# Vocabulary: Thinking With Mathematical Models

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<tr><th>Concept</th><th>Example</th></tr>
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**Function**: a relationship between 2 variables, say (x, y), so that, for any given value of x, a unique value of y can be calculated from an equation or read from a graph or a table.<br>
**Linear Function**: A relationship where the dependent variable changes at a **constant rate** in relationship to the change in the independent variable. The pattern of change can be recognized from a table or graph, and can be described using words or symbolic expressions.<br>
**Non-Linear Function**: A relationship where the dependent variable does not change at a constant rate. This non-constant rate will appear in the table, and will cause the graph to be a curve.

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- Y = 2x and y = 5 – 0.5x are **linear functions.** In general y = mx + b represents a linear function. Y = $0.5x^2$ and y = $2^x$ are examples of non-linear functions.
- Students are familiar with the pattern of a **constant rate** of change in y shown below. (See *Moving Straight Ahead*)

<table>
<tr><td>X</td><td>Y = 2x</td><td>Y = 5 – 0.5x</td></tr>
<tr><td>0</td><td>0</td><td>5</td></tr>
<tr><td>1</td><td>2</td><td>4.5</td></tr>
<tr><td>2</td><td>4</td><td>4</td></tr>
<tr><td>3</td><td>6</td><td>3.5</td></tr>
</table>

- The two types of patterns shown below are **non-linear**. They are investigated further in *Frogs, Fleas and Painted Cubes* and in *Growing, Growing, Growing*.

<table>
<tr><td>X</td><td>Y = $0.5x^2$</td><td>Y = $2^x$</td></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>0.5</td><td>2</td></tr>
<tr><td>2</td><td>2</td><td>4</td></tr>
<tr><td>3</td><td>4.5</td><td>8</td></tr>
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**Algebraic Expression:** A combination of symbols and operations that can be evaluated if the values of the variables are given.<br>
**Linear Equation**: This might be an equation in 2 variables, such as y = 2x + 3, where y changes at a **constant rate** in relation to changes in x (see “function” above), or in one variable, such as 17 = 2x + 3 (which is just a particular case of y = 2x + 3), or 3x – 2 = 2x + 3. (See *Moving Straight Ahead*.)<br>
**Solving Linear Equations:** In the first case above, y = 2x + 3, there is an infinite number of solutions, each of which is pictured as a point on the graph of the line y = 2x + 3. In the other two cases there is just one value of x that makes the equal sign true. (See *Moving Straight Ahead.)*<br>
**Slope**: The slope of a line the ratio of vertical change to horizontal change, or (change in y) divided by (corresponding change in x). (See *Moving Straight Ahead.)*

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- 2x + 3 is an **algebraic expression.** 3x – 2 is a different expression. Notice that we are not asked to FIND a value of x for either of the expressions. We may substitute ANY values of x into these expressions to evaluate the expression. For example, if we insert x = 1, we get 5 for the value of the first expression.
- If we connect 2 expressions with an equal sign, we are asserting they are equal for some value(s) of the variable. For example, the **linear equation**<br>3x – 2 = 2x + 3<br>is only true for one value of x. One efficient way of **solving** this equation would be to do the same operations to both sides, using Properties of Equalities:<br>3x – 2 **– 2x** = 2x + 3 **– 2x**, or,<br>x – 2 = 3.<br>x – 2 **+ 2** = 3 **+ 2**, or,

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<td>Y-Intercept: The point where the graph of a function crosses the y-axis. Since the point is on the y-axis the coordinates are (0, something). So we could say the y-intercept is the value of the dependent variable when x is 0. (See Moving Straight Ahead.)</td>
<td>$x = 5$<br>Or to use a graphical way of solving.<br>(See Moving Straight Ahead)</td>
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<td>Linear Inequality: A comparison between 2 linear expressions, such as $17 > 2x + 3$, or $3x – 2 < 2x + 3$. This time we want to find the solutions that make the inequality sign true.</td>
<td>The rules for solving equations (see above) apply to solving inequalities, except that when we have to multiply or divide both sides of the inequality by a negative the order is reversed. For example:
<ul>
<li>$3x – 2 < 2x + 3$<br>$3x – 2 – 2x < 2x + 3 – 2x$ (subtracting 2x from both sides)<br>$x – 2 < 3$<br>$x – 2 + 2 < 3 + 2$ (adding 2 to both sides)<br>$x < 5$. This means that any value lass than 5 is a solution for the original inequality.</li>
<li>$-3x – 5 < 4x + 2$<br>$-3x -5 -4x < 4x + 2 -4x$<br>(subtracting 4x from both sides)<br>$-7x -5 < 2$<br>$-7x – 5 + 5 < 2 + 5$<br>(adding 5 to both sides)<br>$-7x < 7$<br>$\frac{-7x}{-7} > \frac{7}{-7}$<br>(dividing both sides by -7)<br>(Notice that the inequality sign reversed when both sides were divided by -7. This happens because dividing (or multiplying) by a negative changes positives to negatives and vice versa, and we know that positive integers are in the OPPOSITE order from negative integers, That is, $-3 < -2$, but $3 > 2$.)</li>
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<td>Direct variation: A relationship between 2 variables in which an increase in one variable by a particular factor creates an increase in the other variable by the same factor. A direct variation relationship, between x and y, for example, always has the form y = ax, so this is a particular case of a linear relationship. Since this can also be written as $\frac{y}{x} = a$, we can say that in a direct</td>
<td><ul><li>$y = 0.6x$ shows a direct variation between x and y. As x increases, y also increases. Substituting 2 for x, we get 1.2 for y. If we double the value of x, to 4, we get double the value of y, 2.4. In every case the value of $\frac{y}{x} = 0.6$.</li></ul>
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Y</td><td>0.6</td><td>1.2</td><td>1.8</td><td>2.4</td></tr>
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<td>variation relationship the ratio of the variables is always a constant.</td>
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- Y = 2x + 3 would NOT be a direct variation. When x = 10, y = 23. If we double x, say x = 20, then the corresponding y value is not doubled.

  There is no constant value for $\frac{y}{x}$ .

<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Y</td><td>5</td><td>7</td><td>9</td><td>11</td></tr>
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<td>Inverse Variation: A relationship between 2 variables in which an increase in one variable by a particular factor causes a decrease in the other variable by the same factor. An inverse variation relationship between x and y always has the format y = $\frac{a}{x}$ . Since this can also be written as xy = a we could say that in an inverse variation the product of the variables is always constant. The graph has a characteristic curved shape.</td>
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- y = $\frac{2}{x}$ shows an **inverse variation** between x and y. As x increases, y decreases. When x = 0.1, y = 20. If we multiply the given x value by a factor of 4, say, then the new y-value is one fourth of what it was. y = $\frac{2}{0.4}$ = 5. In every case xy = 2.

<table>
<tr><td>X</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td></tr>
<tr><td>Y</td><td>20</td><td>10</td><td>$6\frac{2}{3}$</td><td>5</td></tr>
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<td>Mathematical Modeling: a process by which mathematical objects and operations can be used as approximations to real-life data patterns. We know that the mathematical model will not fit the real life example exactly, but there is enough of a pattern in the situation to make the model (which in this unit is a linear or non-linear function) fit reasonably well, and make predictions reasonable.</td>
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- Suppose we collect data about the outside temperature as a plane ascends. We would see that the temperature falls as the height increases. But we would have no way of making any accurate prediction unless we look for a **pattern in the data**. If we place the data in a table and look at the pattern of change, we might be able to determine if the relationship is approximately linear or not. We might also make a graph and examine **the shape of the graph**, to determine if the relationship is linear, or if the pattern fits some other relationship we recognize.

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