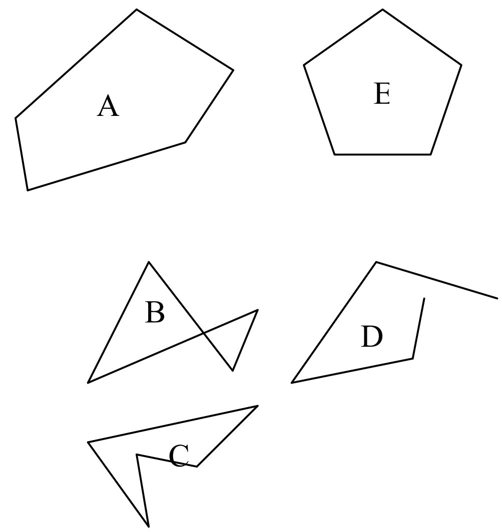
Vocabulary: *Shapes and Designs*

Concept | **Example**

Polygon: From examples of polygons and non-polygons students create the definition of a polygon as a **closed figure made of straight, non-intersecting edges**. The focus is not on learning names for polygons (triangle, quadrilateral, pentagon, hexagon, septagon, octagon etc.), but on reasoning about these figures.



Sorting polygons by properties: One can sort polygons into regular or non-regular polygons; or into concave and convex. (There are other ways to sort polygons.) A **regular** polygon has sides which are all equal and angle measures which are all equal. A **convex** polygon “bulges” out; all angles will be less than 180 degrees. A **concave** polygon collapses in; at least one angle will be more than 180 degrees.

“A” is **a non-regular** pentagon (5 sides). “E” is a **regular** pentagon. “A” and “E” are convex pentagons, while “C” is a concave non-regular pentagon. “B” and “D” are not polygons.

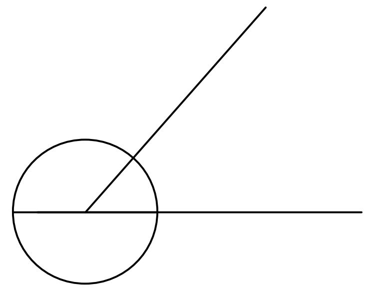
A square is regular; a non-square rectangle is not (because the sides are not congruent)

Angle: An angle can be thought of in three distinct ways: as an amount of rotation, as a wedge shape, as a space between two intersecting edges. For each of these ways of thinking students should be able to identify the **vertex** of the angle, and the **rays** that make the sides or arms of the angle. Students use an **angle-ruler** to measure angles.

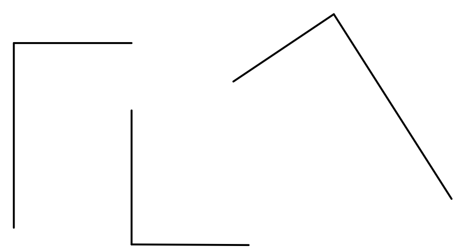
A slice of pizza is a wedge shape. If we slice a pizza into 6 equal pieces then the angle at the center of each wedge will be 60 degrees.

A tree branch comes off the trunk of a tree at an angle. If the angle is small then the branches leave little space between the trunk and the branch (like a poplar tree). If the angle is large then the branches leave a lot of space (like a pine tree).

Benchmark: Students use **90 degrees** as a benchmark. They have to recognize a right angle in various orientations. Other angles such as 45 degrees (half a right angle) or 30 degrees (third of a right angle) or 150 degrees (nearly 2 right angles) can be **estimated by comparing with a right angle.**



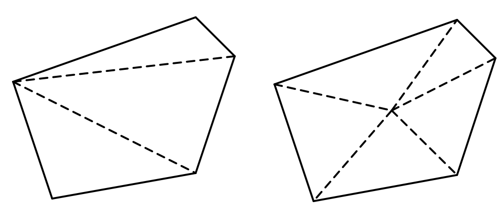
This is a simplified version of an angle ruler. The circular part is marked off as a scale, in degrees. **The vertex** of the angle is at the center of the circular part. The sides of the angle should line up with the line segments on the angle ruler, so that the number of degrees to rotate from one leg to another can be counted from zero. Since the arms of the angle can be thought of as **rays** (fixed starting point, infinite length) the length of the arms is not relevant to the size of the angle.



The above are all examples of 90 degree angles. The orientation and the length of the legs are irrelevant.

Angle sum of polygon: (Students first measured and then reasoned about the angle sum of any polygon.) By splitting any polygon into triangles they arrive at the formula that
Angle sum = (n-2)(180 degrees), where n is the number of sides. This works for any polygon, regular or not.

Each angle of a regular polygon: If all angles are equal then **each angle of a regular polygon = (n – 2)(180)/n degrees.**



In the pentagon on the left we make 3 triangles by drawing diagonals from a single vertex. The angles of these triangles comprise the angles of the pentagon. Thus, the angle sum of the pentagon = 3(angle sum of triangle) = 3(180) = 540 degrees. Students can reason why there are 2 fewer triangles than sides of the original polygon: hence, (n – 2)180.

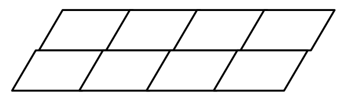
In the pentagon on the right, similar reasoning arrives at angle sum of pentagon = 5(angle sum of triangle) – 360 = 540 degrees. Notice that the central angle has to be subtracted because it is not an angle at any of the vertices of the pentagon.

Note: this reasoning works for regular and non-regular polygons. But if we have a regular pentagon (for example) we can divide the sum by 5 to get the size of **each angle**: 540/5 = 108 degrees.

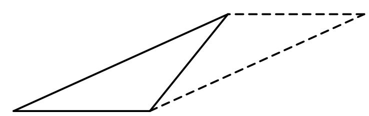
Tesselation: Tesselation means **tiling the plane** with a shape or a combination of shapes, leaving no gaps. The mention of a "plane" implies that whatever pattern has been chosen should go on forever.

Regular polygons that tile: Triangle, quadrilateral, hexagon.

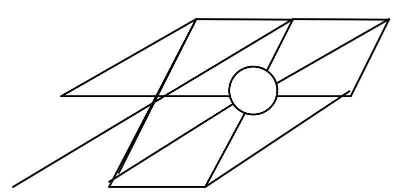
Non-regular polygons that tile: Any triangle or quadrilateral tiles (some pentagons, or other polygons also). The goal is not to memorize this but to understand that the angle sum being a factor of 360 degrees guarantees that the angles of ANY triangle or quadrilateral will fit around a vertex with no gaps.



The above is an example of tiling with a parallelogram. Because opposite sides are parallel and congruent the "next" parallelogram always fits along the side of the "last" parallelogram, creating one continuous set of parallel lines running horizontally across the plane, in this example.



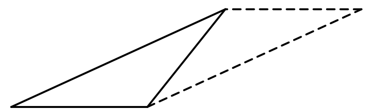
Students find out, by experimenting, that they can start with any triangle and make a copy that has been rotated, as above. This always results in a parallelogram. (At this point students observe this. Later, when they know more about symmetries and parallel lines they can confirm that this will always be true.) So, since ANY triangle can produce a parallelogram, and since ANY parallelogram will tile a plane, we can show that ANY triangle will tile a plane.



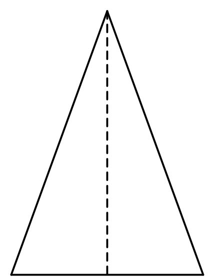
Notice that at the vertex highlighted there are 6 triangles coming together. The 6 angles around this vertex are actually the angles of the original triangle, occurring in pairs. Since the sum of the angles of a triangle is 180 degrees, the sum of these 6 angles is 360 degrees, which is WHY the tiling works.

Symmetries:
A shape that can rotate around a point and still fit inside the same outline has **rotation symmetry.** A shape that can flip over a line and still fit within the same outline has **reflection symmetry.** In this unit this is used to reason informally about certain polygons, and about tessellations. In future units students will investigate symmetry more thoroughly.

A parallelogram has rotation symmetry of 180 degrees around the midpoint of the diagonals:



An isosceles triangle has reflection symmetry:

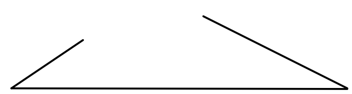


An equilateral triangle has both kinds of symmetry, reflection and rotation.

Conditions for a triangle: A triangle can be made with any three lengths, as long as the **sum of any 2 sides is greater than the third side**. Given three sides that make a triangle the triangle will be **unique**. It may have to be reflected or rotated but the final shape will be congruent to the original. The triangle is **rigid.**

The lengths 3, 5, 5 units will make a triangle. In fact once the triangle has been created it is unique.

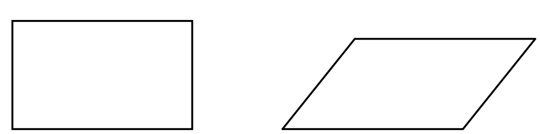
The lengths 1, 3, 5 units will NOT make a triangle; the two sides 1 and 3 units will not add up to enough to “close” the triangle.



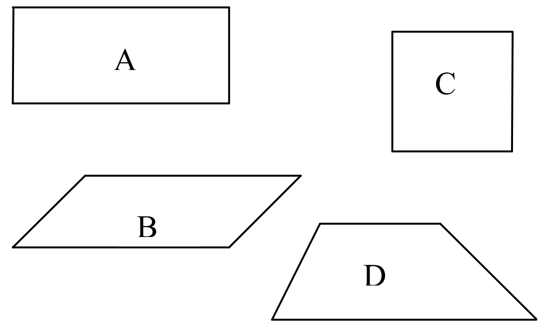
Classification of triangles: A **scalene triangle** has no equal sides; an **isosceles triangle** has 2 equal sides (and angles); an **equilateral triangle** has all sides (and angles) equal.

Conditions for a quadrilateral: By using polystrips students discovered that, given a useable combination of 4 side lengths, many different quadrilaterals can be created, because the angles can change. Unlike a triangle the quadrilateral is not rigid.

The two quadrilaterals below have the same lengths of sides, in the same order. However the angles are different, so the quadrilaterals are not congruent.

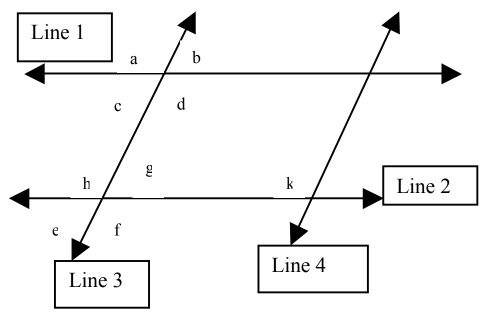


Classification of quadrilaterals: Students use squares, rectangles, parallelograms, trapezoids and kites in their attempts to make tessellations. Each of these is a special kind of quadrilateral. A **parallelogram** has opposite sides parallel (which permits a rotation around the intersection of the diagonals, to fit the same outline). Thus a square and rectangle are special kinds of parallelograms. A **rectangle has right angles**, and so does a square, but a **square also has all sides equal.** Thus a square is a special kind of rectangle. A **trapezoid** has only one pair of opposite sides parallel. The goal is not to memorize the names but to reason about the properties.



“A” is a rectangle, “B” is a parallelogram, C is a square, and “D” is a trapezoid.

Parallel Lines and transversals: Given two parallel lines and a **transversal** line crossing, certain pairs of congruent angles are formed. **Corresponding angles** are equal. **Alternate interior** angles are equal.



If we are given that Line 1 is parallel to Line 2 then we can conclude that a = h (corresponding angles) and b = g (corresponding) and c = e (corresponding) and d = h (alternate interior) and c = g (alternate interior).

Conversely, if we know that f = k then we can deduce that Line 3 is parallel to line 4.
Note: students do a lot more reasoning with parallel lines in later units.