Focus Questions

Background
The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

Description
The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

Purpose
These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.
Unit Goals

**Rational Numbers** Develop an understanding that rational numbers consist of positive numbers, negative numbers, and zero

- Explore relationships between positive and negative numbers by modeling them on a number line
- Use appropriate notation to indicate positive and negative numbers
- Compare and order positive and negative rational numbers (integers, fractions, decimals, and zero) and locate them on a number line
- Recognize and use the relationship between a number and its opposite (additive inverse) to solve problems
- Relate direction and distance to the number line
- Use models and rational numbers to represent and solve problems

**Operations With Rational Numbers** Develop understanding of operations with rational numbers and their properties

- Develop and use different models (number line, chip model) for representing addition, subtraction, multiplication, and division
- Develop algorithms for adding, subtracting, multiplying, and dividing integers
- Recognize situations in which one or more operations of rational numbers are needed
- Interpret and write mathematical sentences to show relationships and solve problems
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations
- Use parentheses and the Order of Operations in computations
- Understand and use the Commutative Property for addition and multiplication
- Apply the Distributive Property to simplify expressions and solve problems
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<td><strong>Focus Question</strong> What algorithm can you use for dividing integers? How are multiplication and division related?</td>
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<td><strong>Focus Question</strong> How can you use a chip model to represent addition and subtraction?</td>
<td><strong>Focus Question</strong> What related sentence is equivalent to $4 + n = 43$ and makes it easier to find the value of $n$?</td>
<td><strong>Focus Question</strong> What patterns do you notice on the game board for the Integer Product Game that can help you?</td>
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| **1.** How do you decide which of two numbers is greater when  
   a. both numbers are positive?  
   b. both numbers are negative?  
   c. one number is positive and one number is negative?  
**2.** How does a number line help you compare numbers?  
**3.** When you add a positive number and a negative number, how do you determine the sign of the answer?  
**4.** If you are doing a subtraction problem on a chip board, and the board does not have enough chips, how can you extend the example to make it fair? | **1.** a. What algorithm(s) will produce the correct result for the sum \( a + b \), where \( a \) and \( b \) each represent any rational number? Show, using a number line or chip board, why your algorithm works.  
   b. What algorithm(s) will produce the correct result for the difference \( a - b \), where \( a \) and \( b \) each represent any rational number? Show, using a number line or chip board, why your algorithm works.  
**2.** How can any difference \( a - b \) be restated as an equivalent addition statement, where \( a \) and \( b \) each represent any rational number? | **1.** a. Give an example of a multiplication problem, involving two integers, in which the product is  
   a. less than 0.  
   b. greater than 0.  
   c. equal to 0.  
   d. In general, describe the signs of the factors for each product in parts (a)–(c).  
**2.** Give an example of a division problem, involving two integers, in which the quotient is  
   a. less than 0.  
   b. Greater than 0.  
   c. Equal to 0.  
   d. In general, describe the signs of the dividend and divisor | **1.** a. What is the Order of Operations? Why is the Order of Operations important?  
   b. Give an example of a numerical expression in which the use of parentheses changes the result of the computation.  
**2.** Describe how the Distributive Property relates addition and multiplication. Give numerical examples. |

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| **3.** | a. What does it mean to say that an operation is **commutative**?  
   b. Describe some ways that the additive inverse of a number is important. |
|----------------------------------|------------------------------------------------------------------|
| **4.** | a. Suppose three numbers are related by an equation of the form $a \cdot b = c$, where $a$, $b$, and $c$ are not equal to 0. Write two related number sentences using multiplication.  
   b. Suppose three numbers are related by an equation of the form $a \div b = c$, where $a$, $b$, and $c$ are not equal to 0. Write two related number sentences using multiplication.  
   c. Which operations on integers are commutative? Give numerical examples to support your answer. |