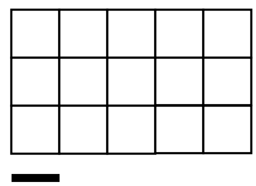
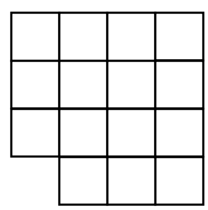
Covering and Surrounding: **Homework Examples from ACE**

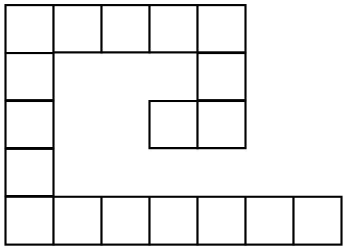
Investigation 1: Questions 5, 8, 21

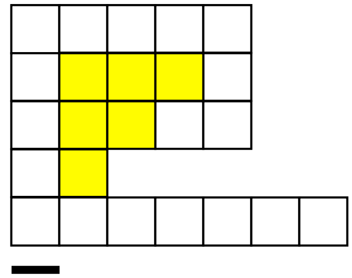
Investigation 2: Questions 6, 7, 11, 27

Investigation 3: Questions 6, 8, 11

Investigation 5: Questions 15, 26

ACE Question	Possible Answer
Inv 1	
5. Draw 2 different shapes, each with an area of 15 square units and perimeter of 16 units.	5. This question reinforces the idea that the relationship between perimeter and area is not simple. The obvious answer is a rectangle with length 5 and width 3.  P = 5 + 3 + 5 +3 = 16. A = 5 x 3 = 15. But if students use their experiment with square tiles they can come up with other, non-rectangular, arrangements of 15 square tiles.  P = 4 + 4 + 3 + 1 + 1 + 3 = 16. Area = 15 square units.
8. Copy the design onto grid paper. Add 6 squares to make a new design with a perimeter of 30 units. Explain how the perimeter changed as you added new tiles to the figure.	8. We need to reduce the perimeter while increasing the area. Students learned that more “compact” figures can cover the same area without increasing the perimeter. This idea means that they have to “fill in” some of the blank space in the center of the shape.



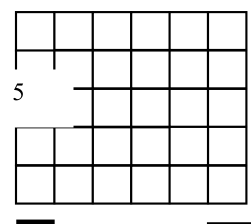


Perimeter, starting where the heavy line segment is drawn = 7 + 1 + 5 + 1 + 3 + 3 + 5 + 5 = 30 units.

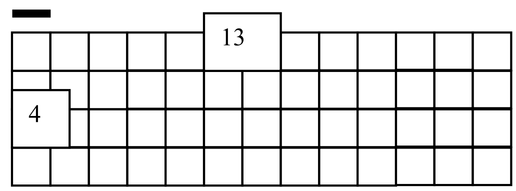
21.
Find the areas and perimeters of the rectangles whose lengths and widths are given in the chart below. Make a sketch of each rectangle and label its dimensions.

Rect	Length	Width	Area	Perimeter
A	5	6		
B	4	13		
C	6.5	8		

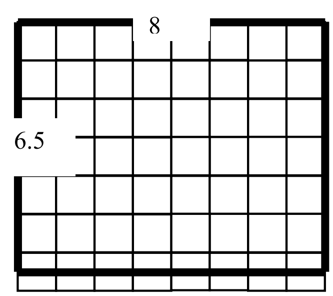
Rect	Length	Width	Area	Perimeter
A	5	6	30	22
B	4	13	52	34
C	6.5	8	52	29



A



B

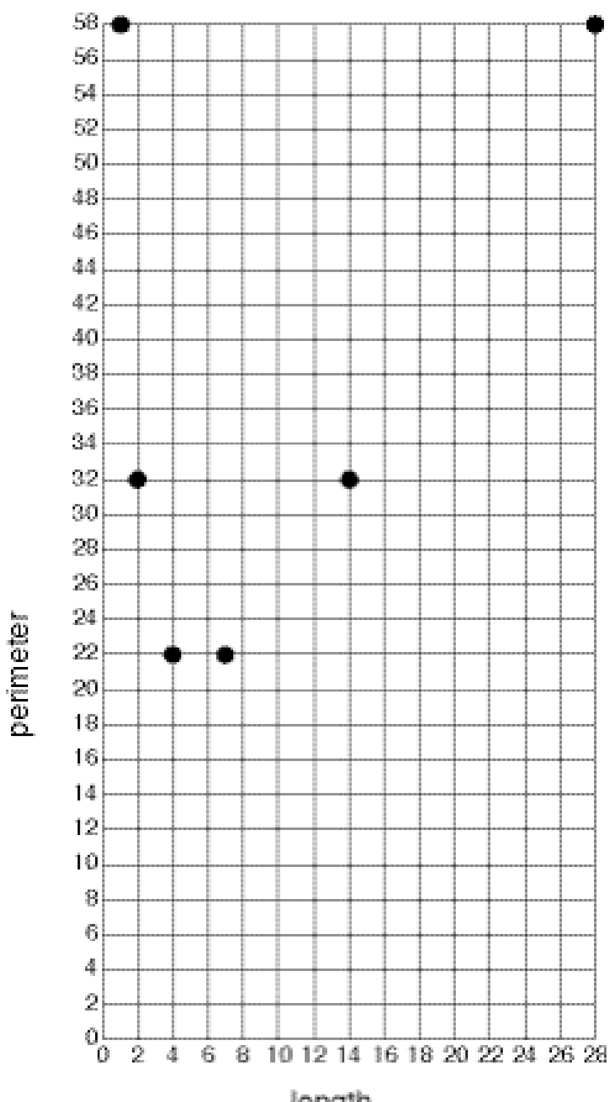


C

Inv 2

6.
The graph at right represents the whole number

6.
Note: students know that for a rectangle

lengths and perimeters for rectangles with a fixed area. a. What is the perimeter of a rectangle whole length is 2 meters? What is its width? b. Describe the rectangle that has the largest perimeter. c. Describe the rectangle that has the smallest perimeter. d. What is the fixed area? Explain how you found the answer. 	A = lw and P = l + w + l + w = 2l + 2w = 2(l + w). a. The point (2, 32) represents a rectangle with length 2 and perimeter 32. So, 2 + w + 2 + w = 32. So w = 14. (Students may also use the fact that l + w = half perimeter, so 2 + w = 16. This makes the solution for w easier.) b. The points (1, 58) and (28, 58) both have the largest perimeter given on this graph. If l = 1 then 1 + w + 1 + w = 58, so w = 28. If l = 28 then 28 + w + 28 + w = 58, so w =1. In both cases the rectangle has dimensions 1 by 28. c. The points (4, 22) and (7, 22) both have the smallest perimeter on this graph. If l = 4 then 4 + w + 4 + w = 22. so w = 7. If l = 7 then 7 + w + 7 + w = 22, so w = 4. In both cases the rectangles have dimensions 4 by 7. d. In part a the rectangle had length 2 and width 14 so area was 28 square units. In part b the rectangle had length 1 and width 28 so area was 28 square units. In part c the area had length 4 and width 7 so area was 28 square units. This graph shows rectangles whose dimensions and perimeters vary but whose areas are all 28 square units.
7. The following 4 by 6 rectangle is drawn on grid paper. Billie started at an edge and cut a path to the opposite corner. She then slid the	7. This question addresses the fundamental idea that "area" is about "covering space." Therefore, if we cut a shape into pieces and

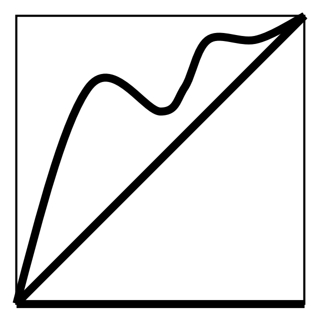
piece onto the opposite edge, making the straight edges match.

Are the area and perimeter of her new figure the same as, less than, or greater than the area and perimeter of the original figure? Explain.

See text

rearrange the pieces we will still cover the same area, though the actual shape looks different.

Students often think that if the areas are the same the perimeters must be the same. The other misconception they have is that the distance across a square is the same no matter how the square is crossed. They often think that a diagonal is the same length as a side of a square. The square below shows 3 bolded "lines" all of different lengths.

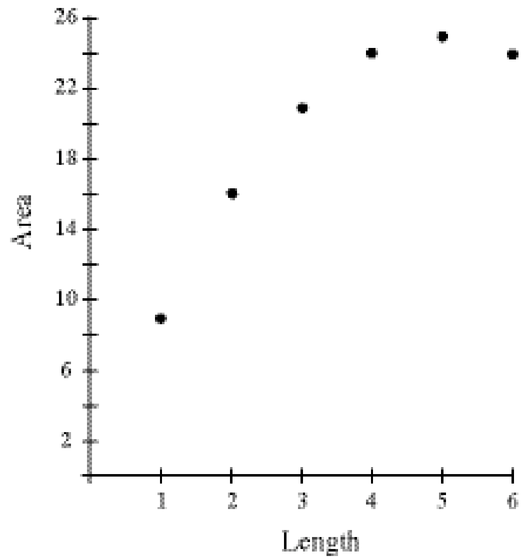


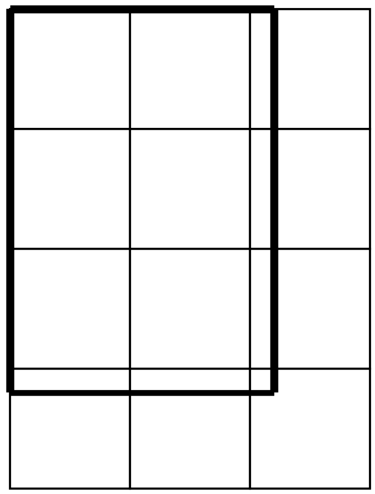
So the resulting shape has a perimeter made of 6 + 6 + 2 curves of unknown length. These curves are each definitely longer than 4 units. So the perimeter has increased.

11.
a. Sketch rectangles with perimeter 20 meters. Record the length, width, area and perimeter in a table.
b. Sketch a graph of the length and area.
c. Describe how to use the table and graph to find the rectangular shape that has the greatest area. The smallest area.

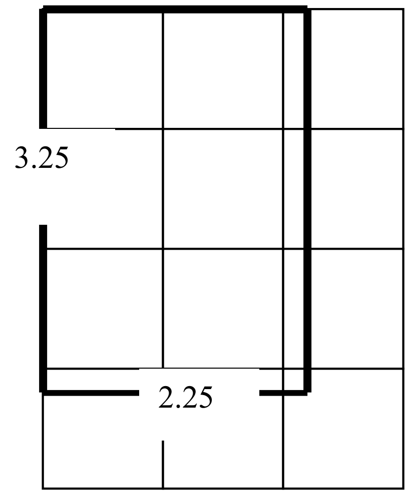
11.
a. If the perimeter is 20 meters then 2(l + w)= 20 so l + w = 10. Some pairs are: (1, 9), (2, 8), (3, 7), (4, 6), (5, 5,) (6, 4) etc.

Length	Width	Perimeter	Area
1	9	20	9
2	8	20	16
3	7	20	21
4	6	20	24
5	5	20	25
6	4	20	24
etc			

	 b. c. The maximum area occurs when length and width are both 5 units. The minimum area occurs when the dimensions are 1 by 9. (If we were permitted to use fractions for lengths of sides then the max would still be 25 but the smallest area would now be impossible to determine. If we choose a fraction less than 1 for the length then the width can still be determined and the area will be less than 9 square units. But we can always find another area even smaller by choosing an even smaller fraction for the length.)
27. a. Find the perimeter and area of the rectangle outlined in bold lines.	a. The area is 2.25 x 3.25 = 7.3125 square units or $7\frac{5}{16}$ square units.



b. Draw another rectangle on grid paper that has the same perimeter as the one above but a different area than the one shown. What is the area of the one you have made? Be sure to label length and width.



The perimeter = 3.25 + 2.25 + 3.25 + 2.25 = 11 units.

b. The equation for perimeter gives a strategy for a way to change the dimensions but retain the total 11 units.

3.25 + 2.25 + 3.25 + 2.25

= 3.5 + 2 + 3.5 + 2 (adding 0.25 to the length and subtracting 0.25 from the width)

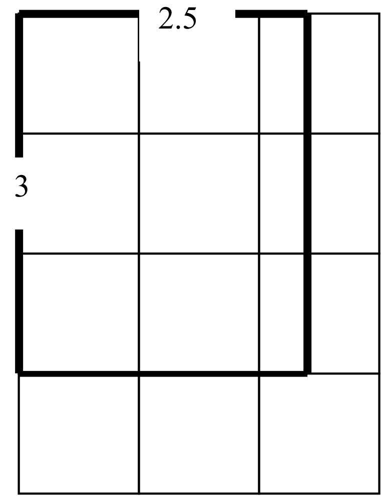
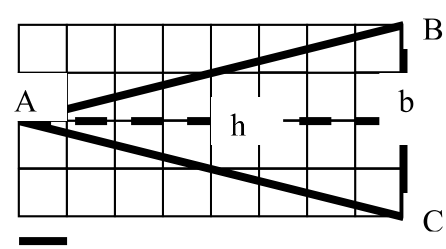
= 3.75 + 1.75 + 3.75 + 1.75

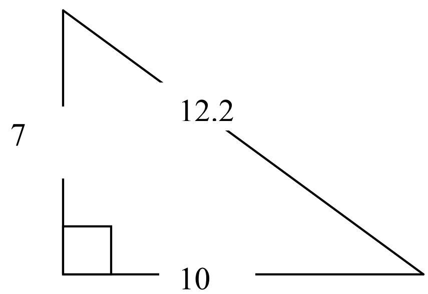
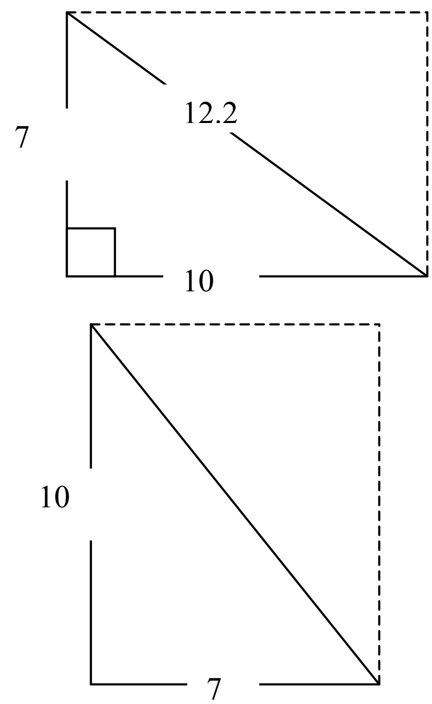
= 4 + 1.5 + 4 + 1.5

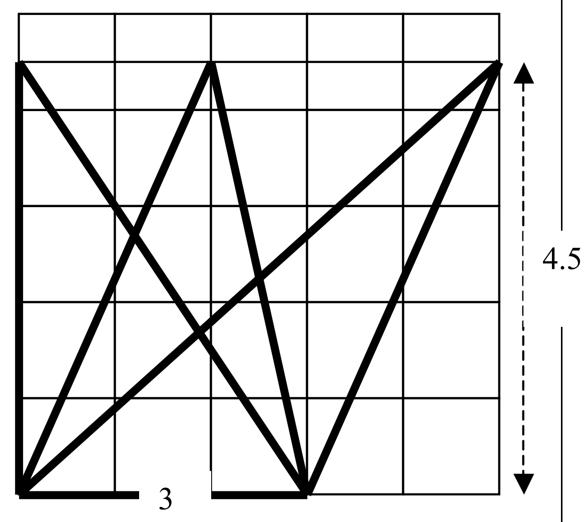
= 4.25 + 1.25 + 4.25 + 1.25

etc.

So some other rectangles with the same perimeter are: 3.5 by 2, 3.75 by 1.75, 4 by 1.5, 4.25 by 1.25 etc. One of these is drawn below.

	 Area of the example drawn = 3 x 2.5 = 7.5 square units.
Inv 3	
6. Calculate the area and perimeter of the triangle and explain your reasoning.	6. Students have several ways to think about the area of the triangle. They might count whole square units covered and then estimate the area covered by the partial squares. Or they might surround this with a 4 by 8 rectangle and observe that the triangle is half of the rectangle. Or they might use the rule that Area of triangle = ($\frac{1}{2}$)(base)(height) and use the base and height shown on the picture below.  Area = 0.5(4)(8) = 16 square units. Perimeter = 4 + length of AB + length of AC. The problem we have with the lengths of aB and AC is that they do not lie on grid lines, and so have to be measured or estimated

	using the edge of a grid square as a unit. Each is approximately 8.25 units long. (In a later unit students learn how to use the Pythagorean Theorem to find an accurate answer for these lengths.) Perimeter = 4 + 8.25 + 8.25 = 20.5 units.
8. Vashan said that if you used 7 feet as the base of the triangle shown below then you would calculate the same area as you did when you used the10 feet base. Do you agree with him? 	8. Vashan is correct. It does not matter which side of a triangle we choose as the base, as long as we then choose as the height the distance from the base to the opposite vertex. The triangle is half of the same 7 by 10 rectangle no matter the orientation. 
11. Melissa was finding the area of a triangle when she wrote: Area = $(\frac{1}{2}) \times 3 \times (4\frac{1}{2})$ a. Make a sketch of a triangle she might have been working with. b. What is the area of the triangle?	11. Apparently Melissa is using a base of 3 and a height of 4.5 for her triangle. But there are many triangles she might be working with. The key is to make the height be the perpendicular distance from the base to the opposite vertex. Shown below are several different triangles with the same base and height (and, therefore, the same area.)



Inv 5

15.

Best Crust Pizzaria sells three different sizes of pizza. The small size has a radius of 4 inches, the medium size has a radius of 5 inches, and the large size has a radius of 6 inches.

a. Make a table with the following headings. Fill in the table and explain how you found the area of the pizzas.

Pizza Size	Diameter	Radius	Circum	Area
Small				
Medium				
Large				

b. Sam claims that the area of the pizza is about $0.75(\text{diameter})^2$. is he correct?

15.

a.

Pizza Size	Diameter	Radius	Circum	Area
Small	8	4	3.14(8) = 25.12	$3.14(4^2)$ = 50.24
Medium	10	5	3.14(10) = 31.4	$3.14(5^2)$ = 78.5
Large	12	6	3.14(12) =37.68	$3.14(6^2)$ = 113.04

The above calculations use 3.14 as an approximation for pi.

b. Using the answers for area from the above table and making comparisons we have:

Pizza Size	Area	$0.75(\text{diameter})^2$
Small	50.24	$0.75(8^2)$ = 48
Medium	78.5	$0.75(10^2)$ = 75
Large	113.04	$0.75(12^2)$ = 108

As you can see the estimates using Sam's formula are a little low. This makes sense

	since the formula used in the first table for area = π(radius)^2. Radius = $\frac{\text{diameter}}{2}$ so we could replace this in the formula and have Area = $\pi(\frac{\text{diameter}}{2})^2 = \frac{\pi}{4}(\text{diameter})^2$. Now $\frac{\pi}{4}$ is more than $\frac{3}{4}$ and so Sam's formula will always give a lower estimate.
26. A rectangular lawn has a perimeter of 36 meters and a circular exercise run has a circumference of 36 meters. Which shape do you think will give Rico's dog the most area to run?	26. This question refers back to the idea in earlier investigations that two shapes with the same perimeter do not necessarily have the same area. Students discovered that, if only rectangles are compared, that the more "square" a rectangle (that is the closer the ratio of sides is to 1:1) the more area it can enclose for a given perimeter. So in this case the "best" rectangle they can make is 9 × 9. (9 + 9 + 9 + 9 = 36) Now we have to do some reasoning with the formula for the circumference. C = pi(diameter) 36 = pi(diameter) So, diameter = $\frac{36}{\pi}$ = 11.5 (approx). So the radius must be 5.75. Now that we know the radius we can figure the area = $\pi(5.75)^2$ = 103.8 (approx). So a circle with circumference 36 meters covers more area than a square with perimeter 36 meters.