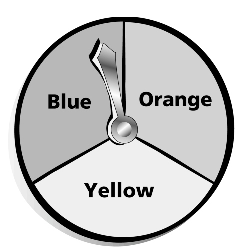
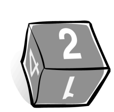
Vocabulary: *How Likely Is It?*

Concept	Example
The Meaning of Probability: The terms *chance* and *probability* apply to situations that have uncertain outcomes on individual trials but a regular pattern of outcomes over many trials. Suppose we are observing a situation and keeping track of the number of times a particular event occurs. Every time that event occurs we might call this a "favorable outcome," such as a basket scored in a free throw situation. We also keep track of the number of times that the event was a possibility but did not occur. Together the "favorable" and "unfavorable" outcomes make the total "possible outcomes." In the basketball situation these would be all the free throw attempts. The outcome is uncertain for each attempt, but, for a given player, a pattern will emerge over the long term that we can call the probability of a free throw for this player. A player with a very high probability (F.T. percentage) of making a free throw can still miss many consecutive baskets. But, assuming his/her skill level has not changed, in the long term the probability of he/she making a free throw will be stable. In general: **Probability is a ratio given by** **P(event) =** $\frac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$ **Where "P(event)" means "Probability this event occurs."** This ratio may be expressed as a fraction, decimal or percent. Some definitions: • If *every* outcome was a favorable	1. If we toss a tack into the air, we know that the tack will land either on its head or its side. If we toss the tack many times, we can use the ratio of the number of times the tack lands on its side to the total number of tosses to estimate the likelihood that the tack will land on its side. Since we find this ratio through experimentation, the ratio is called an ***experimental probability.*** The experimental probability that the tack will land on its side can be expressed as: $P(side) = \frac{\textit{number of times the tack lands on its side}}{\textit{total number of tosses}}$ This is the ***relative-frequency interpretation*** of probability: we count the frequency of the favorable outcomes and compare this to the total number of outcomes. 2. If we toss a coin in the air, we know it will land on showing either a head or a tail. If we count all the outcomes that show "head" and count the total number of trials we will have the ***experimental probability*** P(head). 3. Since we know that for a fair coin there are two equally likely outcomes and one of them is "head," we could say that the ***theoretical probability*** of the coin showing a head is P(head) = $\frac{1}{2}$.

outcome then the probability ratio would be 1. If *no* outcome was a favorable outcome then the probability ratio would be 0. Therefore, $0 \le P(\text{event}) \le 1$ • If we have 2 favorable outcomes, A and B, which do not over lap then the probability of A or B occurring is intended to mean that we will count all occurrences of A and we will count all occurrences of B and add these. $P(A \text{ or } B)= P(A) + P(B)$ Note: For most purposes, **outcome, event and result** are interchangeable terms in this unit. "R**andom**" is used in everyday language to mean "haphazard." In the context of a probability situation "random" means an unbiased, uncertain event. Thus, the outcome of a toss of a coin is random, the outcome of a toss of a number cube is random. Probability is the study of randomness.	
Experimental vs. Theoretical Probability **Experimental Probability**: The probability ratio is calculated by observing and counting favorable outcomes and possible outcomes. We make no assumptions ahead of time. Experimental probability of an event will change from experiment to experiment, and within an experiment as more trials are run. Experimental probability of an event will get closer and closer to the theoretical probability of the same event as the number of trials gets larger and large. **Theoretical Probability:** The probability ratio is calculated by enumerating favorable and possible outcomes, not by observing	4. Suppose three students play a game where they must spin a spinner AND toss a die to win. Annie wins if the spinner comes up blue and the number cube turns up any number but 6, and Tom wins if the spinner comes up "not blue" and the cube turns up 6. Dion wins whenever no one else does. We can find out Annie's **experimental probability** of winning by trying the game out many times and counting how many times she wins.  

them. For example, rolling a die has 6 possible outcomes. If we assume we have a fair die then we can say that theoretically a “3” has a 1 in 6 chance of occurring because each of the six outcomes is **equally likely.**

P(rolling a 3) = $\frac{1 \textit{ (only one "3" on the die)}}{6}$ possible equally likely outcomes = $\frac{1}{6}$.

Game	Winner
1 (Blue, 4)	Annie
2 (Yellow, 6)	Tom
3 (Blue, 6)	Dion
4 (Yellow, 4)	Dion
5 (Blue, 2)	Annie
6 (Blue, 1)	Annie
7 (Orange, 6)	Tom
8 (Orange, 1)	Dion
9 (Orange, 5)	Dion
10 (Blue, 6)	Dion

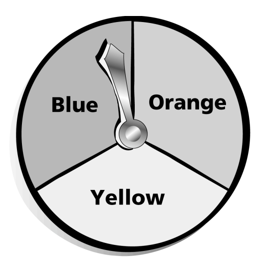
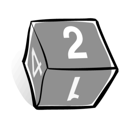
Based on these experimental results Annie has a $\frac{3}{10}$ or 30% chance of winning. Notice that if we had stopped the trials at 5 trials we would have said that Annie had a $\frac{2}{5}$ or 40% chance of winning. We need to do many trials to approach the theoretical probability that Annie will win.

5. To determine the **theoretical probability** that Annie will win we need to make an organized list of all the possibilities and determine if these are equally likely. See Example 9 below.

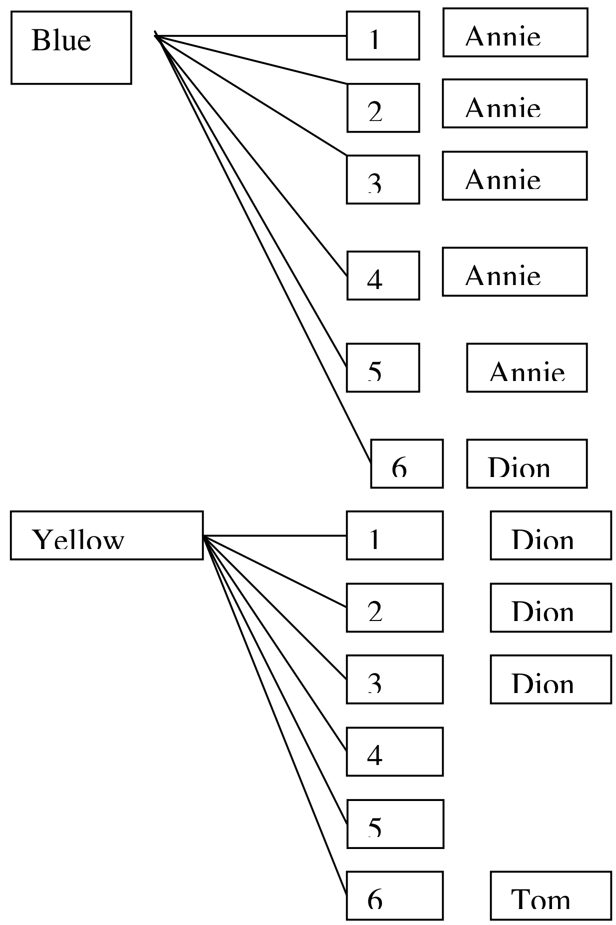
Equally Likely Events: are events that have the same probability of occurring. For example, a head or a tail on a fair coin toss are equally likely, each with a probability of 50%. A 3 or a 4 on a fair die toss are equally likely, each with a probability of $\frac{1}{6}$. Note: just because there are two possible outcomes we can not assume they are equally likely; for example, rain or not rain today covers all possibilities but the probability of rain is not necessarily 50%.

Note: A “**fair”** game is one in which each contestant has an equally likely chance of winning.

6. In the spinner above, each color is **equally likely.** However, since Annie’s winning color is “blue” and Tom wins on “yellow” and “orange” it seems that the game is not fair. We need to investigate further to see if in fact this game is **fair**.

Using Probabilities to make Predictions: We can use the theoretical probability of an event to make an estimate of how many times it will occur in a number of trials. **Predicted # favorable trials = P(event)•(number of trials)**	7. How many times will a multiple of 3 occur if we roll a number cube 600 times? There are two favorable outcomes, 3 and 6, out of 6 equally likely outcomes, therefore, the probability of rolling a 3 or a 6 is $\frac{2}{6}$ or $\frac{1}{3}$. This means that over the long term we can expect that $\frac{1}{3}$ of all outcomes will be favorable. Therefore, if a number cube is tossed 600 times, we can expect that about 200 of these tosses will result in a multiple of 3. Students may do this one by thinking of equivalent fractions. $\frac{1}{3} = \frac{?}{600}$. Note: we may not get a multiple of 3 every third toss, or 1 in 3 tosses, but over the course of MANY tosses we can predict that the number of multiples of 3 occurring will get closer and closer to what is theoretically expected.
Strategies for Finding All Possible Outcomes: • Make an **organized list.** • Make a **Counting Tree.**	8. What is the **theoretical probability** of getting one head and one tail on a 2 tosses of a coin? The **list** of possibilities is HH, HT, TH, TT. These are all **equally likely**. So the probability of one head and one tail is 2 out of 4 or 50%. Note: if we try to find an experimental probability for this we would have to do many trials. 9. See example 4 above.  If we make a list of possibilities of what will happen when we spin the spinner and then toss the cube

	we might start B1, B2, B3, B4, B5, B6, Y1, Y2…… We can organize this list on a counting tree, as shown below. This is not the entire tree, just the top half.



If we completed this tree we would see that Annie has **5 chances out of 18** to win, Tom has 2 chances out of 18 to win, and Dion has 11 chances out of 18 to win. This is **not a fair** game since the rules ensure that Tom, Dion and Annie do not have equal chances to win.

Note: Counting trees and other arrangements are ways for students to understand the possibilities involved in events with multiple parts. They can be used as a basis for understanding the multiplication of probabilities, though they are not intended to be used that way in this

	unit. Annie has a $\frac{1}{3}$ probability of having the first event come out favorably (the spinner) and a $\frac{5}{6}$ probability of having the second event come out favorably. Since these events are independent the probability that Annie wins is $(\frac{1}{3})(\frac{5}{6}) = \frac{5}{18}$. To understand when this multiplication rule can be applied students would have to understand the meaning of independent, which is not dealt with in this unit.
<u>Law of Large Numbers</u> Experimental data gathered over many trials should produce probabilities that are close to the theoretical probabilities. This idea is sometimes called the *Law of Large Numbers*. The Law of Large Numbers applies to mathematically random outcomes.	10. Which statement is a correct application of the Law of Large Numbers? a) If we toss a coin 100,000 times then 50,000 times we will get a head. b) If we have a long run of heads when we toss a coin then the next toss is more likely to be a tail. c) The difference between the numbers of heads and tails gets smaller as the number of tosses gets larger. d) The ratio of heads to tails gets closer to 1:1 as the number of tosses gets larger. Only part d is correct. Part a is not correct because the Law of Large Numbers does not guarantee exactly theoretical results. Part b is not correct because the probability of a tail is $\frac{1}{2}$ for every single event, independent of what happened on prior events. Part c is not correct because the difference between the number of heads and tails could get larger, while the ratio of heads to tails gets closer to 1:1.