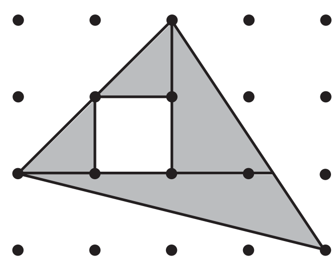
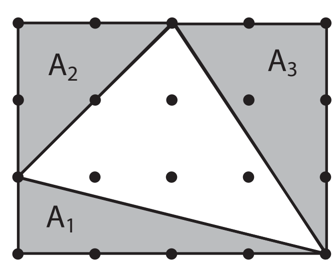
Selected ACE: ***Looking For Pythagoras***

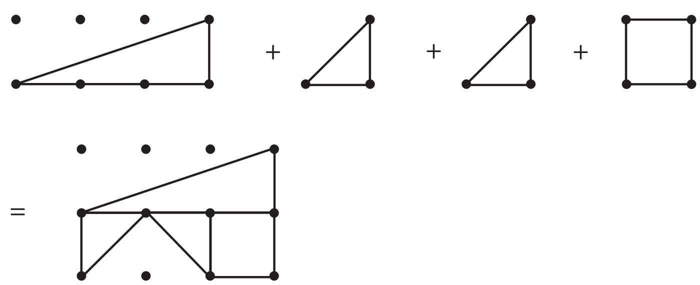
Investigation 1: #20, #32.

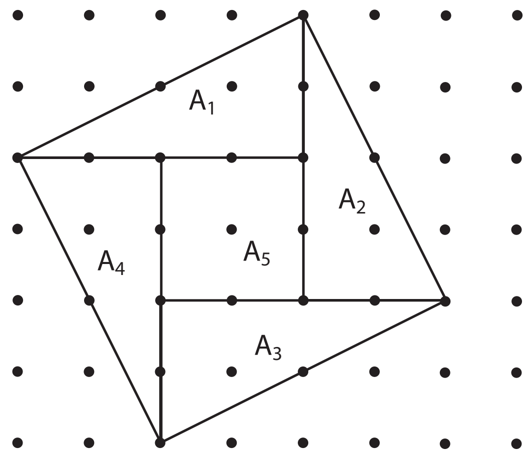
Investigation 2: #18, #38, #42.

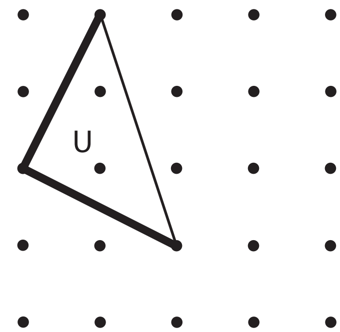
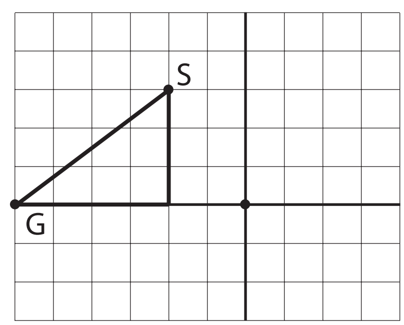
Investigation 3: #8, #14, #18.

Investigation 4: #12, #15, #23.

ACE Problem	Possible solution
Investigation 1	
20. Find the area of the triangle. (See student text.)	Students know that the area of a triangle can be found by using the formula A = 0.5(base x height). (See *Covering and Surrounding*.) The problem with this approach is that neither the base length nor the height is clear in this problem, because they do not align with unit segments on the grid. When students can use the Pythagorean Theorem they can find these diagonal lengths, but for now they have to find another strategy. 20. Students commonly use 2 different strategies to find areas: they subdivide the area into shapes for which they know the area; or they surround the shape by a rectangle and subtract areas from the rectangle.  Subdividing the area as above may not be very helpful if the areas of the shaded triangular shapes are not easy to find.  Surrounding the triangle with a 5-by-3 rectangle and then subtracting the shaded areas (using the formula for the area of

	a triangle) gives the following calculation: Area of triangle = area of rectangle – $(A_1 + A_2 + A_3)$ $= 12 – (2 + 2 + 3)$ = 5 square units.
32. Marcia finds the area of a figure on dot paper by dividing it into smaller shapes. She finds the area of each smaller shape and writes the sum of the areas as _(3) + _ + _ + 1. a. What is the total area of the figure? b. On dot paper, draw a figure Marcia might have been looking at.	32. a. 3.5 square units. b. This problem makes students attend to the format of the expressions. This develops symbol sense. There seem to be 4 areas summed together. The expression "1/2(3)" implies a triangle with area "1/2(base x height)" where the base could be 3 and the height could be 1. The expression "1/2" could be a triangle with area "1/2(base x height)" where the base is 1 and the height is 1. The expression "1" could be a square with side 1. Putting all these clues together we can conclude that the figure *could* be: 
Investigation 2	
18. Tell whether the statement is true. $11 = \sqrt{(101)}$	Students learn in this Investigation that to find the area of a square they must multiply the length of a side by itself, AND, to find the length of the side of a square from its area, they must find the *square root* of the area. 18. This question asks: if a square has area 101 square units then is its side 11 units long? $11^2 = 121$, so if a square had side length 11 units then its area is 121 square units. Therefore, $11 = \sqrt{(101)}$ is not true. Since $10^2 = 100$, $\sqrt{(100)} = 10$; and since $11^2 = 121$, $\sqrt{(121)} = 11$. $\sqrt{(101)}$ must lie between 10 and 11.

38. Which line segment has a length of √17 units? (See student text for figures.)	38. Figure F shows a slanting line segment, which can be thought of as the side of a square. If we build the rest of the square we see it has the area 20 square units. $A_1 + A_2 + A_3 + A_4 + A_5 = 4 + 4 + 4 + 4 + 4 = 20$ square units. Therefore, the line segment is √20 units long.  We need to check the other figures in the same way to find which segment is the side of a square with area 17 square units.
42. a. Which of the triangles below are right triangles? (See student text for figures.) b. Find the area of each right triangle.	42. At this stage students cannot use the Pythagorean theorem to show whether these triangles are right angled or not. But they can use the strategy they have been using to draw a square to decide if two sides of the triangle are perpendicular. In Investigation 1 of this unit, in order to create a square on a slanting line segment, students discussed the use of slope (See *Moving Straight Ahead*). If the given line segment has slope a/b, then the slope of the adjacent side of the square has slope –b/a. (Students may not have formalized this at this time.) a. For figure U we can see that the slopes of the bolded line segments are 2/1 and -1/2. Therefore, these two line segments are perpendicular. This is a right triangle.

	 b. Area of figure U can be found by subdividing or surrounding (see ACE 20 Investigation 1). Or it can be found by noticing that Figure U is half of a square with area 5 square units (not drawn here). So area of figure U = 2.5 square units. The other figures can be investigated the same way.
Investigation 3	
8. Find the flying distance in blocks between the two landmarks, Greenhouse and Stadium, without using a ruler. (See diagram in student text.)	8. The segment joining the two landmarks can be thought of as the hypotenuse of a right triangle, with legs of lengths 4 and 3 units.  $GS^2 = 3^2 + 4^2 = 25$. Therefore, GS = 5 units. The distance between the greenhouse and the stadium is 5 blocks.
14. Show that this triangle satisfies the Pythagorean Theorem. (See student text for figure..)	14. Students might approach this problem either by showing that the triangle is right angled, in which case the Pythagorean Theorem applies. Or, they might find the areas of the squares on the sides, and check that these fit the Pythagorean

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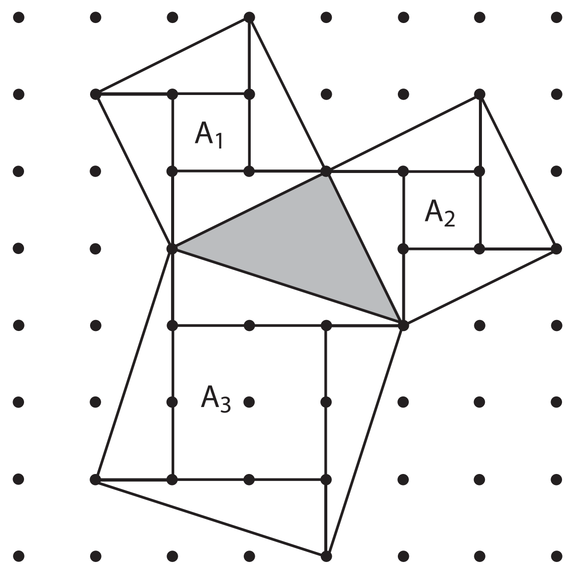
<td>relationship. If they take the first approach they must have a way to show that two sides of the triangle are perpendicular. The relationship between the slopes of perpendicular lines has not been formally stated in any unit thus far (will be formalized in Shapes of Algebra), but some classes may have drawn a conclusion about this relationship in this unit. See ACE 42, Investigation 2.

The second approach is illustrated below.



The area A_3 can be found by subdividing the square.
$A_3 = 1.5 + 1.5 + 1.5 + 1.5 + 4 = 10$ square units.
Likewise, $A_2 = 5$, and $A_1 = 5$ square units.   So, $A_1 + A_2 = A_3$.</td>
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<td>18.
The prism has a base that is a right triangle (See student text for the figure.)
a. What is the length a?
b. Do you need to know the length a to find the volume of the prism? Do you need to know it to find the surface area? Explain.
c. What is the volume?
d. What is the surface area?</td>
<td>18.
a. Applying the Pythagorean Theorem $2.5^2 + 6^2 = a^2$. So, $42.25 = a^2$. So, $a = 6.5$.
b. To find the volume you need to know the area of the base of the prism and the height of the prism. The area of the base is 0.5(2.5 x 6) = 7.5 square units. You don’t need to know a to find this base area. To find the surface area you need to know the areas of all faces. The triangular base areas can be found as above. But the area of one of the rectangular faces is 4 x a square units. So we do need to know a to find surface area.
c. Not answered here.
d. Not answered here.
e. The net should show 3 rectangular faces and 2 triangular</td>
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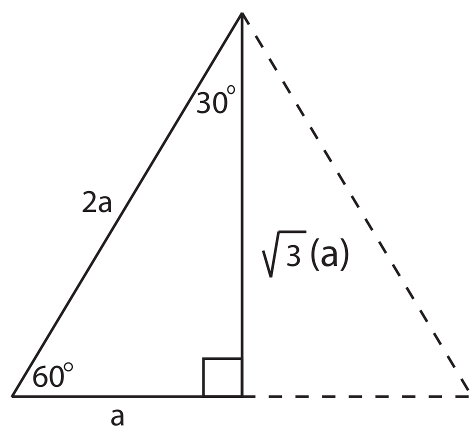
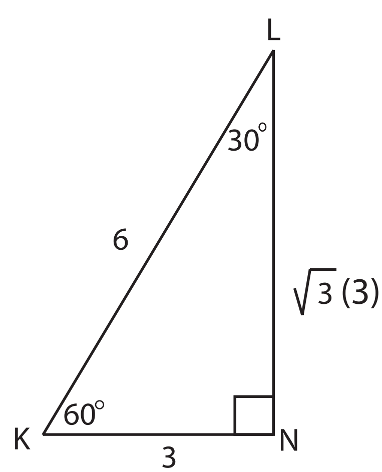
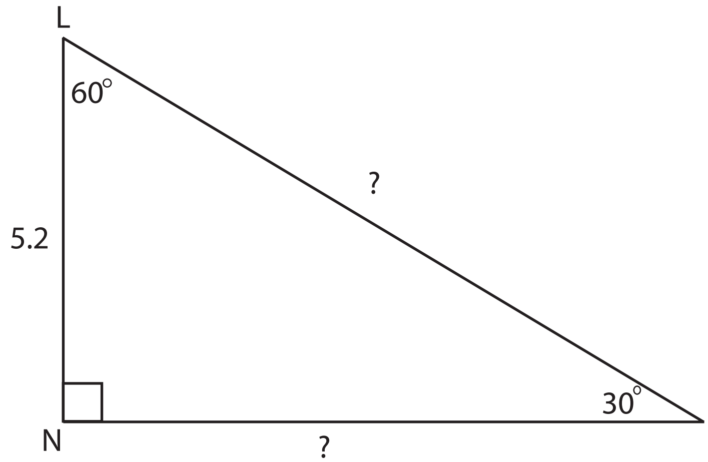
e. Sketch a net.	bases.
Investigation 4	
12. Find the perimeter of triangle KLM. (See student text for figure.)	In this Investigation students applied the Pythagorean theorem to a particular triangle, with angles 30, 60 and 90 degrees. By observing that this triangle is half of an equilateral triangle they were able to conclude that the shortest side is always half of the hypotenuse; and by applying the Pythagorean Theorem they were able to conclude that the longer side is always √3 times the shortest side. These relationships apply to any 30-60-90 triangle, because all such triangles are similar, or scale copies of each other. A general 30-60-90 triangle is pictured below:  12. Applying this to triangle KLN we have:  Now look at triangle MLN. It is also a 30-60-90 triangle, and we know *the shortest* side is √3(3) units, or approximately 5.2

	units.  We can deduce the length of hypotenuse LM and longer leg MN by using the length of the shortest side LN. (Not completed here)
15. Estimate the square root to one decimal place without using the √ key on your calculator. Then, tell whether the number is rational or irrational.	15. $\sqrt{16} = 4$. $\sqrt{9} = 3$. $\sqrt{9} < \sqrt{15} < \sqrt{16}$. So, $3 < \sqrt{15} < 4$. We can see that $\sqrt{15}$ is closer to $\sqrt{16}$ than to $\sqrt{9}$. Therefore, we might try 3.9 as a first approximation. $3.9^2 = 15.21$. $3.8^2 = 14.44$. 3.9 appears to be a better approximation than 3.8. Since there is not exact decimal answer for $\sqrt{15}$ it is an irrational number (that is, the decimal answer neither terminates nor repeats).
23. Write the fraction as a decimal and tell whether the decimal is terminating or	23. In an earlier unit, *Bits and Pieces II*, students learned to think of a fraction in different ways; for example, a fraction might be thought of as parts out of a whole, or as a ratio, or as a

repeating. If the decimal is repeating, tell which digits repeat. 8/99.	division. The last interpretation helps to connect decimals to fractions. $$\begin{array}{r} .080808 \\ 99\overline{)8.00000} \\ \underline{7\ 92} \\ 800 \\ \underline{792} \\ 800 \\ \underline{792} \\ 8 \end{array}$$ 8/99 = 0.080808… The decimal is repeating, and the digits that repeat are “08”. Note: Every fraction can be written as a decimal by dividing as above. Since there can be a finite number of choices for the remainders created by such a division we eventually come to a situation where the remainder is zero, in which case the decimal terminates, or a previous remainder repeats, in which case the decimal answer repeats. For example, when dividing by 99 we could theoretically have any remainder from 0 to 98. After all remainders have been used once one of them must repeat. In fact the only remainder created by the above division is *8*, and so the division process repeats very quickly.