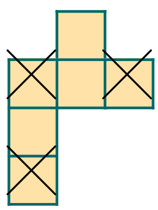
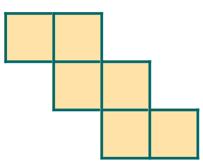
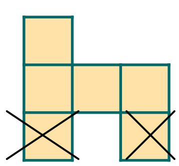
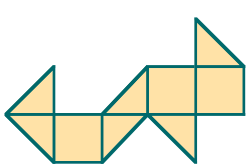
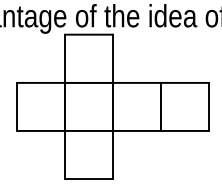
Homework Answers from ACE: *Filling and Wrapping*
ACE Investigation 1: #1 – 4, 10 – 13.
ACE Investigation 2: #4, 22.
ACER Investigation 3: #4 – 6, 13, 19.
ACE Investigation 4: #15, 25, 32.
ACE Investigation 5: #5 – 7, 10.

ACE Question	Possible Answer
ACE Investigation 1	
In Exercises 1–4, tell whether the flat pattern could be folded along the lines to form a closed cubic box. If you are unsure, cut the pattern out of grid paper and experiment. 1.  2.  3.  4. 	This question gives students some practice with visualizing the manipulation of the flat pattern to make the 3-dimensional shape. (They have the opportunity to make this a concrete activity.) It also reinforces the concept that there are many ways to make flat patterns for the same cube. The usual cube or rectangular prism pattern takes advantage of the idea of opposite faces.  1. Does not work because there are two “opposite” faces for the left face (marked with a large X). The furthest right face on the pattern and the face at the bottom of the pattern will overlap. 2. Will work (though it may take a concrete model to convince students). 3. Does not work. Again 2 faces overlap (these are marked with a large X.). 4. Does work. Students will probably have to cut this out to convince themselves that this does work. It is difficult to keep track visually of which face is the bottom, front etc.

10 - 13. Several boxes are described in Exercises 10 -13. For each box

- Make a sketch of the box and label the dimensions.
- Draw a net (flat pattern).
- Find the area of each face.
- Find the total area of all the faces.

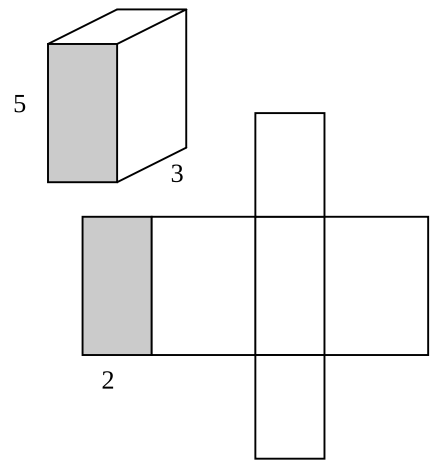
10. a rectangular box with dimensions 2 cm by 3 cm by 5 cm.

11. a rectangular box with dimensions $2\frac{1}{2}$ cm by 2 cm by 1 cm.

12. a cubic box with side length $3\frac{2}{3}$ cm.

13. a cubic box that holds 125 unit cubes.

10. 2 by 3 by 5 rectangular prism.
Note: there are different ways to draw correct patterns for this prism. The front face is shaded on the sketch and on the net to make this easier to compare. If students draw a different sketch they will name the faces differently (“top” etc.)



Top and bottom: 6 sq cm.
Front and back: 10 sq cm.
Left and right side: 15 sq cm.
Total: 62 sq cm.

11. Diagram as above but with different dimensions.
Top and bottom: (2.5 x 2) = 5 sq cm.
Front and back: (2.5 x 1) = 2.5 sq cm.
Left and right: (2 x 1) = 2 sq. cm.
Total area = 19 square cm.

12. Diagram as above but with different dimensions.
Each face has the same area, $3\frac{2}{3}$ x $3\frac{2}{3}$ sq. cm.
So total area = $6(3\frac{2}{3} \times 3\frac{2}{3}) = 80\frac{2}{3}$ square cm.

13. This question asks student to work backwards. If the volume is 125 cubic units, and if the box is a cube,

	then all edges are equal, say *b* units, and *b x b x b* = 125. So each dimension is 5 units. Therefore, there are six 5 by 5 faces. Area of each face is 5 x 5 = 25 square units. The total area = 6(25) = 150 square units.
ACE Investigation 2	
4. Suppose you want to make a box to hold exactly forty 1-inch cubes. a. Give the dimensions of all the possible boxes you could make. b. Which box has the least surface area? Which has the greatest surface area? c. Why might you want to know the dimensions of the box with the least surface area?	4. For this problem students must visualize a box and decide how many cubes are on the base layer, and how many layers high the box is. Since we are talking about whole unit cubes we must look for factors of 40; one factor is the base layer and the other factor is the height, Base layer x Height = 40. Also, Length x Width = Area of Base Layer. There may be more than one choice for Length and Width for a given Base Layer. For example, we know that 5 x 8 = 40, so the base layer could have 5 cubes and the height could be 8 inches. If the base layer has 5 inch cubes then the length must be 5 inches and the width must be 1 inch (or vice versa). On the other hand, we also know that 10 x 4 = 40, so the base layer could be 10 inch cubes and the height could be 4 inches. If the base layer has 10 inch cubes then the length might be 10 inches and the width 1 inch, or the length might be 5 inches and the width 2 inches. a.

Base	L	W	H	V	SA
1	1	1	40	40	162
2	1	2	20	40	124
4	1	4	10	40	108
4	2	2	10	40	88
5	1	5	8	40	106
8	1	8	5	40	
8	2	4	5	40	76

8	2	4	5	40	76
10	**1**	**10**	**4**	**40**	
10	2	5	4	40	
etc					

If we examine this table we will see that some of the boxes created are identical.

L = 1, W = 4, H = 10 will be exactly the same box as L = 1, W = 10 and H = 4. There are only 6 different arrangements. The Surface areas of these have been calculated above.

b. The 2 x 4 x 5 box has the least surface area. The 1 x 1 x 40 box has the greatest surface area.

c. The material for making the box might be expensive. If so, we would want to know the least expensive way to make a box that holds a volume of 40 cubic units. Or we might be concerned with heat loss (or gain). A box that has the least area exposed to the air will be least affected by the air temperature.

22.

a. Each of the boxes you designed in Problem 2.1 had a rectangular base and a height. Use a coordinate graph to show the relationship between the area of the base and the height of each box.

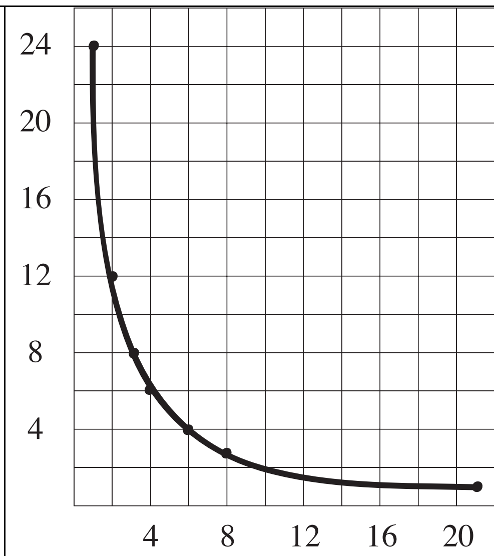
b. Describe the relationship between the height and the area of the base in words.

c. How might your graph be useful to the packaging engineer at ABC Toy Company?

22.

a. For each box, Base x Height = 24. Or Height= $\frac{24}{base}$. Using x for the Base and y for the Height we have y = $\frac{24}{x}$. Some possible values for x and y are shown in this table.

X (Base)	1	2	3	4	6	8	21
Y (Height)	24	12	8	6	4	3	1



b. As the base gets larger the height decreases, but not at a constant rate. For lesser base values a small change in base size makes a big change in the height. For large base values a large change in base size makes a small change in height.

c. The engineer might examine points in the middle of the graph if he/she is interested in having both a small base and a small height, because this would make a smaller total surface area.

ACE Investigation 3

4 - 6. For each of the following problems, decide whether you have found

- a volume,
- a surface area, or
- an area,

and identify whether the computation relates to figure 1, 2, or 3.

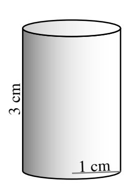


fig. 1

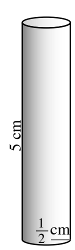


fig. 2

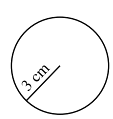
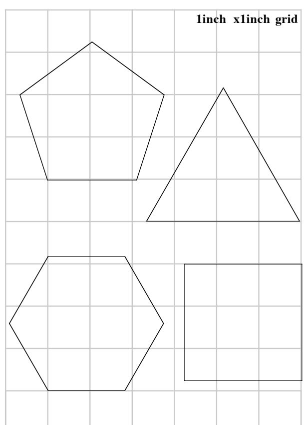


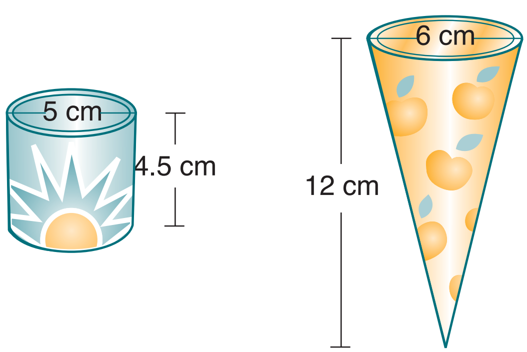
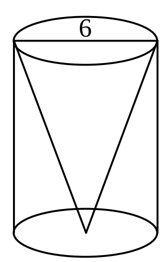
fig. 3

4 - 6. This problem helps students to develop a sense about symbolic expressions, an essential skill for algebraic reasoning. Volume expressions will have 3 variables multiplied (for 3 dimensions). Expressions for area will have 2 variables multiplied (for 2 dimensions). An expression for a total area will be a sum of partial areas, so an addition sign will appear. An expression for a length will have 1 variable (1 dimension). Expressions that involve π may be the area or circumference of a circle.

4. This expression is the sum of 2 parts. The first expression has a factor repeated, the . This makes one think

4. $(\frac{1}{2}\times\frac{1}{2}\times\pi\times 2) + (2\times\frac{1}{2}\times\pi\times 5)$ **5.** $3\times 3\times\pi$ **6.** $1\times 1\times\pi\times 3$	repeated, the _. This makes one think of "A = πr^2", where $r = \frac{1}{2}$. The tall cylinder has a radius of $\frac{1}{2}$. So this part of the expression could be the area of the 2 circular bases. The second part of the expression also has the same factor of $\frac{1}{2}$, but not repeated; this makes one think of "C = $2\pi r$". This part of the expression is the lateral area of the tall cylinder; A = (Circumference)(height). Therefore, this expression is the total surface area of the tall cylinder. 5. This has a single factor repeated and a "π." This is the area of the large circle. 6. This expression has a factor repeated, the "1." This makes one think of "A = πr^2." But there is another factor also. The "3" is the height of the shorter cylinder. Thus, we have a base layer of "1 x 1 x π" multiplied by 3 layers high. This is the volume of the shorter cylinder.
13. **a.** Will all rectangular prisms with the same height and area of the base have the same shape? Explain. **b.** Will all cylinders with the same height and area of the base have the same shape? Explain.	13. a. All rectangular prisms with the same height and base will have the same volume. V = base area x H. BUT the same rectangular base area can be achieved in several different ways, because the area of a rectangle depends on both length and width. For example, a base of 8 square units might be 1 x 8 or 2 x 4. Equal areas do not imply congruent rectangles. (See *Covering and Surrounding*.) Therefore, the prisms might have rectangular bases which look quite different. b. If two cylinders have the same base area and height then their volumes are the same. *V = base area x H*. If the base areas are equal then the circles that make the bases have the same area. The area of a circle depends on

	only the value of the radius. Therefore, if the areas of the circles are equal then the radii must be equal, so the circular bases must be congruent. The cylinders will be congruent, that is exactly the same size and shape.
19. The bases of the prisms for Problem 3.1 are given below. Each prism has a height of 8.5 inches. **a.** Compute the volume of each prism. **b.** Compare these volumes to those you found in Problem 3.1.  	19. Each of the bases below has the same perimeter, because they were all made by folding the same piece of paper, 11 inches long. This refers back to an idea from *Covering and Surrounding*; equal perimeters do not imply equal areas. For every prism the formula is ***volume = base area x height.*** The height is 8.5 inches for each of these prisms because we used an 8.5 x 11 inch piece of paper for each. a. • The square has sides of $\frac{11}{4}$ or 2.75 inches. Therefore, the base area = 7 $\frac{9}{16}$ square inches. The volume = (7 $\frac{9}{16}$) x 8.5 = 64.28125 or 64 $\frac{9}{32}$ cubic units. • The triangle has a base of $\frac{11}{3}$ units. The height appears to be about 3.2 units. (Students do not yet know how to calculate this height using Pythagoras' Theorem.) Therefore, the base area = ($\frac{1}{2}$)($\frac{11}{3}$)(3.2) = 5.87 square units (approx). The volume = (5.87) x (8.5) = 49.9 cubic units. • The pentagonal base is made of a 2 x 2 square + 3 triangles surrounding this square. The areas of the triangles have to be approximated using a method from *Covering and Surrounding*. The sum of the areas of

	these triangles = (0.5)(3.5)(1.25) + (0.5)(2.5)(0.75) + (0.5)(2.5)(0.75) = 4.0625 square units (approx). The base area is therefore 8.0625 square units. The volume = (8.0625)(8.5) = 68.53 cubic units (approx). • The hexagon is made of 6 small equilateral triangles each with side $\frac{11}{6}$ units. (See *Shapes and Designs*.) Again, students will have to approximate the height of each of these as about 1.6 units. The area of each small triangle is therefore (0.5)($\frac{11}{6}$)(1.6) = 1.47 square units (approx). The total base area = 6(1.47) = 8.8 square units approx. The volume = (8.8)(8.5) = 74.8 cubic units approx. *Note: as the same 11 inches is folded to make more and more sides, the base area increases and, therefore, the volume increases.*
ACE Investigation 4	
15. The track-and-field club is planning a frozen yogurt sale to raise money for new equipment. The club needs to buy containers to hold the yogurt. They must choose between the cup and cone shown below. Each container costs the same amount of money. The club plans to charge customers $1.25 for a serving of yogurt. Which  container should the club buy? Why?	15. Students have a formula for the volume of a cone; they found that a cone was one third the volume of a cylinder with the same base and height.  Volume of cylinder = base area x height = π (3^2) x 12 = 339.3 approx. So volume of cone = 113.1 cubic cm. The volume of the cylindrical cup is

	$\pi(2.5^2)(4.5)$ = 88.36 cubic cm. So, the cone is the better buy from the customers' point of view (you get more volume of yogurt for the same $1.25). But, from the club's point of view the cup holds less and so generates more profit (customers pay the same but get less yogurt).
25. Chilly's Ice Cream Parlor is known for its root beer floats. • The float is made by pouring root beer over 3 scoops of ice cream until the glass is filled $\frac{1}{2}$ inch from the top. • A glass is in the shape of a cylinder with a radius of $1\frac{1}{4}$ inches and is $8\frac{1}{2}$ inches tall. • Each scoop of ice cream is a sphere with a radius of $1\frac{1}{4}$ inches. Will there be more ice cream or more root beer in the float? Explain your reasoning.	25. *There are two very different ways to think about this problem. One is to compute the volume for the cylinder-shaped float and subtract the volume of the three spheres of ice cream. The other is to visualize the relationships amongst the cylinder and spheres. The computational method is shown first below.* • The first 2 clues tell us that the total volume of the float is the volume of a cylinder, with height 8 inches and radius 1.25 inches. V = π (1.25^2)(8) = 39.27 cubic ins. • The third clue tells us that the volume of the ice cream = 3(volume of a sphere, with radius 1.25 inches). Students found that the volume of a sphere = ($\frac{2}{3}$)(volume of a cylinder with the same height and radius.) **Notice that the height of a sphere = a diameter,** which in this case = 2.5 inches. So in this case, each sphere of ice cream = ($\frac{2}{3}$)(volume of a cylinder with radius 1.25 and height 2.5) = ($\frac{2}{3}$)(π)(1.25^2)(2.5) = 8.18 cubic inches. Therefore, three scoops have a volume of 24.54 cubic inches. • Subtracting the volume of the ice cream from the total volume of the float we have the volume of the root beer = 39.27 – 25.54 = 14.72 cubic inches. Thus, there is more ice cream than root beer in the

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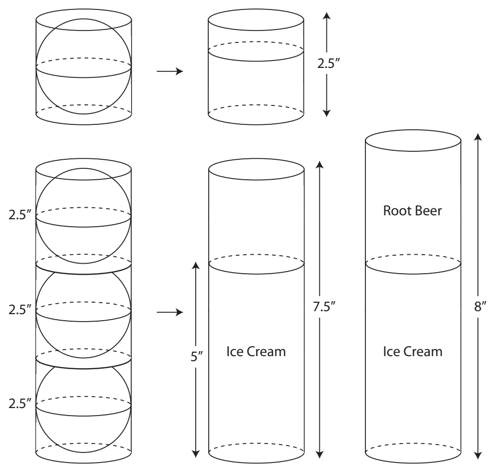
<td>Drawing for alternative method (see explanation to right:


</td>
<td>ice cream than root beer in the float.

Alternative method:
If ONE sphere of ice cream = ($\frac{2}{3}$) of a cylinder with same radius and height, then THREE scoops will have the volume of 2 cylinders with same radius and height. Therefore, 3 scoops of ice cream have the same volume as 2 cylinders with radius 1.25 and height 2.5 inches, or the same as ONE cylinder with radius 1.25 and height 5 inches. The float in question is a cylinder with radius 1.25 and height 8 inches. If 5 inches of this height is packed with ice cream then only 3 inches space is left for the root beer. (See drawing to left)</td>
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<td>32. Kaiya measures the circumference of a sphere and finds that it is 54 cm. What is the volume of the sphere?</td>
<td>32. This question asks students to work backwards. The circumference is known. From that we can deduce the radius. And from the radius we can deduce the volume.

The circumference of a sphere (or circle) = 2 π r = 54 cm. Therefore, $r = \frac{54}{(2\pi)}$ = 8.6 cm (approx).

The volume of a sphere = ($\frac{2}{3}$)(volume of a cylinder with radius 8.6 and height 17.2) = ($\frac{2}{3}$)(π)(8.6^2)(17.2) cubic cm = 2664.3 cubic cm (approx).</td>
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<td>ACE Investigation 5</td>
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<td>5 – 7.
• For each pair of cylinders, decide if they are similar.
• For each pair of similar cylinders describe how many times larger one is than the other.

5. Cylinder 1: height is 10 cm and radius is 5 cm
Cylinder 2: height is 5 cm and radius is 2.5 cm</td>
<td>5. Students investigated similarity in Stretching and Shrinking and in Comparing and Scaling. From these units they know that when two shapes are similar the ratio of corresponding lengths is constant. This ratio is the scale factor. If we call the scale factor K, then they also know that when</td>
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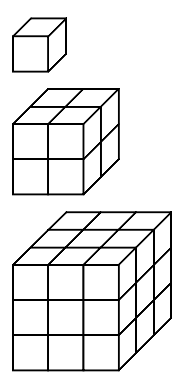
6. Cylinder 1: height is 10 cm and radius is 5 cm Cylinder 2: height is 30 cm and radius is 15 cm **7.** Cylinder 1: height is 10 cm and radius is 5 cm Cylinder 2: height is 15 cm and radius is 10 cm	lengths are multiplied by a factor of K this causes the areas to be multiplied by a factor of K^2. In *Filling and Wrapping* they extend this idea to 3 dimensions. If lengths are multiplied by a factor of K then volumes are multiplied by a factor of K^3. If they have not grasped this from the investigation of scaling up boxes they can calculate the volumes in this question and arrive at the same conclusion. 5. All lengths in cylinder 2 are 0.5 times the corresponding lengths in cylinder 1. Therefore, the cylinders are similar, and the *scale factor is 0.5*. The volume of cylinder 1 = $\pi(5^2)(10)$ = 785.4 cubic cm. The volume of cylinder 2 = $\pi(2.5^2)(5)$ = 98.2 cubic cm. Cylinder 2's volume = $(\frac{1}{8})$ the volume of cylinder 1. (Note: $\frac{1}{8} = (0.5)^3$.) 6. Cylinder 2 lengths are all 3 times the corresponding lengths in cylinder 1. Therefore the cylinders are similar, the scale factor is 3, and the volume of cylinder 2 = 27 times the volume of cylinder 1. (Note: $27 = 3^3$.) 7. The height of cylinder 2 is 1.5 times the height of cylinder 1, but the radius of cylinder 2 = 2 times the radius of cylinder 1. Therefore, corresponding lengths are NOT in the same ratio. So, the cylinders are NOT similar.
10. One cube has edges measuring 1 ft. A second cube has edges measuring 2 ft. A third cube has edges measuring 3 ft. **a.** Make scale drawings of the three cubes. For each cube, tell what length in the drawing represents 1 ft. **b.** Find the surface area of each cube. **c.** Describe what happens to the surface area of a cube when the edge lengths are doubled, tripled, quadrupled, and so on.	10. a. 

	b. • Total area = 6 faces each 1 square unit = 6 square units. • Total area = 6 faces each 4 square units = 24 square units. • Total area = 6 faces each 9 square units = 54 square units. c. When the edge length is doubled the surface area is 4 times as large. When the edge length is tripled the surface area is 9 times as large. Whatever the scale factor that multiplies the lengths, that factor is used twice in the calculation of a face area. Therefore, if the lengths are quadrupled the surface area will be multiplied by 16, and so on. ***In general, if the lengths are multiplied by a factor of K the surface area will be multiplied by a factor of K^2.***