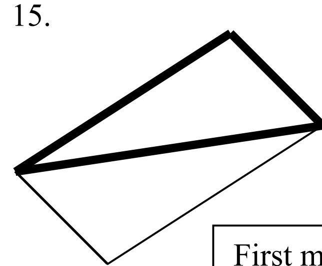
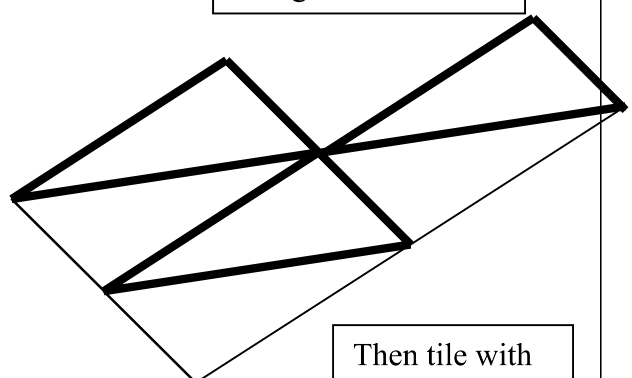
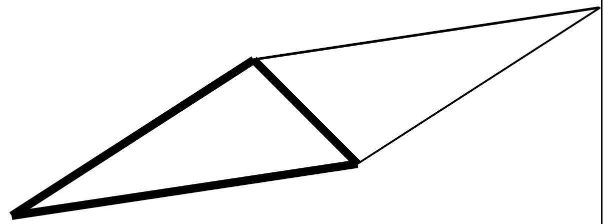
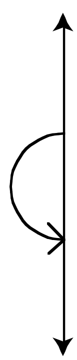
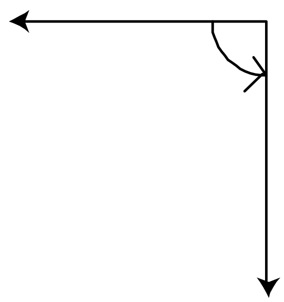
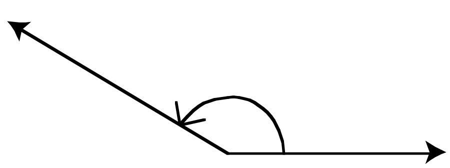
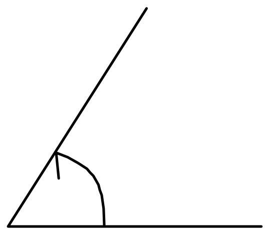
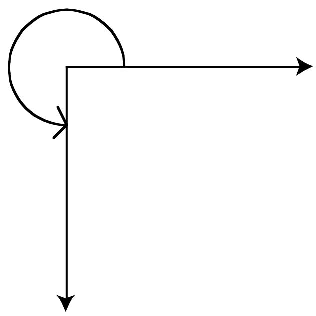
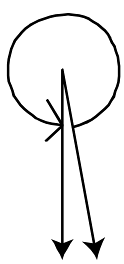
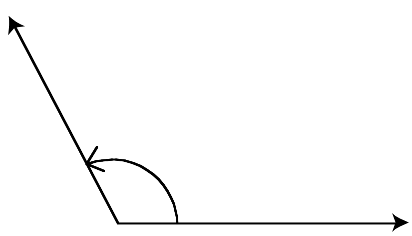
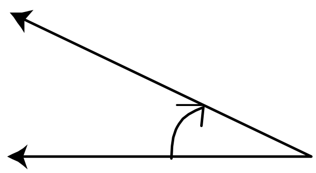
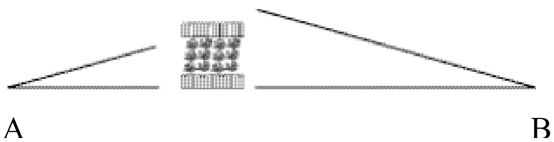
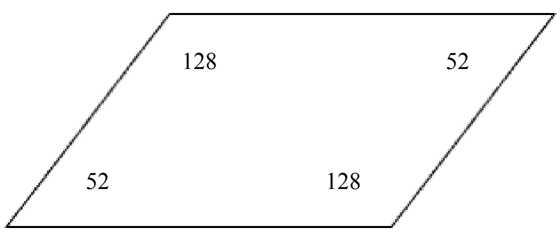
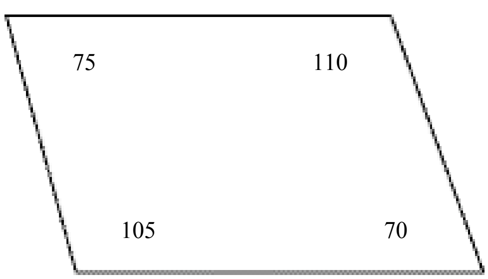
Shapes and Designs: Homework Examples from ACE
Investigation 1: Question 15
Investigation 2: Questions 4, 20, 24
Investigation 3: Questions 2, 12
Investigation 4: Questions 9 – 12, 22.

ACE Question	Possible Answer
ACE Investigation 1	
15. Choose a scalene triangle from your Shapes Set, or draw your own. Find two ways that copies of your triangle can be used to cover, or tile a surface.	15.  First make a parallelogram by rotating the original triangle.  Then tile with the Parallelogram. OR make a different parallelogram by starting with the same triangle and rotating it around the mid point of a different side.

	 Then tile with this parallelogram.
ACE Investigation 2	
4. Without using an angle ruler, decide whether the measure of each angle is closest to 30 degrees, 60 degrees, 90 degrees, 120 degrees, 150 degrees, 180 degrees, 270 degrees, or 360 degrees. a.  **b.**  **c.** 	4. Students use a **right angle**, 90 degrees, as a **benchmark**. Notice that neither of the arms of the angle has to be vertical or horizontal. Students need to be able to identify the **vertex**, no matter the orientation. a. 180 b. 90 c. This is obviously more than 90 and less than 180 degrees. It looks closer to 180 degrees, so 150 would be a good estimate. d. This is less than 90, but closer to 90 than 0 degrees. So 60 would be a good estimate. e. This looks like 3 right angles placed adjacent to each other. So 270 degrees. f. This is almost a complete rotation, so 350 would be a good estimate. g. This is more than 90 but less than 180 degrees. It looks closer to 90 than to 180. So 120 degrees would be a good estimate. h. This is between 0 and 90 degrees, but less than half 90 degrees. So 30 degrees would be a good estimate. (Notice that this rotation is marked in a counterclockwise fashion. From the point of view of students in 6th grade the direction of the rotation is not important. It is the *amount of rotation*

d.  **e.**  **f.**  **g.**  **h.** 	between one arm of the angle and the other that is the focus.)

20. Little Bee left point A for a flower patch and Big Bee left point B for the same flower patch. However, both bees were 15 degrees off course. Little Bee landed on the patch and Big Bee did not. Explain why Big Bee did not hit the patch and Little Bee did.  	20. (This question reminds students that the angle size is not related to the lengths of the arms of the angle.) If we assume that the intended direction is the horizontal line and the actual direction taken is the other arm of the angle, then you can see that the distance between the arms gets wider as the arms get longer. This does not mean that the angle size has changed; angle size depends on the amount of rotation to move one arm into the position of the other arm. But it does mean that Big Bee's position gets further and further from his intended position as the distance along the arm gets larger.
24. Tell if the following are parallelograms and explain why or why not. A.  B.  	24. A. (This LOOKS like a parallelogram, but one of the goals of this CMP unit is to move students towards reasoning about polygons and their properties, so "looks like" is not acceptable as an answer.) We know that the sides of a parallelogram are parallel, so we must show that we have parallel lines here. By extending the sides of the quadrilateral we have straight angles (180 degrees) so we can identify other angle sizes. Specifically, the angle marked with a * is 180 – 52 degrees = 128 degrees. So we have equal angles in alternate interior positions, using the side marked with a broken line as the transversal; therefore the two bolded sides are parallel. The same kind of reasoning gives us that the other pair of sides of the quadrilateral are also parallel. So this is a parallelogram.

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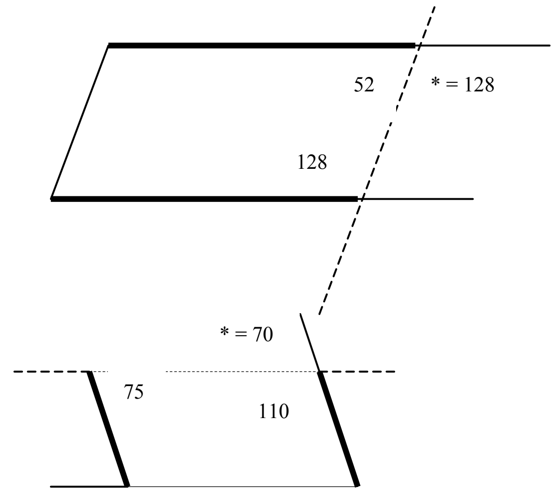
<td>

b. This time extending one side of the quadrilateral and calculating the sizes of angles in alternate interior positions does not produce equal angles. Therefore, this quadrilateral does not have 2 pairs of parallel sides.</td>
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<td>ACE Investigation 3</td>
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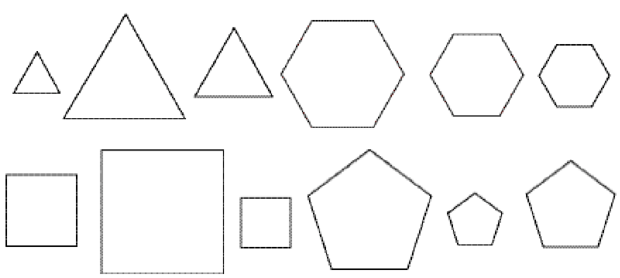
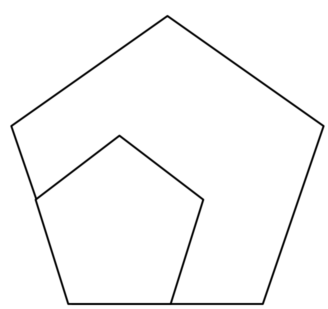
<td>2.
Below are sets of regular polygons of different sizes. Does the length of a side of a regular polygon affect the sum of the interior angle measures?
</td>
<td>2.
Students might reason about this in different ways. They might use the formula they have found in class work to say that the sum of the interior angles of a polygon is (n – 2)180 degrees. Therefore, the angle sum is dependent only on the number of sides (n) not the length of the sides. For example, for a pentagon the angle sum is 3(180) degrees = 540 degrees.

Or they might reason more visually:

The smaller pentagon, for example, fits</td>
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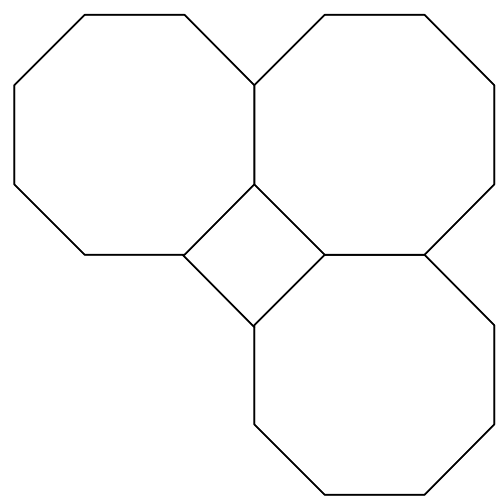
	exactly into the corner of the larger pentagon. The size of the common angle does not depend on the length of the side.
12. Which of the following will tile a plane? a. Copies of a regular heptagon (7 sides) b. Copies of a square and a regular octagon. c. Copies of a regular pentagon and a regular hexagon. d. Copies of a regular hexagon and a square.	12. a. The angle sum of a regular heptagon is $(7-2)180$ degrees = 900 degrees. Therefore each angle is $\frac{900}{7}$ degrees = 128.57 degrees. Since this is not a factor of 360 we know that we can not fit copies of this angle round a common vertex without leaving a gap. This polygon will not make a tiling. b. Using the formula for the angle sum of a polygon we have that each angle of a square is 90 degrees and each angle of a regular octagon is 135 degrees. There is a combination of these angles that would fit together at a common vertex: 135 + 135 + 90 = 360 degrees. This will in fact make a tiling, with 2 octagons and one square at each vertex.  c. The angles of a regular pentagon and a regular hexagon are, respectively 108 and 120 degrees. There is no combination of 108 and 120 degrees that will add to 360 degrees.

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<tr><th>ACE Investigation 4</th><th></th></tr>
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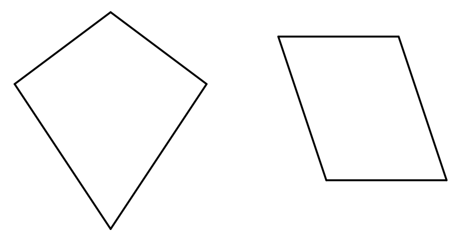
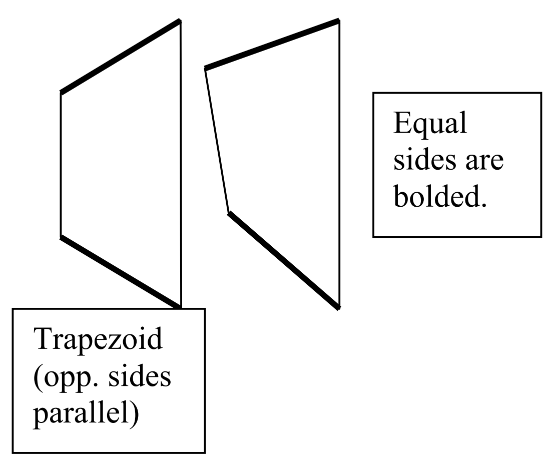
9 – 12

If it is possible to build a quadrilateral, sketch examples that can be made with the given set of side lengths.
Show whether your examples are the only ones that are possible.
If a quadrilateral is not possible explain why.

9. Side lengths of 5, 5, 8 and 8.
10. Side lengths of 5, 5, 6, 14.
11. Side lengths of 8, 8, 8, 8.
12. Side lengths of 4, 3, 5 and 14.
</td><td>
9.
First we should determine that the lengths will make a quadrilateral. That is, is the sum of the shortest three sides longer than the fourth side? With 5, 5, 8 and 8 we know a quadrilateral is possible. In fact, there are two types of shapes possible; if we place the equal sides next to each other we get a kite shape (on the left), and if we place the equal sides opposite each other we get a parallelogram.
Since the angle sizes are not fixed just because the side lengths are fixed, we actually have an infinite number of kites and parallelograms.

10. This time we could have the equal sides opposite each other, in which case we might get a trapezoid (but not necessarily a trapezoid since the angles are not fixed). Again, there are many different possibilities, because the figure is not rigid.


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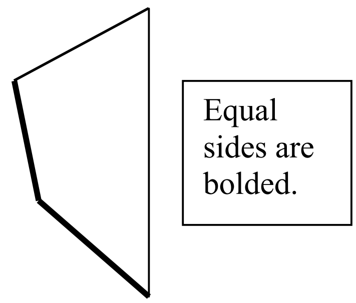
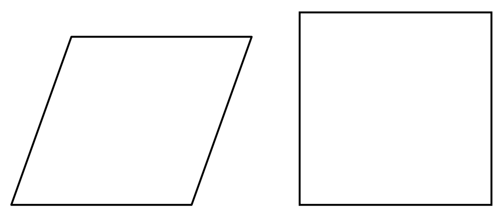
<td>Or we could have the equal sides adjacent to each other. Angles are not fixed, so an infinite number of shapes is possible, none of which is a parallelogram. A trapezoid is possible again.


11.
Since all 4 sides are equal we must have a rhombus or a square (which is a particular kind of rhombus). Since the angles are not fixed the sketch shows only one of the possible rhombuses (on the left).

12.
This time NO quadrilateral is possible, because the sum of the three sides 4 + 3 + 5 is not long enough to span the fourth side.</td>
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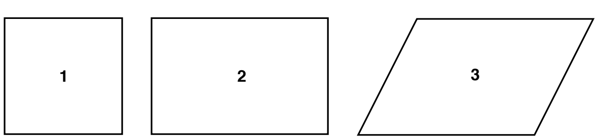
<td>22.
a. In what ways are all three quadrilaterals below alike?
b. In what ways does each quadrilateral differ from the others?

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<td>22.
(This problem makes students look for properties of each quadrilateral, such as angle sizes, side lengths, parallel sides etc.)
a. All 3 quadrilaterals have opposite sides equal and parallel. All 3 quadrilaterals have opposite angles equal. All 3 quadrilaterals have the same height. (Students may not notice this since they have not focused on height at this time.)
b. Quadrilateral 1 has all four sides equal; none of the others has this property. Quadrilateral 3 has no right angles; both of the others have this property. Quadrilateral</td>
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	3 also has unequal diagonals, while the other two have equal diagonals, but since these are not drawn students are unlikely to spot this.