Vocabulary: ***Moving Straight Ahead***

Concept	Example

Patterns of Change in a 2-Variable Table: illustrate how one variable changes in relation to the other. These patterns of change are clues as to the kind of relationship between the variables. In this unit linear relationships are the focus.
Rate of Change: is the amount of change in one variable per *unit* change in the other variable.
Constant Rate of Change: occurs when the dependent variable changes by the same amount *per unit* increment in the independent variable.

*1. Do any of the following tables indicate a **constant rate of change for y?***

Table 1:

X	0	1	2	3	4
Y	3	6	9	12	15

Table 2:

X	0	2	4	6	8
Y	5	9	13	17	21

Table 3:

X	0	1	2	3	4
Y	0	2	5	9	14

Table 4:

X	0	1	3	7	8
Y	0	4	12	28	32

Tables 1, 2, and 4 show y increasing at a **constant rate**. In table 1 the rate of change of y is 3 per unit change in x; in table 2 the rate of change of y is 2 per unit change in x (y increases by 4 units per each increase of 2 units in x); in table 4 the rate is 4 per unit change in x. Table 3 does not show y increasing by a constant amount per unit change in x. Tables in earlier investigations in this unit generally show the independent variable increasing in 1 unit increments, while y increases or decreases.

Linear Relationship between 2 Variables: is evident from
- The table, when the dependent variable changes by constant increments for constant increments in the independent variable, and from
- The graph, when the coordinate points form a straight line, and from
- The equation, when the relationship can be represented by an equation of the form $y = mx$ or $y = mx + b$, and from
- The problem context when there is an underlying constant rate of change.

Connections among Representations: The constant rate of change shown in the table appears

2. Are the relationships represented in the following tables **linear relationships**? If so, what is the **constant rate** of change?

x	*y*
-2	4
-1	1
0	0
1	1
2	4
3	9

x	*y*
-2	3
-1	3
0	3
1	3
2	3
3	3

x	*y*
-3	10
-2	7
-1	4
0	1
1	-2
2	-5

x	*y*
-1	-3
2	3
4	7
8	15
10	19
11	21

The pattern in the first table is not **linear**—as *x* increases one unit there is not a constant rate of change in *y*. This

on the graph as the steepness of the line, negative rates sloping down from left to right, positive slopes sloping up, and faster (greater) rates shown as steeper lines. The constant rate of change also appears, in the equation that describes the relationship, as the coefficient of the x-term.	pattern can be represented as $y = x^2$. (This pattern is studied in the *Frogs, Fleas, and Painted Cubes* unit.) The second pattern is linear. The constant rate of change is 0 and the equation is *y* = 3. The third pattern is also linear. The constant rate of change is -3; the equation is *y* = ⁻3*x* + 1. The fourth pattern is also linear. The constant rate of change is 2; the equation is *y* = 2*x* – 1. (Note in the last table, the increments in *x* are not equal, but **the rate of change depends on increments in *both*** x and y.) If students graph the coordinate pairs in the tables they can see that the points from the second, third and fourth tables lie in **straight lines**, and that the steepness of the line, and direction of the slope reflects the rate of change. (The line is “flat” for table 2, sloping down for table 3, and sloping up for table 4, but not as steep as for table 3.) *3. Suppose the cost to rent a number of bikes from a company is $150 plus $10 per bike. Can we tell from the* ***contex****t whether the relationship between number of bikes and cost is linear or not?* We first recognize that there is a ***constant rate*** being charged for the bicycles, so there will be a linear pattern in the relationship between cost and number of bicycles. The constant rate of change is 10. This constant rate appears in the symbolic representation; we write *C* = $150 + $10*n*, where *C* is the cost in dollars and *n* is the number of bikes. (The rate also appears in a graph of the relationship, as the steepness of the graph. And it appears in a table showing different costs for different numbers of bikes rented.)
Y-Intercept: is the point where the straight line crosses the y-axis (vertical axis). It has x-coordinate 0, and therefore is a representation of the point in the table where the independent variable is 0. To find the y-intercept we can i) Obtain it from the verbal situation, or ii) Read it from a graph or table, or iii) Work backward or forward in a table to find the point (0, *b*), or iv) Find it by substituting the slope and one of the points into the equation *y* = *mx* + *b* and then solving for *b*.	*4. What is* ***the y-intercept*** *in the bike rental problem in example 3 above?* In the bike rental example above we can see that when 0 bikes are rented there is still a base cost of $150, so this is the y-intercept, (0,150).

<table>
<tr>
<td></td>
<td>5. What are some other ways we can find the y-intercept in the bike rental problem above?

The bike rental problem might have been presented in the form of a table or graph that did not show the y-intercept, instead of as a verbal description:
<table>
<tr><td># bikes</td><td></td><td>5</td><td>10</td><td>20</td><td>30</td></tr>
<tr><td>Cost in $</td><td></td><td>200</td><td>250</td><td>350</td><td>450</td></tr>
</table>
Using what they know about ratios, what they see in the table, and assuming that the rental cost per bike is a constant rate, students can reason, using the difference between the first two entries, that the rental cost per bike is $10; then can count backwards from the cost of 5 bikes to find the cost of 0 bikes. Or they might use the difference between the costs for 5 bikes and 10 bikes to reason backwards in a larger step. Either way they arrive at the pair (0, 150) for the y-intercept.</td>
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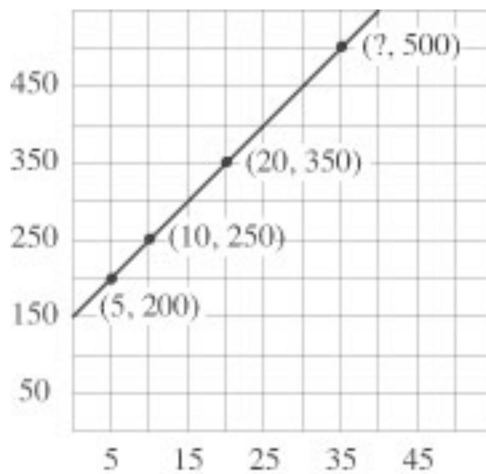
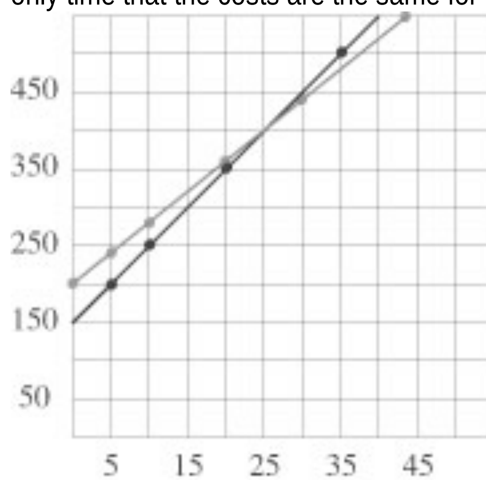
<tr>
<td>Solutions from Table or Graph: Since each coordinate point on a graph of a relationship represents a coordinate pair from the table, and since the regularity of the linear pattern means we can “complete” a table or graph, we can find missing values of y, given x, or missing values of x, given y.</td>
<td>6. How can we figure the cost of renting 7 bikes from the table given in example 5 above?

We can find the cost for renting 7 bikes, even though this information is missing from the table, by counting on from the cost for 5 bikes in constant increments of $10 per additional bike. This is the solution for the question, “What is the cost for 7 bikes?” or the equation, C = 150 + 10(7).</td>
</tr>
</table>

	7. How can we figure how many bikes we can rent for $500 (see examples 3, 4, 5, 6 above) from a graph? We could graph the information in the table in example 5 and extend a line through the points to read other coordinate pairs, such as (?, 500) which would represent the number of bikes that can be rented for $500. This is the solution for the question, “How many bikes can be rented for $500?” or the equation, 500 = 150 + 10x. The crucial understanding is that each point on the graph, and each pair in the table, satisfies the underlying linear relationship that has a constant rate of increase of 10, and a y-intercept of 150. 

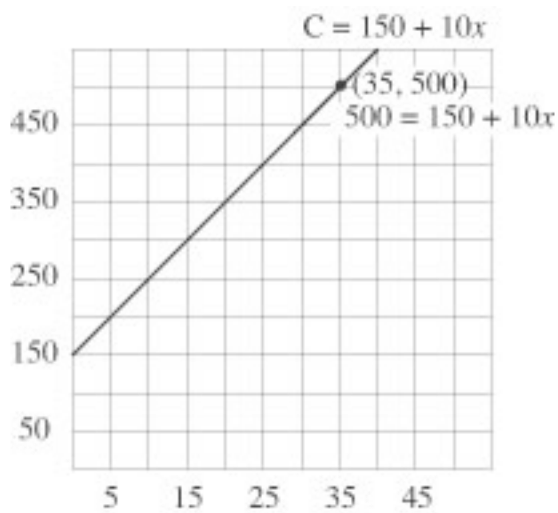
Intersection Point: is the point where 2 lines meet and, therefore, a coordinate pair that fits both relationships represented by the lines.	*8. Compare two bike rental plans, one described by the equations C = 150+ 10x, and the other by C = 200 + 8x. Which rental plan costs more for 30 bikes? Is there ever a number of bikes where the cost is the same for both companies?* If we graph both of these relationships we will have a pair of lines that share the point (25, 400). This point satisfies both relationships, therefore the cost is $400 for 25 bikes for both plans. This is the only point on both lines, so is the only time that the costs are the same for both plans.  For any number of bikes greater than 25 we see from the graph that the line with equation C = 150 + 8x has higher y-values. Therefore, for 30 bikes the plan given by C = 150 + 8x costs more.
Solutions from Symbolic Equation: Since each pair of related values must fit the equation representing the relationship, we can find a missing value of a variable by various strategies, including “undoing,” “balancing,” or “guess and check.” The key to solving equations symbolically is understanding equality; equality is a statement that states two quantities are equal. Equality can be thought of as a “balance.” To solve an equation means that we want to maintain the equality between the two quantities. **Properties of equality**: We can add, subtract, and multiply or divide (by a non-zero number) both sides of the equation and maintain equality.	*9. For the original bike rental plan described by C = 150 + 10x, how many bikes can we rent for $500?* In this question we are asked to **solve for one of the variables, x,** when we are given a value for the other variable, C; or we are asked **to solve a linear equation** with one variable or “one unknown”. We could find the information by using a table or graph as explained above, or we can find the value of the variable by solving 500 = 150 + 10x by symbolic methods. Students might reason by “working backwards” or “undoing” that if 10x + 150 = 500 then 10x must be 350 (cost of x bikes without base charge), and so x (number of bikes) must be 35.

	10. Compare the two bike rental plans, C = 150 + 10x and C = 200 + 8x. Which one is cheaper for 30 bikes? We can make this comparison graphically as explained above, or symbolically by **solving C = 150 + 10x and C = 200 + 8x simultaneously.** This means that we want to know when the Costs are equal; we want to find the value of x that makes 150 + 10x = 200 + 8x. "Working backwards" is not as productive as "balancing" for this equation format, where the variable is on both sides. By subtracting 150 from both sides we have 10x = 50 + 8x. By subtracting 8x from both sides we have 2x = 50. By dividing by 2 we have x = 25. This means that when the number of bikes is 25, the costs for both plans are equal. For any number of bikes greater than 25 the plan with the smaller rate of change (or lesser slope) will be cheaper. (In this unit we only solve equations of the following types: $6 – 3x = 10$; $5 + 17x = 12x – 9$; $2(x + 3) = 10$)

Connections among representations of solutions: Each pair of related values in a table of a linear relationship appears as a point on a graph of the line, and as a solution that makes the equation representing the relationship true. Thus each point on a line represents a particular linear equation and its solution, and vice versa. (An intersection point of 2 lines simultaneously satisfies 2 relationships and 2 equations.)

11. Find the solution to the question "How many bikes can be rented (on the original plan, C = 150 + 10x) for $500?" graphically and from a table.

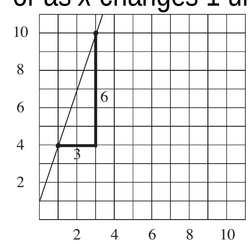
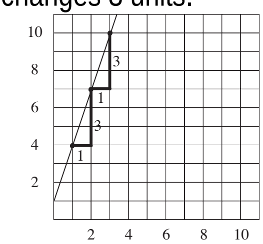
The **solution appears as a single point** on the graph of C = 150 + 10x. The solution is (35, 500); we can rent 35 bikes for $500.



The **solution also appears as a pair in an extended table.** Students can complete the table counting in increments of 1 bike and $10 at a time to (35, 500), or in greater increments, say 5 bikes and $50 at a time.

# bikes	0	1	….	?
Cost in $	150	160	….	500

Note: students have three ways to picture and solve 500 = 150 + 10x. They learn to choose the most convenient way; symbolic methods are quick, but only when the equations are simple; graphical methods always work, but may be hard to read exactly; table methods work but may be tedious.

Slope of a Line from Table or Graph: can be calculated by finding the ratio of the change in the dependent variable to the corresponding change in the independent variable, or the ratio of the vertical change on the graph (rise) to the corresponding horizontal change (run). The slope is a visual representation of the rate at which y is changing in relation to x. **Parallel Lines:** have the same slope. **Perpendicular lines**: have slopes which are opposite reciprocals of each other.	*12. Suppose the points* $(1, 4)$ *and* $(3, 10)$ *lie on a line. Find the slope of that line.* The slope is the ratio: $\frac{\text{change in vertical distance}}{\text{change in horizontal distance}}$ $=\frac{10-4}{3-1}$ $=\frac{6}{2}=\frac{3}{1}=3.$ Note that the connection to constant rate is: As *x* goes from 1 to 3 the change is 2. (horizontal change) As *y* goes from 4 to 10 the change is 6 (vertical change) That is, as *x* changes 2 units, *y* changes 6 units or as *x* changes 1 unit, *y* changes 3 units.  OR 

Equation of Line: can be found given slope and intercept, or slope and a point, or 2 points. Any two points on a line define the slope, which is constant for any particular line, and there is only one line with a particular slope through any given point.

13. Suppose we know that a line has slope 3 and passes through (7,2). What is the ***equation of the line?***

We can "count backwards" to the y-intercept, using the fact that the y increases by 3 for every increase of 1 in *x*. This gives us the y-intercept (0, -19). We then know the "*m*" and the "*b*" in the format "*y* = *mx* + *b*."

X	0	…	6	7
Y	-19	…	-1	2

Or we could use the symbolic representation y = mx + b, where we know that "m" is 3, and we know that (7,2) is on the line. Thus, y = 3*x* + b because m = 3, and 2 = 3(7) + b because (7, 2) satisfies this equation. Solving this gives b = -19.

The **equation of the line** with slope 3 , passing through (7, 2) is *y* = 3*x* – 19.

14. What is the ***equation of the line*** *which passes through the two points, (1, 4) and (3, 10).*

The slope, using the ratio of ***rise/run***, = $\frac{10-4}{3-1} = 3$. Counting backwards we have the *y*-intercept is (0, 1). Thus the equation is y = 3*x* + 1.

X	0	1	2	3
Y	?	4	?	10

Or, students might also use a graph to find the *y*-intercept. They extend the line to intercept the *y*-axis. They might use the ratio definition of slope to work from a point on the graph back or forward until they hit the *y*-axis.

Or they might find the slope is 3, as above and then substitute either of the given points, say (1, 4), into the equation y = 3*x* + b. This gives
4 = 3(1) + *b*. From this equation we have *b* = 1.
Thus the equation is y = 3*x* + 1.