8-1: Thinking with Mathematical Models

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Linear and Nonlinear Relationships Recognize and model linear and nonlinear relationships in bivariate data
- Represent data patterns using graphs, tables, word descriptions and algebraic expressions
- Use mathematical models to answer questions about linear relationships
- Investigate the nature of linear variation in contexts
- Write linear functions from verbal, numerical, or graphical information
- Analyze, approximate, and solve linear equations
- Model situations with inequalities expressed as “at most” and “at least” situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships
- Compare inverse variation relationships with linear relationships

Data Analysis Measure variation in data and strength of association in bivariate data
- Use data patterns to make predictions
- Fit a line to data that show a linear trend and measure goodness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear relationship between the two variables.
- Use correlation coefficients informally to describe the strength of the linear relationship illustrated by scatter plots.
- Distinguish between categorical and numerical variables.
- Use 2-way tables and analysis of cell frequencies and relative frequencies to help in deciding whether two categorical variables are related.
- Use standard deviation to measure variability in data distributions
### 7-8 Samples and Populations: Focus Questions (FQ) and Mathematical Reflections

<table>
<thead>
<tr>
<th>Investigation 1</th>
<th>Investigation 2</th>
<th>Investigation 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem 1.1</strong></td>
<td><strong>Problem 2.1</strong></td>
<td><strong>Problem 3.1</strong></td>
</tr>
<tr>
<td>Comparing Performances: Using Center and Spread</td>
<td>Asking About Honesty: Using a Sample to Draw Conclusions</td>
<td>Solving an Archeological Mystery: Comparing Samples Using Box Plots</td>
</tr>
<tr>
<td>FQ: Given a set of results, how might you use measures of center and variability (spread) to judge overall performance?</td>
<td>FQ: What is a population? What is a sample? What is a sampling plan?</td>
<td>FQ: How might you analyze samples from known and unknown populations to determine whether the unknown population has one or more attributes in common with the known population?</td>
</tr>
<tr>
<td>Problem 1.2</td>
<td>Problem 2.2</td>
<td>Problem 3.2</td>
</tr>
<tr>
<td>Which Team Is Most Successful? Using the MAD to Compare Samples</td>
<td>Selecting a Sample: Different Kinds of Samples</td>
<td>Comparing Heights of Basketball Players: Using Means and MADs</td>
</tr>
<tr>
<td>FQ: What strategies might you use to evaluate numerical outcomes and judge success?</td>
<td>FQ: How could you select a sample of your school population to survey?</td>
<td>FQ: How can you determine whether differences in sample data are large enough to be meaningful, or just due to naturally occurring variability from one sample to another?</td>
</tr>
<tr>
<td>Problem 1.3</td>
<td>Problem 2.3</td>
<td>Problem 3.3</td>
</tr>
<tr>
<td>Pick Your Preference: Distinguishing Categorical Data From Numerical Data</td>
<td>Choosing Random Samples: Comparing Samples Using Center and Spread</td>
<td>Five Chocolate Chips in Every Cookie: Using Sampling in a Simulation</td>
</tr>
<tr>
<td>FQ: How might you compare results to see if each sample responded to a survey in a similar way? How can using percentages help you make comparisons?</td>
<td>FQ: How could you use statistics of a random sample of data to make predictions about an entire population?</td>
<td>FQ: How can you simulate a real-world problem? How can you analyze the data that you collect from that simulation to draw conclusions?</td>
</tr>
<tr>
<td>Problem 1.4</td>
<td>Problem 2.4</td>
<td>Problem 3.4</td>
</tr>
<tr>
<td>Are Steel-Frame Coasters Faster Than Wood-Frame Coasters? Using the IQR to Compare Samples</td>
<td>Growing Samples: What Size Sample to Use?</td>
<td>Estimating a Deer Population: Using Samples to Estimate the Size of a Population</td>
</tr>
<tr>
<td>FQ: How might you decide whether steel-frame coasters or wood-frame coasters are faster?</td>
<td>FQ: Can you make good statistical estimates with less work by selecting smaller samples? How does sample size relate to the accuracy of statistical estimates?</td>
<td>FQ: How can you estimate the size of a large population?</td>
</tr>
</tbody>
</table>

### Mathematical Reflections

**Investigation 1**

1a. A new term is used in this Investigation: sample. What do you think sample means?

1b. Suppose you have data from a 7th-grade class. The data are answers to the questions:
   - What is your favorite movie?
   - How many movies do you watch per week?
   - Which statistic can you use to summarize the results of the data?
   - How could you use the data to predict the number of students in the entire 7th-grade who would say they watch two movies per week?

2a. How do graphs of distributions help you compare data sets?

2b. How do measures of center help you compare data sets?

2c. How do measures of spread help you compare data sets?

3. When does it make sense to compare groups using counts, or frequencies? When does it make sense to compare groups using percents, or relative frequencies? Explain.

**Investigation 2**

1. Why are data often collected from a sample rather than from an entire population?

2. Describe four plans for selecting a sample from a population. Discuss the advantages and disadvantages of each plan.

3a. How are random samples different from convenience, voluntary-response, and systematic samples?

3b. Why is random sampling preferable to the other sampling plans?

3c. Describing three plans for selecting a random sample from a given population. What are the advantages and disadvantages of each plan?

4. Suppose you select several random samples of size 30 from the same population.

4a. When you compare the samples to each other, what similarities and differences would you expect to find among the measures of center and spread?

4b. When you compare the samples to the larger population, what similarities and differences would you expect to find among the measures of center and spread?

5. How has your idea of the term sample changed from what you wrote in Mathematical Reflections, Investigation 1?

**Investigation 3**

1a. How can you use statistics to compare samples? How can you use samples to draw conclusions about the populations from which they are selected?

1b. In what ways might a data distribution for a sample be similar to or different from the data distribution for the entire population?

2a. How can you use box plots, medians, and IQRs to compare samples? Give an example.

2b. How can you use means and MADs to compare samples? Give an example.

2c. How can you use statistics to decide whether differences between samples are expected due to natural variability or reflect measurable differences in underlying populations?

3a. How can you use simulations to generate samples?

3b. How can you use data from a capture-tag-recapture simulation to estimate the actual size of a population?

4. The process of statistical investigation involves posing questions, collecting and analyzing data, and making interpretations to answer the original questions. Choose a Problem from this Investigation. Explain how you used the process of statistical investigation to solve the Problem.