Vocabulary: *Variables and Patterns*

<table>
<tr><th>Concepts</th><th>Examples</th></tr>
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Variable: A changing quantity or a symbol representing a changing quantity. (Later students may use variables to refer to matrices or functions, but for now variables are changing *quantities*.)

Table of Data: Information about a situation which has been organized into a 1-variable or 2-variable table, prior to further analysis of the pattern of change in the variables.

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1. A child's height might be represented as h and a parent might record changes in the height as time passes. Height is a **variable quantity.** We can use any symbol or letter to represent height but usually we make the letter relate to the real situation.

2. *Blood pressure might be represented by b* and a patient's weight by w, and a doctor might record changes in blood pressure and compare these to the changes in a patient's weight. Blood pressure is a **variable quantity.** Often the changes in one variable are recorded as time passes, but sometimes, as in the case of the blood pressure example, neither of the two variable quantities recorded is time.

3. Suppose you drove your child to hockey practice and kept track of miles driven as time passes. You know that sometimes you travel many miles in a short time, when on a highway perhaps, and sometimes you travel only a few miles in a long time, when stopped frequently by lights, for example. A **table of data** might look like this for a 20 minute journey:

<table>
<tr><td>Time in minutes, t</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td></tr>
<tr><td>Distance in miles, d</td><td>0</td><td>1</td><td>5</td><td>7</td><td>8</td></tr>
</table>

Since the time variable is recorded every 5 minutes you can see that the first and last 5 minute sections were slowest.

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Patterns of Change: Various patterns of change occur in common situations, linear being the pattern focused on in this unit. Analyzing patterns of change in the variables tells us something about the relationship and permits us to extend the pattern and predict further values of the variables.

Relationship: Often there is a clear and describable relationship between 2 variables. We can describe this relationship *in words*, such as, "When A increases by 1 B increases by 3,"

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4. Compare these three tables. What can we say about the ***patterns or relationships*** *shown?*

<table>
<tr><td>A</td><td>B</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>

<table>
<tr><td>C</td><td>D</td></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>8</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>

<table>
<tr><td>E</td><td>F</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>?</td></tr>
</table>

In all three tables the second variable is ***increasing***, but the patterns are different. The first table shows

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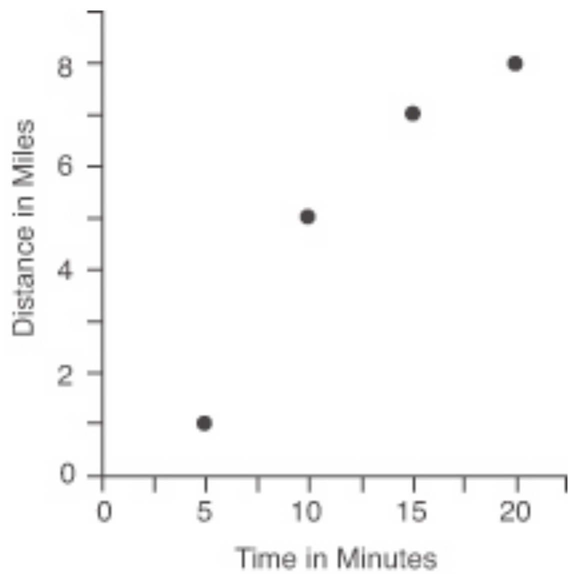
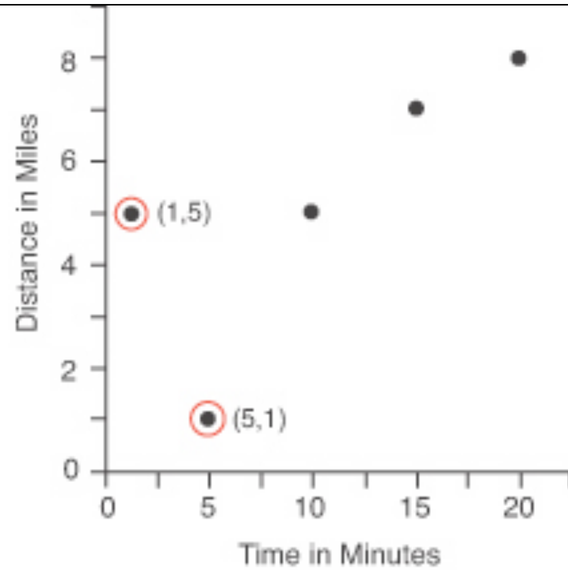
or, "When C increases by 1 D doubles its last value." Or we may see *a formula or rule* to describe the relationship. Sometimes the relationship is not so regular, and all we can say is that there is an *increasing or decreasing relationship*, for example.	that B increases by *adding 3* every time A increases by 1. We say that B is increasing at a **constant rate** (a *linear relationship*), and we feel reasonably confident about predicting the value of B when A is 4. The second table also displays a regular pattern of growth for D, but it is not an additive pattern. Nevertheless we feel reasonably confident that we can predict the value of D when C is 4. The last table shows no regular pattern of change in F, so we would not attempt a prediction.

Coordinate Pair or Ordered Pair: a pair of numbers recorded in parentheses, to show corresponding values of two related variables. Two Dimensional Graph: is a picture of a relationship that might have been previously shown in a table. Every point on the graph represents an *ordered pair* from the table. Axes: In every ordered pair the first of the numbers is the *x-coordinate* and the second of the numbers is the *y-coordinate* (unless some other variable names have been assigned). This convention is useful when we record the coordinate pair as a point on a graph, because by convention we assign the first coordinate to the *horizontal axis*, and the second to the *vertical axis*.	*5. Does the **order** matter in a coordinate pair?* (5,1) and (10,5) and (15, 7) indicate the (times, distances) in the example of driving your child to hockey practice. The order matters and is established in the table. (5,1) means that in 5 minutes 1 mile has been covered. (1,5) would mean that in 1 minute 5 miles had been traveled. *6. Graph the **coordinate pairs** in example 3.* Below is a graph of the time-distance data for the “driving to hockey practice example.”  To graph (5,1) we count 5 units right on the horizontal axis (minutes) and 1 unit up on the vertical axis (miles), and find the point that matches both of these instructions.

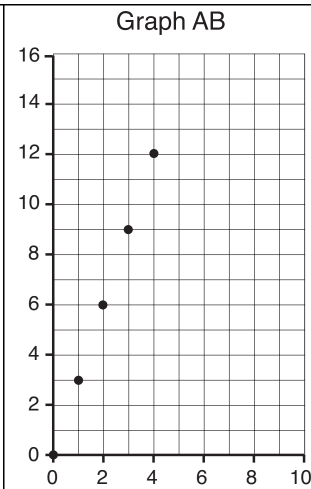


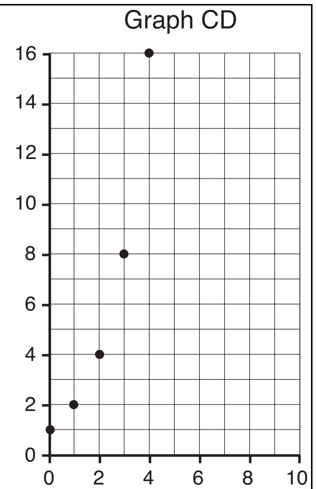
7. *How do we decide which variable should be measured on the horizontal axis, and which on the vertical axis?*

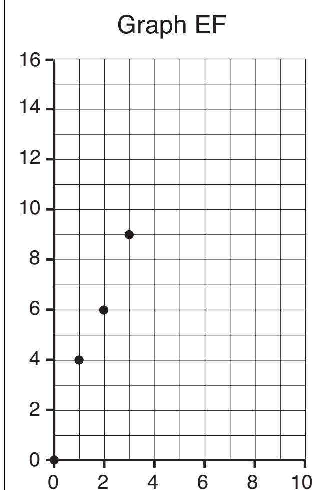
In the "driving to hockey practice" example, the distance driven **depends** on time passing, so distance is on the vertical axis. If your doctor recorded your blood pressure and weight he/she would place weight on the horizontal axis, and blood pressure on the vertical axis to show how blood pressure responds to or **depends o**n weight. He may ask you to "manipulate" the weight variable by losing weight, to see if blood pressure responds. The **dependent variable is recorded on the vertical axis**, **and the independent variable on the horizontal axis.**

Shape of the graph: Tells a story of how changes in one variable are related to changes in another variable. It is often clear from the *shape* of a graph that there is an *underlying pattern or relationship* even when the numbers in the table are sufficiently complicated to obscure the pattern. Only simple *linear change* is investigated in this unit. This is the foundation for other patterns of change and other shapes of graph.

Scale on Axes: Selecting a scale depends on having a good feel for the *range* of values. Sometimes the table is a good starting point and helps dictate scale. The axes are always marked in equal increments.







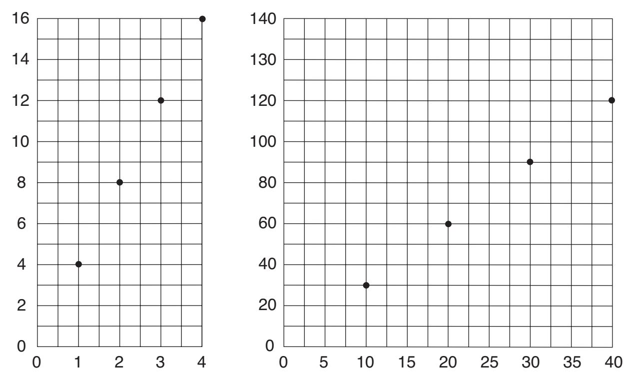
8. The graphs above show the relationships between (A, B), (C,D) and (E,F) as in the tables from example 4 in the prior section. It is clear from the regular *shapes* of the first 2 graphs that there are *underlying patterns*.

9. How does scale *affect the pattern of change on the graph?*
Shown below are 2 examples of linear patterns of change. Both tables and graphs show a ***constant*** *rate of increase* in the dependent variable; at first sight the rate of change appears to be much greater in the first graph because the y values appear to be increasing faster, as shown by the steepness of the line of points. Scale is an issue here because the axes are marked differently, and so we must be

careful in thinking about the rate of change in Y. In fact the rate of change in the first graph/ table is 4 units increase for Y for each 1 unit increase for X. The rate of change in the second table/ graph is 30 units increase in Y for each 10 units increase in X, or 3 units change in Y per 1 unit change in X.

X	1	2	3	4
Y	4	8	12	16

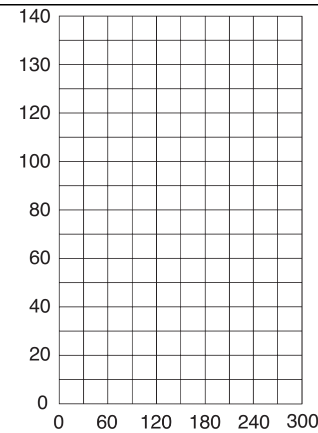
X	10	20	30	40
Y	30	60	90	120



10. Choose a **scale** *to graph this data about heart rate after exercise:*

Time (seconds)	0	30	60	120	300
Heart Rate/ minute	140	120	90	80	70

The horizontal axis would need to show a **range of values** from 0 to 300. Marking this axis in 10 *equally spaced* **increments** of 30 would be convenient. The vertical axis would have to show a range of values from 70 to 140. Generally, it is safer to show **the origin** (0,0), so, even though this “wastes” space the vertical axis might be marked from 0 to 140; increments of 10 would make a convenient scale.

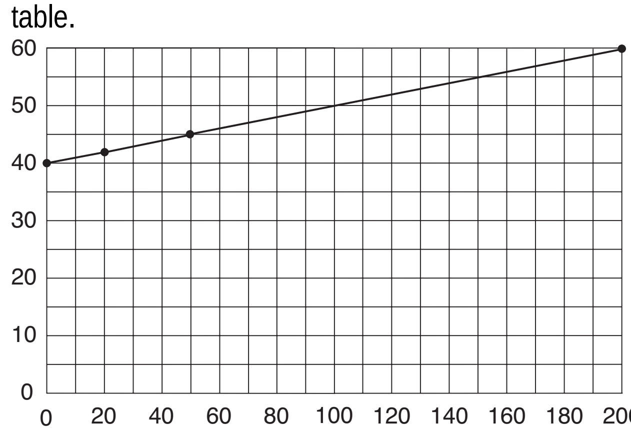


Discrete or Continuous Graph: All of the above graphs are *discrete* graphs. This means they are shown as disconnected points. However, when the pattern seems very strong, and when we feel we can predict meaningful points between the given data points we may want to connect the points with a *continuous* line or smooth curve.

11. If a car rental company says that the rate is $40 a day and $0.10 a mile and shows some examples in a table, extend the pattern to show other possible charges, beyond or between the given information.

Miles driven in one day	0	20	50	200
Cost for 1 day	40	42	45	60

By graphing the coordinate pairs given in the table we can see a clear pattern. It makes sense to use this pattern to predict other coordinate pairs that would exist between these points, for other possible "miles driven." By connecting the points we make a line that passes through intermediate points (such as (30, 43) and (35, 43.5), and also predicts other points such as (250, 65) which are beyond this table.



If we want to figure out the cost to drive 100 miles

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<td>we can work this out symbolically
(40 + 100(0.10) = 50)
or we can look on the graph for the point with the x-coordinate 100. (100, 50).

12. Does it make sense to connect the given data points in the table below?
<table>
<tr><td>Number of movie tickets</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Cost</td><td>5.50</td><td>11.00</td><td>16.50</td><td>22.00</td></tr>
</table>

There would be no point in asking about a point between (1, 5.50) and (2, 11.00) because there is no meaningful point there. We cannot buy a fraction of a ticket. These points are discrete points</td>
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<tr>
<td>Equation or symbolic formula: When a pattern is very regular we can often find a rule or equation to describe it. The advantage of the rule is that we can predict other ordered pairs. In this unit all equations are linear equations, either one-step or two-step.</td>
<td>13. In the movie ticket example above, Cost = 5.50 times number of tickets, or, briefly, C = 5.5t. This is a one-step equation. We can substitute any value for t and multiply by 5.5 to find the cost.

14. The car rental example above would be described by C = 40 + 0.10(m). This is a two-step equation. We can substitute any value for m, then multiply by 0.10 and add 40 to find the cost.</td>
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<td>Distance, rate and time context: Many of the examples in this unit are in the distance, rate and time context. “Rate” is a centrally important idea that occurs in many contexts (rental rate per mile, rate per movie ticket, heart rate per minute) but is more easily understood in this familiar context.</td>
<td>15. If the rate is constant at 10 miles per hour:
<table>
<tr><td>Time (hours)</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Distance (miles)</td><td>0</td><td>10</td><td>20</td><td>30</td></tr>
</table>
Write an equation for this relationship.

The relationship is linear (distance grows at a constant rate, by adding 10 miles each hour) and can be captured by the rule or equation, D = 10t. If the rate was a constant 20 mph then the rule would be D = 20t.

In general, D = rt if the rate is constant.</td>
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Using Graphing Calculators: to organize and represent data and to analyze linear relationships.	