Growing, Growing, Growing: Homework Examples from ACE

Investigation 1: #13, 23, 39.
Investigation 2: #4.
Investigation 3: #17.
Investigation 4: #7, 12.
Investigation 5: #5, 27, 50.

ACE Question	Possible Answer
ACE Investigation 1	

ACE Question

13.

Decide whether this number is greater or less than one million, without using a calculator. Try to decide without actually multiplying. Explain how you found your answer. Use a calculator to check.

3^{10}

Possible Answer

13.

This question focuses on the meaning of exponential notation. There are several ways students might think of this. One way is:

3^{10} means 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3 x 3.
One million is 10 x 10 x 10 x 10 x 10 x 10.
If we group 3^{10} as (3 x 3) x (3 x 3) x (3 x 3) x (3 x 3) x (3 x 3), we see that this is the same as 9^5 or 9 x 9 x 9 x 9 x 9.
Now the comparison is clearer:
10 multiplied by itself 6 times is clearly greater than 9 multiplied by itself 5 times.

Checking by calculator: 3^{10} = 59049.

ACE Question

23.

Zak's wealthy uncle wants to donate money to Zak's school for new computers. He suggests three possible plans for his donations.

Plan 1: He will continue this pattern until day 12.

Day	1	2	3	4
Donation	1	2	4	8

Plan 2: He will continue this pattern until day 10.

Day	1	2	3	4
Donation	1	3	9	27

Plan 3: He will continue this patterns until day 7.

Day	1	2	3	4
Donation	1	4	16	64

a. Copy and extend each table to show how much money the school would receive each day.

Possible Answer

23.

a. It is important that students recognize that these are all exponential patterns: the size of the donation grows by *multiplying* by a given factor each day. The factors are 2, 3, 4 respectively.

Day	1	2	3	….	12
Donation	1	2	4	….	2048

Day	1	2	3	…	10
Donation	1	3	9	…	19683

Day	1	2	3	….	7
Donation	1	4	16	….	4096

b. $d = 2^{n-1}$; $d = 3^{n-1}$; $d = 4^{n-1}$.
Students may have difficulty with the exponent. It is clear that the donations in table 1, for example, are all powers of 2. The problem is that the exponent does not exactly match the day number.

b. For each plan, write an equation for the relationship between the day number n and the number of dollars donated d.
c. Which plan would give the school the greatest total of money?
d. Zak says there is more than one equation for the relationship in plan 1. He says that $d = 2^{n-1}$ and $d = 0.5(2^n)$ both work. Is he correct? Are there two equations for each of the other plans?

Thus, on day 4 the donation is 2^3 or 8, not 2^4. Students met the same pattern when they investigated the "ruba" problem in class.

c.

d. Shown below is a table in which $d = 0.5(2^n)$.

Day	1	2	3	….	12
Donation	1	2	4	….	2048

It appears that this equation gives the same answers as $d = 2^{n-1}$. Just checking for matches for 12 days is not absolutely conclusive. Students should ask themselves WHY these two equations give the same results. What is the effect of the initial "0.5"? This divides each of the results by 2, effectively making the same result as multiplying by one fewer factor of 2. Following that idea students should be able to produce equations for the other two plans.

39.

Sarah used her calculator to keep track of the number of rubas in Problem 1.2. She found that there will be 2,147,483,648 rubas on square 32.

a. How many rubas will there be on square 33? On square 34? On square 35?
b. Which square would have the number of rubas shown here?
2,147,483,648 • 2 • 2• 2• 2• 2• 2• 2• 2• 2
c. Use your calculator to do the multiplication in part b. Do you notice anything strange about the answer your calculator gives? Explain.
d. Calculators use shorthand notation for showing very large numbers. For example, if you enter 10^ 12 on your calculator, you may get 1E12. This is shorthand for the number 1.0×10^{12}. The number 1.0×10^{12} is written in scientific notation. For a number to be in scientific notation it must be in the form:

(a number greater than 1 but less than 10) x (a power of 10).

Write

39.

Students have met scientific notation in a previous unit, *Data Around Us*.

a. In Problem 1.2 the multiplying factor is 2. We need to take the number of rubas on square 32 and multiply by 2 to get to the number of rubas on square 33, and again by 2 to get to the number of rubas on square 34 etc. Thus the numbers of rubas are respectively: 4,294,967,296; 8,589,934,592; 17,179,869,184.
b. Because this shows the number of rubas on square 32 multiplied by 9 more factors of 2 the result is the number of rubas on square 41.
c. 1.099511628E12. (Students answers will vary depending on the number of digits shown on their calculator screens.)
d. $1.099511628 \times 10^{12}$. (Actually this is not exact, because the calculator screen did not have enough space to show all the digits in the answer. The exact answer would be: $1.099511628576 \times 10^{12}$.)
e. $2^{10} = 1024$ OR 1.024×10^3.
$2^{20} = 1048576$ OR 1.048576×10^6. Etc,
f. When you have the number in standard notation

2,147,483,648 • 2 • 2• 2• 2• 2• 2• 2• 2• 2 in scientific notation.

e. Write the numbers 2^{10}, 2^{20}, 2^{30}, and 2^{35} in both standard and scientific notation.

f. Explain how to write a large number in scientific notation.

you place a decimal point so that the number has a value between 1 and 10. This new number has the same digits as the original number but the place values of the digits have changed. Next you have to choose the correct number of factors of 10 to multiply by to adjust the place values of the digits. For example, if your number in standard form has a leading digit with a place value of "million", then you will need a 6 factors of 10 in the scientific notation form.

ACE Investigation 2

4.

As a biology project, Talisha is studying the growth of a beetle population. She starts her experiment with 5 beetles. The next month she counts 15 beetles.

a. Suppose the beetle population is growing linearly. How many beetles can Talisha expect to find after 2, 3, and 4 months?

b. Supose the beetle population is growing exponentially. How many beetles can Talisha expect to find after 2, 3, and 4 months?

c. Write an equation for the number of beetles b after m months if the beetle population is growing linearly. Explain what information the variables and numbers represent.

d. Write an equation for the number of beetles b after m months if the beetle population is growing exponentially. Explain what the information the variables and numbers represent.

e. How long will it take for the beetle population to reach 200 if it is growing linearly?

f. How long will it take for the beetle population to reach 200 if it is growing exponentially?

This question compares linear and exponential growth. The equations look similar but the operation that shows growth is ADDITION in a linear equation, and MULTIPLICATION in an exponential equation.

4.

a. A linear pattern of growth would mean that the same number of beetles is ADDED every month, in this case 10.

Month, m	0	1	2	3	4
Beetles, b	5	15	25	35	45

b. An exponential pattern of growth would mean that the number of beetles is MULTIPLIED by the same factor every week, in this case 3.

Month, m	0	1	2	3	4
Beetles, b	5	15	45	135	405

c. $b = 5 + 10m$. The "5" represents the original number of beetles. The "*10*" represents the rate at which the number of beetles is growing. "1*0m*" represents the number of beetles added to the original *5* after any number, *m*, months.

d. $b = 5(10)^m$. The "*5"* still represents the original number of beetles. The "*10*" represents the growth factor. The "*m"* represents the number of times we have to multiply by *10* to find the number of beetles after any number, *m*, months.

e and f. Students might extend the table to find the answers.

ACE Investigation 3

17.

Currently 1000 students attend Greenville Middle School. The school can accommodate 1, 300 students. The school board estimates that the student population will grow by 5% per year for the next several years.

a. In how many years will the population outgrow the present building?

b. Suppose the school limits growth to 50 students per year. How many years will it take for the population to outgrow the school building?

17. This question uses both *growth rate* and *growth factor* language. It is quite usual to think of some quantity, for example, a population or a sum of money in a bank account, growing by some *percentage*. The idea *of percentage growth rate* includes the idea of MULTIPLYING, for example by 5%, and then ADDING this amount of growth to the original to produce a new result. You can combine these two operations into one by MULTIPLYING by (1 + 0.05). The multiplication by "1" indicates that the original amount is still there, waiting to be added to the amount of growth.

Note: 1.05 is the ***growth factor. 5% is the growth rate.***

a. Students might make a table and extend it until the number of students is greater than 1300. Making the table is more efficient if the growth factor 1.05 is used. (Answers are rounded here)

Year	0	1	2	3	4	5	6
Stud ents	1000	1000 x 1.05 = 1050	1000 x 1.05^2 = 1103	1158	1216	1276	1340

b. This supposes that the growth is at a constant rate; 50 is ADDED every year. Students might make a table to answer this, or they might write and solve a *linear* inequality:

$1000 + 50y > 1300$.

ACE Investigation 4

7.

Hot coffee is poured in a cup and allowed to cool. The difference between coffee temperature and room temperature is recorded every minute for 10 minutes.

T	0	1	2	3	4	5	6	7	8
D	80	72	65	58	52	47	43	38	34

a. Plot the (time temperature difference) data. Explain what the patterns in the table tell you about the rate at which the coffee cools.

b. Approximate the decay factor for this

7.

It is important to think about the two *variables* here. The second row shows how different the coffee temperature is from the room temperature. The coffee will continue to cool until this difference is zero.

a. The plot is not shown here. The pattern of change shown in the table will reflect in the shape of the graph, that is, the temperature falls fastest to begin with, so the difference between coffee and room temps will change faster to begin with; but, as the two temps get more alike, the rate of

relationship. c. Write and equation for the relationship between time and temperature difference. d. About how long will it take for the coffee to cool to room temperature? Explain.	cooling will slow. The graph should have a steep slope for low values of time, but a less steep slope as time passes. b. To approximate the decay factor we look for a number that multiplies each temperature difference to get the next temperature difference. We can get this by looking at the ratios $\frac{72}{80}$, $\frac{65}{72}$, $\frac{58}{65}$, $\frac{52}{58}$, $\frac{47}{52}$ etc. These ratios are 0.9, 0.903, 0.94, 0.90, 0.904 etc. It looks like 0.9 would be a good approximation of the decay factor. **Note: this is the same as saying that the temp difference decreases by 10% each time.** c. $D = 80(0.90)^T$. d. Students can continue the table until the difference is zero.
12. Answer parts a and b without using your calculator. a. Which decay factor represents the faster decay, 0.8 or 0.9. b. Order the following from least to greatest: 0.9^4, 0.9^2, 90%, $\frac{2}{10}$, $\frac{2}{9}$, 0.8^4, 0.84.	12. This question goes to the heart of what a *decay factor* means. If we multiply a quantity by 0.8 then the result is that we lose 20% of the original quantity, and retain 80% of the original quantity. If the decay factor is 0.9 then we lose 10% of the original quantity. Thus a decay factor of 0.8 loses more of the original quantity than a decay factor of 0.9. a. 0.8 represents faster decay than 0.9. b. Multiplying anything by 0.9 loses 10% of the original quantity. Thus repeatedly multiplying by 0.9 keeps making the quantity smaller. Thus, 0.9^4 is less than 0.9^2, which is less than 0.9 or 90%. $\frac{2}{10}$ is 0.2, which seems clearly smaller than 0.9^4 and 0.9^2 and 0.9. Students should also be able to compare $\frac{2}{10}$ and $\frac{2}{9}$ without using a calculator. They should also be able to compare 0.8^4 and 0.9^4. They should be able to mentally compute $0.9^2 = 0.81 < 0.84$. Given all these comparisons they should be able to arrange all the quantities: $\frac{2}{10} < \frac{2}{9} < 0.8^4 < 0.9^4$ ……..

ACE Investigation 5	
5. Predict the ones digit for the standard form of the number 12^{10}.	5. $12^1 = 12$, $12^2 = 144$, $12^3 = 1728$, $12^4 = 20736$, $12^5 = 248832$, $12^6 = 2985984$, etc. The ones digits are in a sequence, 2, 4, 8, 6, 2, 4, 8, 6, 2, 4, 8, 6, 2…. Every fourth power of 12 ends in a "6". So, 12^4 ends in a "6", 12^8 ends in a "6", 12^{12} ends in a "6" and so on. 12^{100} comes in this sequence so 12^{100} also ends in a "6." Note: This is the same pattern as the ones digits for powers of 2, which students investigated in class. This makes sense if you think about how "long multiplication" is set up, or if you think about the Distributive property. $(12)^2 = (10 + 2)(10 + 2)$, so the only term in the answer without a factor of 10 will be the 2 x 2 term. Similarly $(12)^3 = (10 + 2)(10 + 2)(10 + 2)$, and the units term will be 2 x 2 x 2.
27. Tell whether the statement is true or false: $\frac{5^{12}}{5^4} = 5^3$	27. This question gets to the meaning of exponential form. 5^{12} means 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5 x 5. 5^4 means 5 x 5 x 5 x 5. So, $\frac{5^{12}}{5^4} = \frac{(5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)}{(5 \times 5 \times 5 \times 5)}$ We can then rewrite this fraction in an equivalent form by dividing the same factor, 5 x 5 x 5 x 5, out of the numerator and denominator, to get: $\frac{5^{12}}{5^4} = \frac{(5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)}{1} = 5^8$
50. Grandville has a population of 1000. Its population is expected to decrease by 4% a year for the next several years. Tinytown has a population of 100. Its population is expected to increase by 4% a year for the next several years. Will the populations of the two towns	50. This asks students think about 2 different exponential relationships: the first is an exponential decay relationship, modeling a 4% loss each year, $P = 1000(0.96)^y$; and the second is an exponential growth relationship modeling a 4% increase every year, $P = 100(1.04)^y$. Students

ever be the same? Explain.	should be able to picture these two relationships as graphs, without actually using a calculator. The first “starts” at P = 1000 and decreases; the second “starts” at P = 100 and increases. The curves will eventually cross. (In fact, if we graph these on a calculator they two populations coincide after about 30 years.)