Unit Goals

**Exponential Functions** Explore problem situations in which two or more variables have an exponential relationship to each other

- Identify situations that can be modeled with an exponential function
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation
- Represent an exponential function with a table, graph, or equation
- Make connections among the patterns of change in a table, graph, and equation of an exponential function
- Compare the growth/decay rate and growth/decay factor for an exponential function and recognize the role each plays in an exponential situation
- Identify the growth/decay factor and initial value in problem situations, tables, graphs, and equations that represent exponential functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function
- Determine the values of the independent and dependent variables from a table, graph, or equation of an exponential function
- Use an exponential equation to describe the graph and table of an exponential function
- Predict the $y$-intercept from an equation, graph, or table that represents an exponential function
- Interpret the information that the $y$-intercept of an exponential function represents
- Determine the effects of the growth (decay) factor and initial value for an exponential function on a graph of the function
- Solve problems about exponential growth and decay from a variety of different subject areas, including science and business, using an equation, table, or graph
- Observe that one exponential equation can model different contexts
- Compare exponential and linear functions

**Equivalence** Develop understanding of equivalent exponential expressions

- Write and interpret exponential expressions that represent the dependent variable in an exponential function
- Develop the rules for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation
### 8-3 Growing, Growing, Growing: Focus Questions (FQ) and Mathematical Reflections

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| | FQ: In what ways are the relationships represented in a chessboard and ballot-cutting situations similar? Different? | Problem 3.3 Making a Difference: Connecting Growth Rate and Growth Factor | Problem 4.2 Fighting Fleas: Representing Exponential Decay | FQ: How are the rules for integral exponents related to rational exponents? How are the rules for exponents useful in writing equivalent expressions with exponents?
| | Problem 2.1 Killer Plant Strikes Lake Victoria: y-Intercepts Other Than 1 | Problem 3.4 Making a New Offer: Growth Factors | Problem 4.3 Cooling Water: Modeling Exponential Decay | Problem 5.4 Operations with Scientific Notation |
| | FQ: What are several rules for working with exponents and why do they work? | FQ: How is the growth factor in this Problem similar to that in the previous Problems? How is it different? | FQ: How can you find the initial population and decay factor for an exponential decay relationship? |
| | Problem 2.2 Growing Mold: Interpreting Equations for Exponential Functions | Problem 3.5 Making a Difference: Connecting Growth Rate and Growth Factor | Problem 5.5 Revisiting Exponential Functions | FQ: How does scientific notation help to solve problems? |
| | FQ: How is the growth factor and initial population for an exponential function represented in an equation that represents the function? | FQ: How does the difference show up in a table, graph, and equation? | FQ: What are the effects of a and b on the graph of y = a(b^x), b ≠ 0. |
| | Problem 2.3 Studying Snake Populations: Interpreting Graphs of Exponential Functions | Problem 3.6 Making a Difference: Connecting Growth Rate and Growth Factor | Problem 5.6 Revisiting Exponential Functions | FQ: How can you use a table, a graph, and an equation that represent an exponential function to find the y-intercept and growth factor for the function? Explain. |
| | FQ: How is the growth factor and initial population for an exponential function represented in a graph that represents the function? | FQ: How can you find the initial population and decay factor for an exponential decay relationship? |
| | Problem 3.7 Making a Difference: Connecting Growth Rate and Growth Factor | FQ: How are the growth factor and growth rate for an exponential function related? When might you use each in an exponential growth pattern? |
| | Problem 4.1 Making Smaller Ballots: Introducing Exponential Decay | FQ: How can you recognize an exponential decay function from a contextual setting, table, graph, and equation that represents the function? |
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### Mathematical Reflections

1. Describe an exponential growth pattern. Include key properties such as growth factors.

2. How are exponential functions similar to and different from the linear functions you worked with in earlier Units?

3. Compare exponential and linear functions. Include in your comparison information about their patterns of change, y-intercepts, whether the function is decreasing or increasing, and any other information you think is important. Include examples of how they are useful.

### Mathematical Reflections

1. Suppose you know the initial value for a population and the yearly growth rate. 1a. How can you determine the population several years from now? 1b. How is a growth rate related to the growth factor for the population? 1c. How can you use this information to write an equation that models the situation?

2. Suppose you know the initial value for a population and the yearly growth factor. 2a. How can you determine the population several years from now? 2b. How can you determine the yearly growth rate?

3. Suppose you know the equation that represents the exponential function relating the population p and the number of years n. How can you determine the doubling time for the population?