whole relationship where the whole is not necessarily out of 100, but is scaled or partitioned to be "out of 100" or “per 100.”
- Apply a variety of partitioning strategies to solve problems.

**Ratios as comparisons**. Understand ratios as comparisons of two numbers.

- Use ratios and their associated rates to compare quantities.
- Distinguish between difference (additive comparison) and ratio (multiplicative comparison).
- Distinguish between fractions as numbers and ratios as comparisons.
- Apply a variety of scaling strategies to solve problems involving ratios and unit rates.
- Understand that a unit rate is a ratio in which one of the quantities being compared has a value of 1; use rate language in the context of a ratio relationship.
- Scale percents to predict new outcomes.

**Equivalence**. Understand equivalence of fractions and of ratios, and use equivalence to solve problems.

- Understand that equivalent fractions represent the same amount, distance or location; develop strategies for finding equivalent fractions.
- Understand that comparing situations with different-sized wholes is difficult unless we use some common basis of comparison.
- Use partitioning and scaling strategies to generate equivalent fractions and ratios, and to solve problems.
- Develop meaningful strategies for representing fraction amounts larger than one or less than zero as both mixed numbers and improper fractions.
- Understand that equivalent ratios represent the same relationship between two quantities; develop strategies for finding and using equivalent ratios.
- Build and use rate tables of equivalent ratios to solve problems.

# 6-2 Comparing Bits and Pieces: Focus Questions (FQ) and Mathematical Reflections

| **Investigation 1**<br>Making Connections | **Investigation 2**<br>Connecting Ratios and Rates | **Investigation 3**<br>Extending the Number Line | **Investigation 4**<br>Working With Percents |
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| **Problem 1.1**<br>**Fundraising: Comparing With Fractions and Ratios**<br>FQ:: What are two ways to compare a $500 fundraising goal to a $200 fundraising goal?<br><br>**Problem 1.2**<br>**Fundraising Thermometers: Introducing Ratios**<br>FQ: How does a "for every" statement show a ratio comparison?<br><br>**Problem 1.3**<br>**Equivalent Fractions on the Line**<br>FQ: When you fold fraction strips, what relationships do you see emerge that show how the numerator and denominator change to make equivalent fractions?<br><br>**Problem 1.4**<br>**Measuring Progress: Finding Fractional Parts**<br>FQ: How can fraction strips help you find part of a number?<br><br>**Problem 1.5**<br>**Comparing Fundraising Goals: Using Fractions and Ratios**<br>FQ: What does it mean for two fractions to be equivalent? What does it mean for two ratios to be equivalent? | **Problem 2.1**<br>**Equal Shares: Introducing Unit Rates**<br>FQ: What does a unit rate comparison statement tell us?<br><br>**Problem 2.2**<br>**Unequal Shares: Using Ratios and Fractions**<br>FQ: How are part-to-part relationships related to part-to-whole fractions?<br><br>**Problem 2.3**<br>**Making Comparisons with Rate Tables**<br>FQ: How do rate tables help us find equivalent ratios? | **Problem 3.1**<br>**Extending the Number Line: Integers and Mixed Numbers**<br>FQ: How can the number line help you think about fractions greater than 1 and less than 0?<br><br>**Problem 3.2**<br>**Estimating and Ordering Rational Numbers: Comparing Fractions to Benchmarks**<br>FQ: When comparing two relational numbers, what are some useful strategies for deciding which is greater?<br><br>**Problem 3.3**<br>**Sharing 100 Things: Using Tenths and Hundredths**<br>FQ: How does what you know about fractions help you understand decimals?<br><br>**Problem 3.4**<br>**Decimals on the Number Line**<br>FQ: How do we use what we know about fractions to estimate and compare decimals?<br><br>**Problem 3.5**<br>**Earthquake Relief: Moving from Fractions to Decimals**<br>FQ: Why does it make sense to divide the numerator of a fraction by the denominator to find an equivalent decimal representation? | **Problem 4.1**<br>**Who is the Best? Making Sense of Percents**<br>FQ: How is a percent bar useful in making comparisons with decimals?<br><br>**Problem 4.2**<br>**Genetic Traits: Finding Percents**<br>FQ: How can partitioning be used to express one number as a percent of another number?<br><br>**Problem 4.3**<br>**The Art of Comparison: Using Ratios and Percents**<br>FQ: In what way is a percent like a ratio and like a fraction? |
| **Mathematical Reflections**<br><br>1a. Write three comparison statements about the same situation, one using difference, one using a fraction, and one using a ratio.<br><br>1b. Explain what you think a ratio is.<br><br>2a. What does it mean for two fractions to be equivalent? For two ratios to be equivalent?<br><br>2b. What are some useful ways of finding equivalent fractions and equivalent ratios? | **Mathematical Reflections**<br><br>1a. How can you determine a unit rate for a situation?<br>1b. Describe some ways that unit rates are useful.<br><br>2a. What strategies do you use to make a rate table?<br>2b. Describe some ways that rate tables are useful.<br><br>3. How are your strategies for writing equivalent ratios the same as or different from writing equivalent fractions? | **Mathematical Reflections**<br><br>1a. Not every fraction refers to a quantity between 0 and 1. Give some examples of numbers that are greater than 1 or less than 0.<br>1b. How is a number and its opposite represented on a number line?<br><br>2a. What are some strategies for deciding which of two numbers is greater? Give examples.<br>2b. When comparing two positive whole numbers with different numbers of digits, such as 115 and 37, the one with more digits is greater. Does this rule work for comparing decimals? | **Mathematical Reflections**<br><br>1. Describe strategies for finding a percent of a known quantity.<br><br>2. What strategies can you use to find the percent of one quantity to another quantity?<br><br>3. How are percents used to make a comparison?<br><br>4. Describe other strategies that you can use to make comparisons. |

# 6-3: Let's Be Rational

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Numeric Estimation** Understand that estimation can be used as a tool in a variety of situations including checking answers and making decisions, and develop strategies for estimating results of arithmetic operations

- Use benchmarks and other strategies to estimate results of operations with fractions
- Use estimates to check the reasonableness of exact computations
- Give various reasons to estimate and identify when a situation calls for an overestimate or an underestimate
- Use estimates and exact solutions to make decisions

**Fraction Operations** Revisit and continue to develop meanings for the four arithmetic operations and skill at using algorithms for each

- Determine when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem
- Develop ways to model sums, differences, products, and quotients with areas, fraction strips, and number lines
- Use knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying, and dividing fractions
- Write fact families with fractions to show the inverse relationship between addition and subtraction, and between multiplication and division
- Compare and contrast dividing a whole number by a fraction to dividing a fraction by a whole number
- Recognize that when you multiply or divide a fraction, your answer might be less than or more than the numbers you started with
- Solve real-world problems using arithmetic operations on fractions

**Variables and Equations** Use variables to represent unknown values and equations to represent relationships

- Represent unknown real-world and abstract values with variables
- Write equations (or number sentences) to represent relationships among real-world and abstract values
- Use fact families to solve for unknown values

# 6-3 Let's Be Rational: Focus Questions (FQ) and Mathematical Reflections

| Investigation 1<br>Extending Addition and Subtraction of Fractions | Investigation 2<br>Building Multiplication With Fractions | Investigation 3<br>Dividing With Fractions | Investigation 4<br>Wrapping Up the Operations |
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| **Problem 1.1**<br>**Getting Close: Estimating Sums**<br>FQ: What are some strategies for estimating the sums of fractions?<br><br>**Problem 1.2**<br>**Estimating Sums and Differences**<br>FQ: How do you know if your estimate is an underestimate or overestimate? What information does an underestimate or overestimate tell you?<br><br>**Problem 1.3**<br>**Land Sections: Adding and Subtracting Fractions**<br>FQ: What are some strategies for adding and subtracting fractions?<br><br>**Problem 1.4**<br>**Visiting the Spice Shop: Adding and Subtracting Mixed Numbers**<br>FQ: What are some strategies for adding and subtracting mixed numbers? | **Problem 2.1**<br>**How Much of the Pan Have We Sold?**<br>**Finding Parts of Parts**<br>FQ: How does an area model relate to multiplying fractions?<br><br>**Problem 2.2**<br>**Modeling Multiplicative Situations**<br>FQ: What strategies can you use to multiply all combinations of factors including whole numbers, fractions, and mixed numbers?<br><br>**Problem 2.3**<br>**Changing Forms: Multiplication With Mixed Numbers**<br>FQ: How can you use number properties and equivalent fractions to multiply rational numbers? | **Problem 3.1**<br>**Preparing Food: Dividing a Fraction by a Fraction**<br>FQ: What does it mean to divide a fraction by a fraction? What strategies help you divide a fraction by a fraction?<br><br>**Problem 3.2**<br>**Into Pieces: Whole Numbers or Mixed Numbers Divided by Fractions**<br>FQ: What does it mean to divide a whole number or mixed number by a fraction? What strategies help you divide a whole number or mixed number by a fraction?<br><br>**Problem 3.3**<br>**Sharing a Prize: Dividing a Fraction by a Whole Number**<br>FQ: What does it mean to divide a fraction by a whole number? What strategies help you divide a fraction by a whole number?<br><br>**Problem 3.4**<br>**Examining Algorithms for Dividing Fractions**<br>FQ: What is an efficient algorithm for division problems involving fractions and mixed numbers? | **Problem 4.1**<br>**Just the facts: Fact Families for Addition and Subtraction**<br>FQ: How do fact families help you solve equations such as $\frac{4}{5} - N = \frac{3}{8}$?<br><br>**Problem 4.2**<br>**Multiplication and Division Fact Families**<br>FQ: How do fact families help you solve equations such as $\frac{2}{9} \div N = \frac{2}{3}$?<br><br>**Problem 4.3**<br>**Becoming an Operations Sleuth**<br>FQ: How do you know when a particular operation is called for to solve a problem? How do you represent the problem with a number sentence? |
| **Mathematical Reflections**<br>1a. What are some situations in which estimating a sum or difference is useful? Why is estimation useful in these situations?<br><br>1b. When is it useful to overestimate? When is it useful to underestimate?<br><br>2. When should you use addition to solve a problem involving fractions? When should you use subtraction?<br><br>3. Suppose you are helping a student who has not studied fractions. Explain to him or her how to add and subtract fractions. Give an example of the type you think is easiest to explain. Give an example of the type you think is hardest to explain. | **Mathematical Reflections**<br>1. Explain and illustrate what *of* means when you find a fraction *of* another number. What operation do you use when you find parts of parts?<br><br>2a. If you forget the algorithm for multiplying fractions, how might you use rectangular models to help you multiply fractions?<br>2b. Describe an algorithm for multiplying any two fractions.<br>2c. Describe when it might be useful to estimate a product.<br><br>3. Use examples to explain the following statement: "When you multiply a fraction by another fraction, your answer might be less than both factors, more than one of the factors, or more than both factors." | **Mathematical Reflections**<br>1. When solving a problem, how do you recognize when division is the operation you need to use?<br><br>2a. How is dividing a whole number by a fraction similar to or different from dividing a fraction by a whole number?<br><br>2b. Explain your strategy for dividing one fraction by another fraction. Does your strategy also work for divisions where the dividend or divisor is a whole number or a mixed number? Explain.<br><br>3. When dividing a whole number by a whole number greater than 1, the quotient is always less than the dividend. For example, 15 ÷ 3 = 5, and 5 is less than 15 (the dividend). Use examples to explain the following statement:<br><br>"When you divide a fraction by another fraction, your answer might be greater than the dividend or less than the dividend." | **Mathematical Reflections**<br>1. How do you decide which operation to use when you are solving a problem?<br><br>2. How is the relationship between addition and subtraction like the relationship between multiplication and division? How is it different?<br><br>3. While working with fact families, you thought about decomposing numbers.<br><br>3a. What does it mean to decompose a number?<br><br>3b. How do fact families help you figure out the value for *N* in a sentence such as $N \div 2\frac{1}{2} = 1\frac{1}{4}$? |

# 6-4: Covering and Surrounding

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Area and Perimeter** Understand that perimeter is a measure of linear units needed to surround a two-dimensional shape and that area is a measure of square units needed to cover a two-dimensional shape

- Deepen the understanding of area and perimeter of rectangular and nonrectangular shapes
- Relate area to covering a figure
- Relate perimeter to surrounding a figure
- Analyze what it means to measure area and perimeter
- Develop and use formulas for calculating area and perimeter
- Develop techniques for estimating the area and perimeter of an irregular figure
- Explore relationships between perimeter and area, including that one can vary considerably while the other stays fixed
- Visually represent relationships between perimeter and area on a graph
- Solve problems involving area and perimeter of rectangles

**Area and Perimeter of Parallelograms and Triangles** Understand that the linear measurements of the base, height, and slanted height of parallelograms and triangles are essential to finding the area and perimeter of these shapes

- Analyze how the area of a triangle and the area of a parallelogram are related to each other and to the area of a rectangle
- Recognize that a triangle can be thought of as half of a rectangle whose sides are equal to the base and height of the triangle
- Recognize that a parallelogram can be decomposed into two triangles. Thus the area of a parallelogram is twice the area of a triangle with the same base and height as the parallelogram
- Know that the choice of base of a triangle (or parallelogram) is arbitrary but that the choice of the base determines the height
- Recognize that there are many triangles (or parallelograms) that can be drawn with the same base and height
- Develop formulas and strategies, stated in words or symbols, for finding the area and perimeter of triangles and parallelograms
- Find the side lengths and area of polygons on a coordinate grid
- Solve problems involving area and perimeter of parallelograms and triangles
- Solve problems involving area and perimeter of polygons by composing into rectangles or decomposing into triangles

**Surface Area of Prisms and Pyramids and Volume of Rectangular Prisms** Understand that the surface area of a three-dimensional shape is the sum of the areas of each two-dimensional surface of the shape and that the volume of a rectangular prism is a measure in cubic units of the capacity of the prism

- Extend the understanding of the volume of rectangular prisms
- Relate volume to filling a three-dimensional figure
- Extend understanding of the strategies for finding the volume of rectangular prisms to accommodate fractional side lengths
- Relate finding area of two-dimensional shapes to finding the surface area of three-dimensional objects
- Develop strategies for finding the surface area of three-dimensional objects made from rectangles and triangles
- Solve problems involving surface area of prisms and pyramids and volume of rectangular prisms

# 6-4 Covering and Surrounding: Focus Questions (FQ) and Mathematical Reflections

| **Investigation 1**<br>Designing Bumper Cars: Extending and Building on Area and Perimeter | **Investigation 2**<br>Measuring Triangles | **Investigation 3**<br>Measuring Parallelograms | **Investigation 4**<br>Measuring Surface Area and Volume |
|---|---|---|---|
| **Problem 1.1**<br>**Designing Bumper Car Rides: Area and Perimeter**<br>FQ: What are the formulas for finding the area and perimeter of a rectangle? Explain why they work.<br><br>**Problem 1.2**<br>**Building Storm Shelters: Constant Area, Changing Perimeter**<br>FQ: For a fixed area, what are the shape and perimeter of the rectangles with the greatest and least perimeters?<br><br>**Problem 1.3**<br>**Fencing in Spaces: Constant Perimeter, Changing Area**<br>FQ: For a fixed perimeter, what are the shape and area of the rectangles the greatest and least area? | **Problem 2.1**<br>**Triangles on Grids: Finding Area and Perimeter of Triangles**<br>FQ: What is a formula for finding the area of a triangle?<br><br>**Problem 2.2**<br>**More Triangles: Identifying Base and Height**<br>FQ: Does it make any difference which side is used as the base when finding the area of a triangle?<br><br>**Problem 2.3**<br>**Making Families of Triangles: Maintaining the Base and the Height**<br>FQ: What can you say is true and what can you say is not true about triangles that have the same base and height?<br><br>**Problem 2.4**<br>**Designing Triangles Under Constraints**<br>FQ: What conditions for a triangle produce triangles that have the same area? Do they have the same shape? Explain. | **Problem 3.1**<br>**Parallelograms and Triangles: Finding Area and Perimeter of Parallelograms**<br>FQ: What is a strategy for finding the area of a parallelogram? Explain why the strategy works.<br><br>**Problem 3.2**<br>**Making Families of Parallelograms: Maintaining the Base and the Height**<br>FQ: What can you say about two parallelograms that have the same base and height?<br><br>**Problem 3.3**<br>**Designing Parallelograms Under Constraints**<br>FQ: Under what conditions will two or more parallelograms have the same area? Do these parallelograms have the same shape? Explain.<br><br>**Problem 3.4**<br>**Polygons on Coordinate Grids**<br>FQ: How can you find the area of a polygon drawn on a coordinate graph? On grid paper? | **Problem 4.1**<br>**Making Rectangular Boxes**<br>FQ: What is a strategy for finding the surface area of a rectangular prism? Explain why the strategy works.<br><br>**Problem 4.2**<br>**Filling the Boxes: Finding Volume**<br>FQ: What is a strategy for finding the volume of a rectangular prism? Explain why the strategy works.<br><br>**Problem 4.3**<br>**Designing Gift Boxes: Finding Surface Area**<br>FQ: What is a strategy for finding the surface area of three-dimensional object? Explain why the strategy works. |
| **Mathematical Reflections**<br>1a. Explain what area and perimeter of a figure means.<br><br>1b. Describe a strategy for finding the area and perimeter of any two-dimensional shape.<br><br>1c. Describe how you can find the area of a rectangle. Explain why this method works.<br><br>1d. Describe how you can find the perimeter of a rectangle. Explain why this method works.<br><br>2a. Consider all the rectangles with the same area. Describe the rectangle with the least perimeter. Describe the rectangle with the greatest perimeter.<br><br>2b. Consider all the rectangles with the same perimeter. Describe the rectangle with the least area. Describe the rectangle with the greatest area.<br><br>2c. Explain how graphing relationships between length and perimeter or length and area helps explain patterns between area and perimeter. | **Mathematical Reflections**<br>1a. Describe how to find the area of a triangle. Explain why your method works.<br><br>1b. Describe how to find the perimeter of a triangle. Explain why your method works.<br><br>2a. Does the choice of the base affect the area of a triangle? Does the choice of the base affect the perimeter of a triangle? Explain why or why not?<br><br>2b. What can you say about the area and perimeter of two triangles that have the same base and height? Give evidence to support your answer?<br><br>3. How is finding the area of a triangle related to finding the area of a rectangle? How is finding the perimeter of a triangle related to finding the perimeter of a rectangle? | **Mathematical Reflections**<br>1a. Describe how to find the area of a parallelogram. Explain why your method works.<br><br>1b. Describe how to find the perimeter of a parallelogram. Explain why your method works.<br><br>2a. Does the choice of the base change the area of a parallelogram? Does the choice of the base change the perimeter of a parallelogram? Explain why or why not?<br><br>2b. What can you say about the shape, area, and perimeter of two parallelograms that have the same base and height? Give evidence to support your answer?<br><br>3. How is the area of a parallelogram related to the area of a triangle and a rectangle? How is the perimeter of a parallelogram related to the perimeter of a triangle and a rectangle? | **Mathematical Reflections**<br>1a. What information do you need to find the volume of a rectangular prism? Describe a strategy to find the volume of a rectangular prism.<br><br>1b. What information do you need to find the surface area of a rectangular prism? Describe a strategy to find the surface area of a rectangular prism.<br><br>2a. Describe a strategy for finding the surface area of three-dimensional shapes made from rectangles and triangles.<br><br>2b. How does knowing the area of two-dimensional figures help you find the surface area of a three-dimensional shape? |

# 6-5: Decimal Ops

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Numeric Estimation** Understand that estimation can be used as a tool in a variety of situations, including as a way to check answers and make decisions

- Use estimates to solve problems and check answers

**Decimal Operations** Revisit and continue to develop meanings for the four arithmetic operations on rational numbers, and practice using algorithms to operate on decimals

- Recognize when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem
- Use place value to develop understanding of algorithms and to relate operations with decimals to the same operations with fractions
- Extend understanding of multiplication and division of multidigit whole numbers
- Develop standard algorithms for multiplying and dividing decimals with the aid of, at most, paper and pencil
- Find a repeating or terminating decimal equivalent to a given fraction
- Solve problems using arithmetic operations on decimals, including finding unit rates

**Variables and Number Sentences** Use variables to represent unknown values and number sentences to represent relationships between values

- Write number sentences to represent relationships between both real-world and abstract values
- Use fact families to write and solve equivalent number sentences
- Use multiplication sentences to check division sentences

**Percents** Develop understanding of percents through various contexts, such as sales tax, tips, discounts, and percent increases

- Develop models for percent problems
- Write and solve number sentences involving percents

# 6-5 Decimal Ops: Focus Questions (FQ) and Mathematical Reflections

| **Investigation 1**<br>Decimal Operations and Estimation | **Investigation 2**<br>Adding and Subtracting Decimals | **Investigation 3**<br>Multiplying and Dividing Decimals | **Investigation 4**<br>Using Percents |
|---|---|---|---|
| **Problem 1.1**<br>**Out to Lunch: Matching Operations and Questions**<br>FQ: What signals in a real-world problem tell you which operation to use?<br><br>**Problem 1.2**<br>**Getting Close: Estimating Decimal Calculations**<br>**FQ:** When you work with decimal computations, what strategies can you use to estimate the results?<br><br>**Problem 1.3**<br>**Take a Hike: Connecting Ratios, Rates, and Decimals**<br>FQ: How can you express a unit rate as a decimal and use it to solve problems? | **Problem 2.1**<br>**Getting Things in the Right Place: Adding Decimals**<br>FQ: What's the Difference? Subtracting Decimals<br><br>**Problem 2.2**<br>**What's the difference? Subtracting Decimals**<br>FQ: How do you subtract one decimal number from another?<br><br>**Problem 2.3**<br>**Connecting Operations: Fact Families**<br>FQ: Do fact families apply to operations with decimal numbers? | **Problem 3.1**<br>**It's Decimal Times(s): Multiplying Decimals I**<br>FQ: How do you find the product of any two decimal numbers?<br><br>**Problem 3.2**<br>**It Works Every Time: Multiplying Decimals II**<br>FQ: What algorithm can be used to find any decimal product?<br><br>**Problem 3.3**<br>**How Many Times? Dividing Decimals I**<br>FQ: How can a decimal division problem be written in equivalent fraction and whole number form?<br><br>**Problem 3.4**<br>**Going the Long Way: Dividing Decimals II**<br>FQ: How can you carry out a decimal division using a method similar to long division of whole numbers?<br><br>**Problem 3.5**<br>**Challenging Cases: Dividing Decimals III**<br>FQ: How can you complete a long division problem that doesn't give a whole number quotient? That is, how do you express remainders in decimal form? | **Problem 4.1**<br>**What's the Tax on This Item?**<br>FQ: How do you find the tax and the total cost of an item from a given selling price and tax rate? How do you find the base price from a given tax rate and amount?<br><br>**Problem 4.2**<br>**Computing Tips**<br>FQ: How do you find the tip and the total cost of a restaurant meal from a given meal price and tip rate? How do you find the meal price from a given tip percent and amount?<br><br>**Problem 4.3**<br>**Percent Discounts**<br>FQ: How do you find the discount and the total cost of an item from a given selling price and discount rate? How do you find the base price from a given discount rate and amount? How can you express a change in a given amount as a percent change?<br><br>**Problem 4.4**<br>**Putting Operations Together**<br>FQ: How do you decide which operations to perform when a problem involves decimals and percents? |
| **Mathematical Reflections**<br><br>1. How do you know when solving a problem that involves decimals requires addition? Subtraction? Multiplication? Division?<br><br>2. Describe a strategy that you use when estimating with decimals. Explain why it is helpful to you.<br><br>3. What is a unit rate? Describe how unit rates are useful. | **Mathematical Reflections**<br><br>1. How does interpreting decimals as fractions help you make sense of adding and subtracting decimals? Give an example to show your thinking.<br><br>2. How does the place-value interpretation of decimals help you add and subtract decimals? Give an example to show your thinking.<br><br>3. Describe algorithms for adding and subtracting any two decimal numbers. | **Mathematical Reflections**<br><br>1. What algorithm can be used to multiply any two decimal numbers? Explain why your algorithm works, and give an example that shows how it works.<br><br>2a. What algorithm can be used to divide any two decimal numbers? Explain why your algorithm works, and give an example that shows how it works.<br><br>2b. How can you predict whether a quotient will be a terminating decimal or a repeating decimal?<br><br>3a. What is the fact-family connection between decimal multiplication and division?<br><br>3b. How can you check the result of a division calculation?<br><br>3c. How can you check the result of a multiplication calculation? | **Mathematical Reflections**<br>1a. How do you find the tax on a purchase and calculate the final bill? Give an example, then write and solve a number sentence to illustrate your strategy.<br><br>1b. How do you find the price of a discounted item if you know the percent of the discount? Give an example, then write and solve a number sentence to illustrate your strategy.<br><br>1c. How do you find the cost of a purchase if you know the percent and the amount of the tax on the purchase? Give an example, then write and solve a number sentence to illustrate your strategy.<br><br>1d. How can you find the percent one number is of another? For example, what percent of 35 is 7? Write and solve a number sentence to illustrate your answer.<br><br>1e. How are all the number sentences in parts (a)-(d) the same?<br><br>2. How do you recognize when addition, subtraction, multiplication, and/or division of decimals is required to solve a problem? |

# 6-6: Variables and Patterns

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Variables and Patterns (Relationships)** Develop understanding of variables and how they are related

- Explore problem situations that involve variables and relationships
- Identify the dependent and independent variables and describe how they are related in a situation
- Interpret the "stories" told by patterns in tables and coordinate graphs of numeric $(x, y)$ data
- Represent the pattern of change that relates two variables in words, data tables, graphs, and equations
- Investigate situations that change over time
- Examine increasing and decreasing patterns of change
- Compare linear and nonlinear patterns of change by using tables or graphs
- Use tables, graphs, and equations to find the value of a variable given the value of the associated variable
- Explore relationships that require graphing in all four quadrants
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent patterns of change relating two variables and make connections across those representations
- Write an equation to express the relationship between two variables in one and two operations: $y=mx$, $y=b+x$, and $y=b+mx$
- Calculate average speed and show how it is reflected in a table or graph and vice versa
- Recognize and express direct proportionality relationships with a unit rate ($y=mx$) and represent these relationships in rate tables and graphs
- Solve problems that involve variables

**Expressions and Equations** Develop understanding of expressions and equations

- Use properties of operations, including the Distributive Property and the Order of Operations, to write equivalent expressions for the dependent variable in terms of the independent variable
- Use tables, graphs, or properties of numbers such as the Distributive Property to show that two expressions are equivalent
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity
- Interpret and evaluate expressions in which letters stand for numbers and apply the Order of Operations as needed
- Recognize that equations are statements of equivalence between two expressions
- Solve linear equations of the forms $y=ax$, $y=b+x$, and $y=b+ax$ using numeric guess and check, tables of $(x,y)$ values, and graphs or fact families
- Write an inequality and associate it with an equation to find solutions and graph the solutions on a number line

# 6-6 Variables and Patterns: Focus Questions (FQ) and Mathematical Reflections

| **Investigation 1**<br>Variables, Tables and Graphs | **Investigation 2**<br>Analyzing Relationships among Variables | **Investigation 3**<br>Relating Variables with Equations | **Investigation 4**<br>Expressions, Equations, and Inequalities |
|---|---|---|---|
| **Problem 1.1**<br>**Getting Ready to Ride: Data Tables and Graphs**<br>FQ: How can you construct a graph from a table of data that depicts change over time? How is the pattern of change represented in the graph?<br><br>**Problem 1.2**<br>**From Atlantic City to Lewes: Time, Rate, and Distance**<br>FQ: What are the advantages and disadvantages of tables and graphs in representing and describing the patterns of change in a variable over time?<br><br>**Problem 1.3**<br>**From Lewes to Chincoteague Island: Stories, Tables, and Graphs**<br>FQ: Which representation of data – table, graph, or written notes- seems to better show patterns of change in distance over time, and why?<br><br>**Problem 1.4**<br>**From Chincoteague to Colonial Williamsburg: Average Speed**<br>FQ: How do you calculate average speed for a trip? How do a table and graph of (time, distance) data show speed? | **Problem 2.1**<br>**Renting Bicycles: Independent and Dependent Variables**<br>FQ: How do you analyze and compare the relationship between variables given in different representations?<br><br>**Problem 2.2**<br>**Finding Customers: Linear and Non-Linear Patterns**<br>FQ: How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data?<br><br>**Problem 2.3**<br>**Predicting Profit: Four Quadrant Graphing**<br>FQ: How are the variables, *tour income* and *tour profit,* related to each other? How do you plot data points with one or both coordinates negative?<br><br>**Problem 2.4**<br>**What's the Story? Interpreting Graphs**<br>FQ: When the relationship between dependent and independent variables is displayed in a graph, what can you learn about the relationship from a rising graph, a level graph, and a falling graph? | **Problem 3.1**<br>**Visit to Wild World: Equations with One Operation**<br>FQ: In what kinds of situations will the equation between dependent and independent variables be in the form<br>$y = x + k$? $y = x - k$? $y = kx$? $y = x/k$?<br><br>**Problem 3.2**<br>**Moving, Texting, and Measuring: Using Rates and Rate Tables**<br>FQ: What can you tell about the relationship between dependent and independent variables in an equation of the form $y = mx$? How is that relationship shown in a table and a graph of sample (x, y) values? Why is the point (1, m) on every graph?<br><br>**Problem 3.3**<br>**Group Discounts and a Bonus Card: Equations with Two Operations**<br>FQ: How do you calculate values of y from an equation like $y = 3x + 5$ when values of x are given? How about $y = 5 + 3x$? When do you need such equations that involve two operations?<br><br>**Problem 3.4**<br>**Getting the Calculation Right: Expressions and Order of Operations**<br>FQ: When an equation relating two variables involves two or more operations, how do you use the equation to find values of the dependent variable from given values of the independent variable? | **Problem 4.1**<br>**Taking the Plunge: Equivalent Expressions I**<br>FQ: Is it possible to have two different, but equivalent, expressions for a given situation? Explain.<br><br>**Problem 4.2**<br>**More Than One way to Say it: Equivalent Expressions II**<br>FQ: What does it mean to say that two algebraic expressions are equivalent?<br><br>**Problem 4.3**<br>**Putting it All Together: Equivalent Expressions III**<br>FQ: How can expressions such as $3x + 7x$ *or* $3(x + 2)$ be written in equivalent form?<br><br>**Problem 4.4**<br>**Finding the Unknown Value: Solving Equations**<br>FQ: What strategies can you use to solve equations in the forms $x + a = b$, $x - a = b$, $ax = b$, and $x \div a = b$ $(a \neq 0)$?<br><br>**Problem 4.5**<br>**It's Not Always Equal: Solving Inequalities**<br>FQ: How can you represent and find solutions for inequalities? |
| **Mathematical Reflections**<br>1. You can show patterns of change over time with tables, graphs, and written reports.<br>1a. What are the advantages and disadvantages of showing patterns with tables?<br>1b. What are the advantages and disadvantages of showing patterns with graphs?<br>1c. What are the advantages and disadvantages of showing patterns with written reports?<br>2a. How do you see patterns in the speed of a moving object by studying (time, distance) data in tables?<br>2b. How do you see patterns in the speed of a moving object by studying (time, distance) data in coordinate graphs? | **Mathematical Reflections**<br>1. The word variable is used often to describe conditions in science and business.<br>1a. Explain what the word variable means when it is used in situations like those you studied in this investigation.<br>1b. When are the words independent and dependent used to describe related variables? How are they used?<br>2. Suppose the values of a dependent variable increase as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following?<br>2a. a table of values for the two variables?<br>2b. a graph of values for the two variables?<br>3. Suppose the values of a dependent variable decrease as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following?<br>3a. a table of values for the two variables<br>3b. a graph of values for the two variables | **Mathematical Reflections**<br>1. What strategies help in finding equations to express relationships?<br>2 For relationships given by equations in the form $y = mx$:<br>2a. How does the value of y change as the value of x increases?<br>2b. How is the pattern of change shown in a table, graph, and equation of the function?<br>3a. In this unit, you have represented relationships between variables with tables, graphs, and equations. List some advantages and disadvantages of each of these representations.<br>3b. If the value of one variable in a relationship is known, describe how you can use a table, graph, or equation to find a value of the other variable. | **Mathematical Reflections**<br>1. What does it mean to say that two expressions are equivalent? How can you test the equivalence of two expressions?<br>2. What does it mean to *solve* an equation? What strategies are available for solving equations?<br>3. What does it mean to *solve* an inequality? What will graphs of such solutions look like for inequalities in the form $ax > b$ and $a + x < b$ (Assume $a$ and $b$ are both positive numbers).<br>4. Describe how expressions, equations, inequalities, and representations are used in this Unit. How are they related? |

# 6-7: Data About Us

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Statistical Process** Understand and use the process of statistical investigation

- Ask questions, collect and analyze data, and interpret data to answer questions
- Describe data with respect to its shape, center, and variability or spread
- Construct and use simple surveys as a method of collecting data

**Attributes of Data** Distinguish data and data types

- Recognize that data consist of counts or measurements of a variable, or an attribute; these observations comprise a distribution of data values
- Distinguish between categorical data and numerical data, and identify which graphs and statistics can be used to represent each kind of data

**Multiple Representations for Displaying Data** Display data with multiple representations

- Organize and represent data using tables, dot plots, line plots, ordered-value bar graphs, frequency bar graphs, histograms, and box-and-whisker plots
- Make informed decisions about which graphs or tables can be used to display a particular set of data
- Recognize that a graph shows the overall shape of a distribution, whether the data values are symmetrical around a central value, and whether the graph contains any unusual characteristics such as gaps, clusters, or outliers

**Measures of Central Tendency and Variability** Recognize that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread)

- Distinguish between and compute measures of central tendency (mean, median, and mode) and measures of spread (range, interquartile range (IQR), and mean absolute deviation (MAD))
- Identify how the median and mean respond to changes in the data values of a distribution
- Relate the choice of measures of central tendency and variability to the shape of the distribution and the context
- Describe the amount of variability in a distribution by noting whether the data values cluster in one or more areas or are fairly spread out
- Use measures of center and spread to compare data distributions

# 6-7 Data About Us: Focus Questions (FQ) and Mathematical Reflections

| Investigation 1<br>What's in a name? Organizing, Representing, and Describing Data | Investigation 2<br>Who's in Your Household? Using the Mean | Investigation 3<br>What's Your Favorite…? Measuring Variability | Investigation 4<br>What Numbers Describe Us? Using Graphs to Group Data |
|---|---|---|---|
| **Problem 1.1**<br>**How Many Letters Are in a Name?**<br>FQ: What are "data"? How do you represent data using a frequency table or a line plot? How can you compare two distributions of data?<br><br>**Problem 1.2**<br>**Describing Name Lengths: What Are the Shape, Mode, and Range?**<br>FQ: What are the measures of central tendency and variability (or spread)? How do you compare and use mode and range?<br><br>**Problem 1.3**<br>**Describing Name Lengths: What is the Median?**<br>FQ: How do you identify and use the median? How can you compare two distributions of data using the medians? | **Problem 2.1**<br>**What's a Mean Household Size?**<br>FQ: How do you go about finding a number that is a good estimate of typical household size based on the given data?<br><br>**Problem 2.2**<br>**Comparing Distributions With the Same Mean**<br>FQ: How do you interpret, compute, and use the mean?<br><br>**Problem 2.3**<br>**Making Choices: Mean or Median?**<br>FQ: How do the median and the mean respond to the data in a distribution? How do you choose which measure of center to use when describing what is typical?<br><br>**Problem 2.4**<br>**Who Else is in Your Household? Categorical and Numerical Data**<br>FQ: How do you distinguish different types of data? What statistics are used with different types of data? | **Problem 3.1**<br>**Estimating Cereal Serving Sizes: Determining the IQR**<br>FQ: What information does the interquartile range provide about how data vary in a distribution?<br><br>**Problem 3.2**<br>**Connecting Cereal Shelf Location and Sugar Content: Describing Variability Using the IQR**<br>FQ: How is the interquartile range used to make comparisons among distributions?<br><br>**Problem 3.3**<br>**Is It Worth the Wait? Determining and Describing Variability Using the MAD**<br>FQ: What information does the mean absolute deviation provide about how data vary in a distribution? | **Problem 4.1**<br>**Traveling to School: Histograms**<br>FQ: How can you use a histogram to help you interpret data?<br><br>**Problem 4.2**<br>**Jumping Rope: Box-and-Whisker Plots**<br>FQ: How can you interpret data using a box-and-whisker plot?<br><br>**Problem 4.3**<br>**How Much Taller Is a 6th Grader Than a 2nd Grader? Taking Variability Into Consideration**<br>FQ: How can you compare and contrast data represented by dot plots, histograms, and box plots? |
| **Mathematical Reflections**<br><br>1. The process of carrying out a statistical investigation involves asking a question, gathering and analyzing data, and interpreting the results to answer the question. Choose a data set from this Investigation. Use the data set to answer each question below.<br>What was the question asked?<br>How were the data collected?<br>How were the data analyzed and represented?<br>How did the results from the analysis help you answer the question?<br><br>2. You can represent a set of data using displays such as a data table, a frequency table, and a dot or line plot. Explain how these displays are related.<br><br>3. The median and mode are two measures of the center of a data distribution. The range is a measure of variability, or how spread out the data are.<br>3a. What does each measure of center tell you about the data set?<br>3b. Can the mode and the median for a data set have the same value? Can they have different values? Explain your answers.<br>3c. How does the range tell you how much the data vary?<br><br>3d. Suppose we add a new data value to the set of data. Does this new value affect the mode? The median? The range? Explain.<br><br>4. What strategies can you use to make comparisons among data sets | **Mathematical Reflections**<br><br>1. Describe a method for calculating the mean of a set of data. Explain why your method works.<br><br>2. You have used three measures of center – mode, median, and mean – to describe distributions.<br>2a. Why do you suppose they are called "measures of center"?<br>2b. What does each tell you about a set of data?<br>2c. How do you decide which measure of center to use when describing a distribution?<br>2d. Why might you want to include both the range and a measure of center when reporting a statistical summary?<br><br>3a. One student says you can only use the mode to describe categorical data, but you can use the mode, median, and mean to describe numerical data. Is the student correct? Explain.<br><br>3b. Can you find the range for categorical data? Explain. | **Mathematical Reflections**<br><br>1. Explain and illustrate the following words.<br>1a. Range<br>1b. Interquartile Range<br>1c. Mean absolute deviation<br><br>2a. Describe how you can use the range to compare how two data distributions vary.<br><br>2b. Describe how you can use the IQR to compare how two data distributions vary.<br><br>2c. Describe how you can use the MAD to compare how two data distributions vary. | **Mathematical Reflections**<br><br>1. Describe how you can display data using a histogram.<br><br>2. Describe how you can display data using a box plot.<br><br>3a. How can you use histograms to compare two data sets?<br><br>3b. How can you use box plots to compare two data sets?<br><br>4. Numerical data can be displayed using more than one type of graph. How do you decide when to use a dot plot, bar graph, histogram, or box plot? |