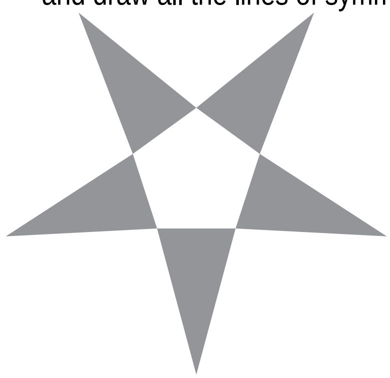
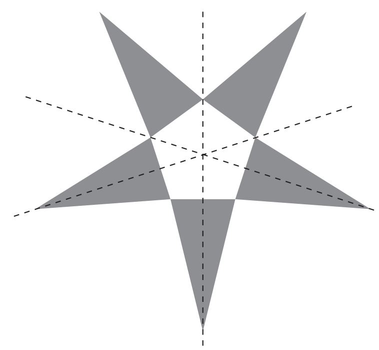
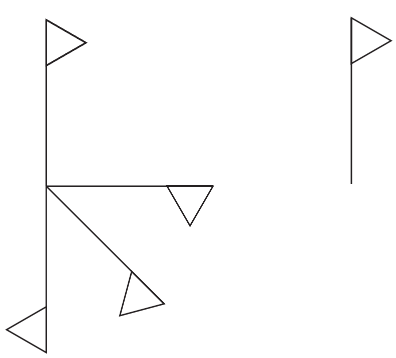
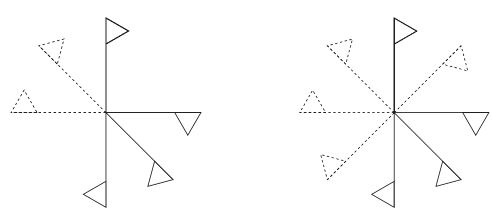
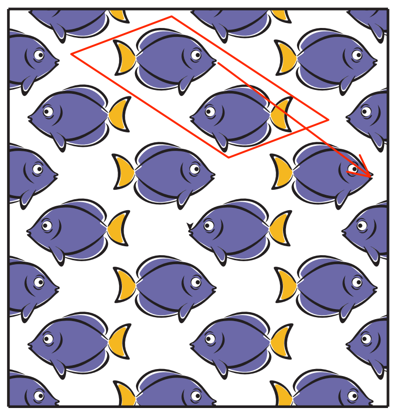
Selected ACE: *Kaleidoscopes, Hubcaps, Mirrors*
Investigation 1: #7, 14, 28
Investigation 2: #9
Investigation 3: #6, 16
Investigation 4: #10, 14, 18
Investigation 5: #5, 9, 11, 15.

ACE Problem	Possible solution
Investigation 1	
7. Tell whether the design has reflection symmetry. If it does, sketch the design and draw all the lines of symmetry. 	7. The design has reflection symmetry. There are in fact 3 ways to “fold” this design so that one half falls exactly on the other. These three lines of symmetry are shown below.  Note: this design also has rotation symmetry. The angle of rotation is one fifth of a rotation or 360/5 = 72 degrees.
14. Use the flag shape at the right as a basic design element. Complete a design with rotation symmetry and give the angle of rotation. 	14. There are two ways to complete a design with rotation symmetry. These are shown below.  The angle of rotation of the first is 180 degrees; the angle of rotation of the second is 45 degrees. (There are other rotation symmetries for the second diagram: 90 degrees, 135 degrees, 180 degrees, 225 degrees, 270 degrees, 315 degrees, in other

	words any multiple of 45 degrees less than 360 degrees.)
28. Identify the basic design element for the wallpaper design. Then, describe how this basic design element can be copied and translated to produce the pattern. Include diagrams with arrows and measures of distances. (See student text for design.)	28. There is more than one way to choose a basic design element. However, the design element must contain 2 fish, one looking in each direction, because there is no way that ONE fish can be translated to fit on top of a fish facing the opposite way. Outlined below is one possible design element. The broken lines with arrow heads indicate the distance and direction of the translation. Remember that the whole design is actually being translated, so that the white space around the fish in the basic element is in fact part of the basic element. 
Investigation 2	
9. Use copies of the figure below for the drawings in parts a – c. (See student text for diagram.) a. Draw the image of the square ABCD under a reflection in line m. b. Draw the image of square ABCD under a 45º rotation about point A. c. Drw the image of the square ABCD under the translation that slides point D to point D´.	9. a. To find the reflection in line m we need to locate the reflection images of each of the vertices of ABCD. D´ is ON the line of symmetry, coinciding with D. C´ is the same distance from the line of symmetry as C, and CC´ is perpendicular to line m. Likewise, both AA´ and BB´ are perpendicular to line m.

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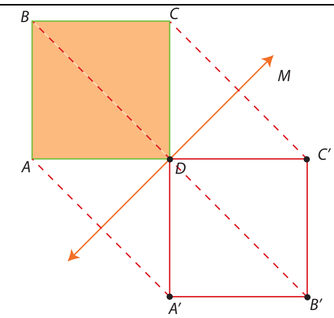
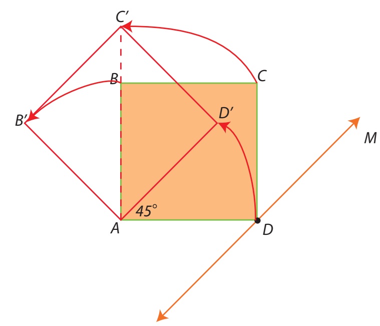
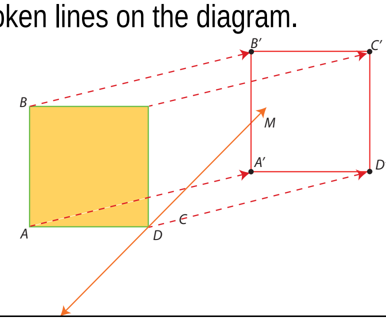
<td>

b.
To draw the image under a 45º rotation we need to find the rotation images of points A, B, C, and D. We need to find D´ so that angle D´AD is 45º, and D´A = DA. Likewise, we need angle B´AB = 45 and B´A = BA; and angle C´AC = 45 and C´A = CA. These are shown below.


c.
The translation we need is defined by the distance and direction DD´. We need to find B so that BB´ = DD´ and BB´ is parallel to DD´, and AA´ = DD´ and AA´ is parallel to DD´ etc. These distances are shown as broken lines on the diagram.

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<td>Investigation 3</td>
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<td>6.
a. The figure below is a rhombus. Identify all the symmetries.
(See student text for figure)
b. List all the sets of congruenet triangles in</td>
<td>6.
a. Because we know this is a rhombus we can identify some equal distances. We know that VW = VY = WX = XY (sides of a rhombus). We also know that WS = YS (S is midpoint of</td>
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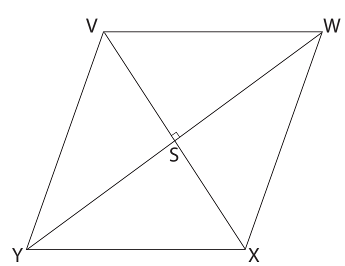
the figure and give evidence for the congruence. Record your findings in a table.

diagonals), but WS is not the same length as VS (we can see that the diagonals are not equal lengths). We can use these equal distances, with some other facts to check out some possible symmetries.

- Since WS ≠ VS, V can NOT be the image of W under a 90 degree rotation around S. However, since YS = WS, and since angle YSW is 180 degrees, Y is the image of W under a 180 degree rotation around S. Likewise, X is the image of V under a 180 degree rotation around S. In fact, students already identified that a parallelogram has 180 degree rotation symmetry around the intersection point of the diagonals, and a rhombus is just a special kind of parallelogram. So the rhombus has 180º rotation symmetry around S.
- Since WS = YS and since WSY is a straight line perpendicular to VX, we can say that Y is the image of W under a reflection in VX. Under the same reflection in VX, V is its own image and X is also its own image. So, VWXY has reflection symmetry in VX. By a similar argument we can show that VWXY has reflection symmetry in WY. (Note: a general parallelogram does not have reflection symmetry across a diagonal, but a rhombus has the additional property that the diagonals are perpendicular to each other.)



b.

The sets of triangles are found by matching points and their images under either a rotation or reflection transformation as in part a. For example, under a 180 degree rotation, W   Y, V   X, S   S. So, if we combine vertices WSV to make a triangle, then the image under 180 degree rotation is YSX.

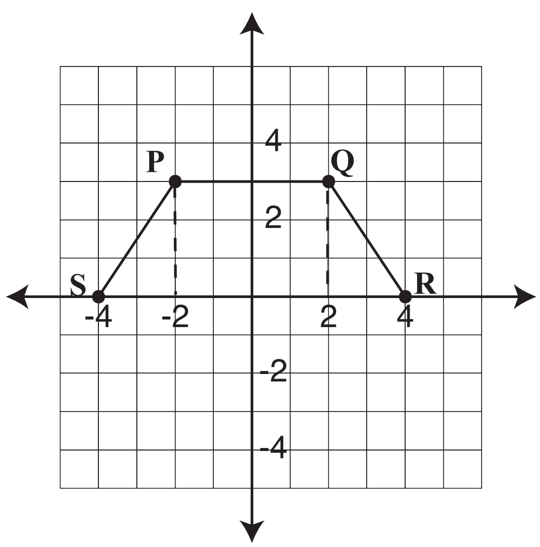
Sets of Congruent Triangles	Evidence for Congruence
VWS, XYS VYS, XWS VXW, XVY YWV, WYX	Rotation symmetry around S (or we could show how 3 sides of one triangle match 3 sides of the other)
VWS, VYS VWS, XWS Etc.	Reflection symmetry in VX Reflection symmetry in WY

16.

a. Trapezoid PQRS is shown below. Explain how you know that sides PS and QR are congruent.

(See student text)

b. Use what you know about symmetry and congruence to show that the two base angles are congruent.



16.

a. Students will have various ways to reason about this.

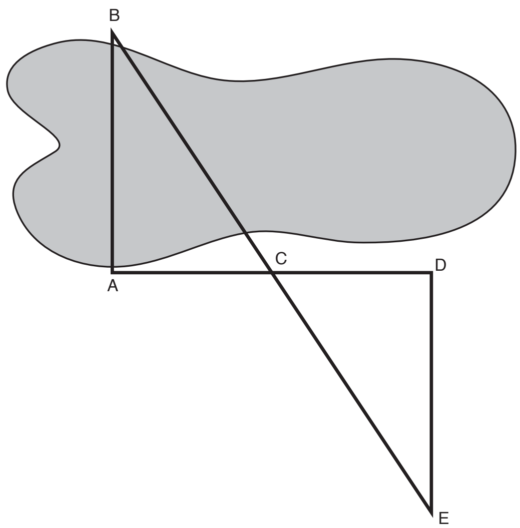
- They might use symmetry. We can see that P is the reflection image of Q in the y-axis. (PQ is perpendicular to the y-axis and both P and Q are 2 units from the y-axis, which satisfies the definition of a reflection transformation.) Likewise, S is the reflection image of R in the y-axis. So, under a reflection in the y-axis PS is the image of QR.
- Or they might use the Pythagorean theorem. Applying this to the right triangles made by dropping perpendiculars from P and Q to the x-axis we have $PS^2 = 3^2 + 2^2$. And the calculation for QR is identical.
- Or they might compare the triangles formed by dropping perpendiculars to the x-axis. They can match 2 sides in one triangle with 2 sides in the other (lengths 2 and 3 units) and the included angle is a right angle. So the triangles are congruent.

b. Not completed here.

Investigation 4

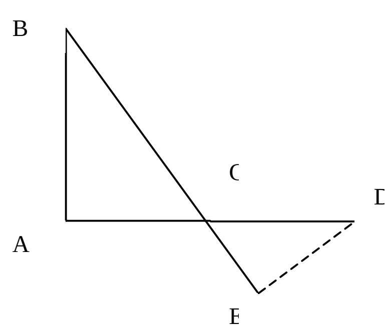
10.

Alejandro wants to measure the distance directly across a pond from A to B. He uses string and some stakes to create the setup shown in the diagram below. None of his string can cross the pond. What information does Alejandro need to build into his setup to find the length of AB?

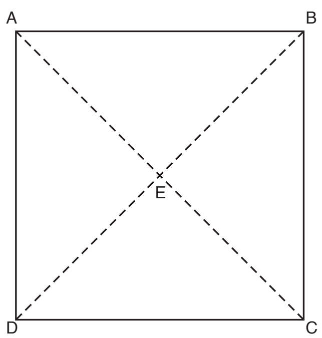


10.

Alejandro needs to build enough information into his setup to ensure that he has 2 congruent triangles. Since he can not stretch string across the pond, the lengths of AB and BC can not be among the known measurements. It is not possible to show two triangles congruent without using at least one pair of matching sides; therefore, Alejandro must match AC with a length in the other triangle. Alejandro must also match 2 angles in one triangle with two angles in the other. This will allow him to use the “AAS” set of conditions that students discovered in Investigation 3. Now, how can he place his stakes so that two pairs of angles can also be matched? Well, if he places stakes at A, B and C to start with, where AB is the required distance across the pond, and C is ANY point on the same shore as A, then he can place another stake at D, so that CD = AC (which he can measure with string) and A, C and D are in a straight line. Now Alejandro has one pair of matching sides (CD = AC) in the two triangles, the “S” in the “AAS” condition. If he also places a stake at E so that B, C and E are in a straight line, and joins C and E with string, then he can deduce that angle BCA = angle ECD because these are vertically opposite angles. (This is an “A” in the AAS condition.) But these angles would be equal even if the triangles were obviously NOT congruent, as below.



So, how does Alejandro find the correct

	placement for point E? He needs to find another pair of angles to match in the two triangles. Since the position of stake E has not yet been satisfactorily determined, he can not use the angle at E as a possible match. This leaves angle BAC and angle EDC as the only possible match. What does Alejandro know about angle BAC? It looks like this angle should be 90 degrees, from the original diagram, so when Alejandro placed the stake at point C he should have checked this angle. In other words, C is not ANY point on the same shore as A. Summary: Alejandro must 1. Place stakes at A, B and C so that AB is the required distance across the pond AND angle BAC is a right angle. 2. Place another stake at D so that CD = AC, and A, C and D are in a straight line, using string to measure. 3. Place a stake at E so that B, C and E are in a straight line, AND 4. Adjust the position of E so that angle EDC matches angle BAC (90 degrees). If he does this then the two triangles will be congruent, using the AAS condition, and he can measure DE to find the actual length of AB.
14. Use what you know about congruent triangles to show that the diagonals of a square are congruent.  	14. Since our goal is to prove that AC is the same length as BD, we need to find two triangles that might be proved congruent, one with AC as a side, and one with BD as a side. It makes sense to concentrate on triangle ABD and triangle BAC. What do we know about these two triangles? We know that two sides in each of the triangles are sides of the square; but we don't know that the third sides of the triangles match, because the third sides are the diagonals that we are trying to prove equal. So we need to find a pair of matching angles. Not completed here.

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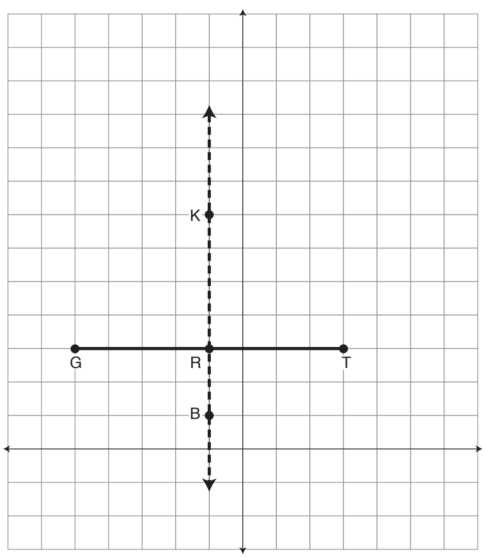
18.
In the diagram below, KB intersects GT at right angles and divides it into two congruent segments. Line KB is called the perpendicular bisector of GT.
a. Is the distance from point K to point G the same as the distance from point K to point T? Explain.
b. Is the distance from point B to point G the same as from point B to point T? Explain.
c. Are there any other points on the line KB that are the same distances from point G and from point T?


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18.
a. Students might have different ways to reason about the lengths KG and KT. They might use the Pythagorean theorem to calculate these lengths, since the triangles KRG and KRT are right angled. OR they might use reflection symmetry, noting that we can see that G and T are the same perpendicular distance (4 units) from KR, so G is the reflection image of T in KR. Since K is its own reflection image in KR, we can say that KG is the reflection image of KT. OR they might use a congruence condition for triangles to match the two triangles KRT and KRG; we can see that KR is a side in both triangles and GR matches TR and angles KRG and KRT are both right angles.
b. The same kind of reasoning as in part *a* will match the distance GB with the distance TB.
c. If students have been using congruent triangles to match distances in parts *a* and *b*, they should be able to see that they could use exactly the same congruence proof for two triangles GRX and TRX, where X is ANY point on the perpendicular bisector. Likewise, if X is ANY point on the perpendicular bisector of GT then GX is the reflection image of TX. We do not need to know specific distances in order to match the lengths GX and TX.
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<td>Investigation 5</td>
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5.
What are the coordinates of the image of point A under a translation in which (2, 4) is the image of (1, 2)?
(See student text for figure.)
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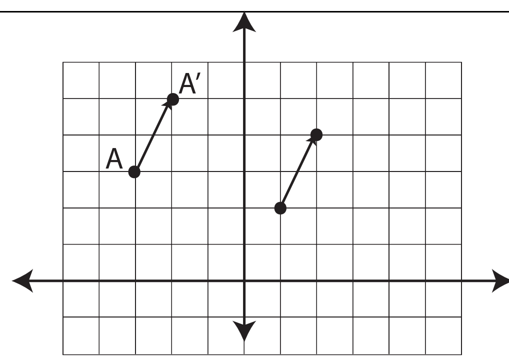
5.


The arrow joining (1, 2) to (2, 4) indicates the distance and direction of the translation. If we apply the same translation to point A we get the image A´(-2, 5).
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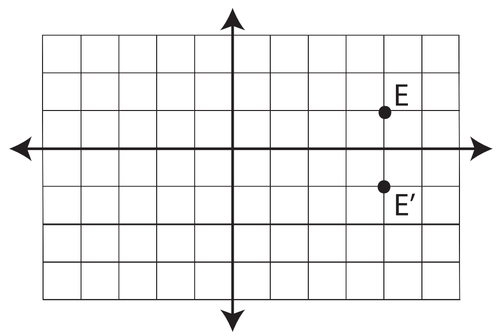
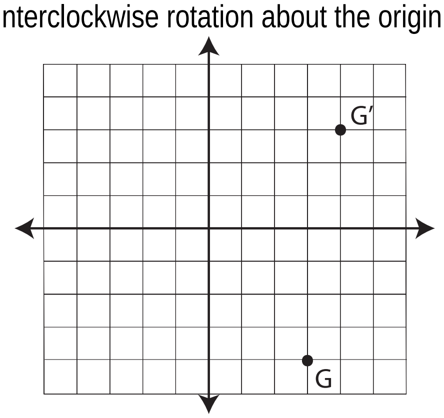
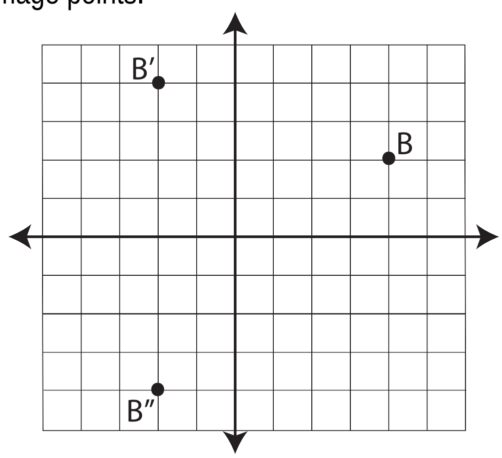
	Note: students can either do this translation visually, or they may observe that the effect of the translation given can be described as (right 1, up 2) or, stated in terms of coordinates, (x, y) → (x + 1, y + 2). Therefore, A(-3, 3) →A´(-3 + 1, 3 + 2)
9. What are the coordinates of the image of point E under a reflection in the x-axis? (See student text for figure.)	9. The point E´ is the reflection image of E. Notice that E´ is the same distance from the x-axis as E.  Note: Students can either do this reflection visually, or they can apply the general rule for a reflection over the x-axis: (x, y) → (x, -y). Applying this to point E we have: E(4, 1) → E´(4, -1).
11. What are the coordinates of the image of point G under a 90° counterclockwise rotation about the origin? (See student text for figure.)	11. The point G´ is the image of G under a 90º counterclockwise rotation about the origin.  Note: Students may either do this rotation visually, or they may apply the general rule for a 90º counterclockwise rotation about (0, 0): (x, y) → (-y, x).

	Applying this to point G: G(3, -4) ⟶ G´(4, 3).
15. a. Copy the figure below onto grid paper. Draw the final image that results from rotating the polygon ABCD 90° counterclockwise about the origin and then reflecting the image in the x-axis. b. Make a new copy of the figure. Draw the final image that results from reflecting the polygon ABCD in the x-axis and then rotating the image 90° counterclockwise about the origin. c. Are the final images you found in parts *a* and *b* the same? Explain.	15. a. In order to create the image of the whole polygon ABCD, students will probably create images for each of the vertices, either visually or applying coordinate rules. Shown below is the result of applying to point B first a rotation of 90° counterclockwise around the origin, followed by a reflection in the x-axis. This creates the image B´´. The images for points A, C and D are not shown here. Students should complete the drawing of the image polygon by finding all 4 image points.  Students may achieve the result of this combination of transformations visually, but it is easier to do this by applying the general coordinate rules for these transformations to the point A. In general: (x, y) ⟶ (-y, x) ⟶ (-y, -x). Therefore: B(4, 2) ⟶ B´(-2, 4) ⟶ B´´(-2, -4). b. Not completed here. c. Comparing the results of applying the general rules for the two transformations in different orders we have: (x, y) ⟶ (-y, x) ⟶ (-y, -x) and (x, y) ⟶ (x, -y) ⟶ (y, x), which are not identical.