Understanding Fractions as Parts of a Whole:
This meaning of “fraction” focuses on partitioning an object or a set of objects into **equal size parts** or groups and making a comparison of some parts to the whole object or set. The **numerator** (top of the fraction) indicates the number of parts chosen, and the **denominator** (bottom of the fraction) indicates the number of parts in the whole. Thus the denominator indicates the size of the parts.

If there are 7 girls, 8 boys and 18 adults in the audience at a school play then \( \frac{7}{33} \) of the audience are girls. The **whole** is the audience (“of the audience”), each person is a “part” and the girls comprise 7 parts out of 33 parts. 7 is the **numerator** and 33 is the **denominator**.

If we have to share a candy bar with 4 sections (the **whole**) between 3 people we need to subdivide the whole into enough **equal parts** to make this possible. The parts have to be the same size, not shape.

Each person gets \( \frac{1}{3} \) or \( \frac{4}{12} \) of the candy bar. \( \frac{1}{3} \) indicates the bar is divided into 3 parts, and each person gets 1 part. \( \frac{4}{12} \) indicates that the bar is divided into 12 parts and each person gets 4 parts. (There are other ways to arrange the 4 parts.)

\( \frac{1}{3} \) of

Is not the same quantity as \( \frac{1}{3} \) of

Because the “whole” is a different size.

**Understanding Fractions as Measures of Quantities:** This meaning of “fraction” focuses on a fraction as a number, “between” whole measures.

**Understanding Fraction as an Indicated Division:**
\( \frac{a}{b} \) can be evaluated by doing the computation \( a+b \). This makes a link between decimals and...
fractions. For example, \( \frac{3}{8} = 3 \div 8 = 0.375 \).

12 so each person gets \( \frac{1}{12} \) of the whole. This means dividing the whole (3 dollars or 3 apples) into 12 parts and giving 1 part (\( \frac{1}{4} \) of apple, or $0.25) to each person.

Equivalence of Fractions:
Fractions may have different names but represent equal values or equal parts. **Common factors** and **common multiples** help to find other way to name the same fractional part.

<table>
<thead>
<tr>
<th>4/12</th>
<th>1/3</th>
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\( \frac{4}{12} \) is the same as \( \frac{1}{3} \).

6 and 9 have a **common factor** of 3. \( \frac{6}{9} \) is the same as (2 groups of 3) / (3 groups of 3) and can be rewritten as \( \frac{2}{3} \).

Comparison of fractions: Fractions which represent parts of the same whole, or quantities, can be compared and ordered by size.

**Benchmark fractions**, such as \( \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \) offer a quick way to compare and order.

**Common multiples** are helpful in creating common denominators, which makes comparison simpler.

Mixed Numbers and Improper Fractions:
represent quantities that may not be a whole number, but are greater than 1.

Understanding and Comparing Decimals:
Decimals are special fractions with denominators of 10 and powers of 10. Since the decimal place indicates the power of 10 in the denominator this is a natural extension of place value for whole numbers.

\[
\frac{1}{10} \text{ is exactly the same as } 0.1; \quad \frac{1}{100} \text{ is exactly the same as } 0.01. \text{ Thus, } 256.182 \text{ means } 256 \text{ and } \frac{182}{1000} \text{ or } 2 \text{ hundreds } + 5 \text{ tens } + 6 + \frac{8}{100} + \frac{2}{1000}.
\]

\[
0.05 > 0.009 \text{ because } \frac{5}{100} > \frac{9}{1000}. \text{ Or we could rewrite both as } 0.050 \text{ and } 0.009, \text{ so we have to compare } \frac{50}{1000} \text{ and } \frac{9}{1000}.
\]

Understanding and comparing percents:

Percents are fractions with denominator 100. Percents are useful when we want to compare fractional parts of two wholes that are different sizes.

Which pays a greater part of their earnings in tax: a person who earns $1,000,000 and pays $90,000, or a person who earns $48,000 and pays $12,000 in tax? The two fractions are \( \frac{90000}{1000000} \) or \( \frac{9}{100} \), and \( \frac{12000}{48000} \) or \( \frac{1}{4} \) or \( \frac{25}{100} \). As percents these are 9% and 25%. So the second person pays a larger part of his earnings.

Connecting Fractions and Decimals and Percents:

Various models and strategies facilitate changing representations from decimal to fraction to percent easier.

A hundreds grid, representing the whole, allows students to represent fractions, decimals (to the second decimal place) and percents. The shaded area below represents \( \frac{4}{5} \cdot 0.80, \text{ or } 80\% \).