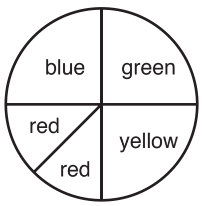
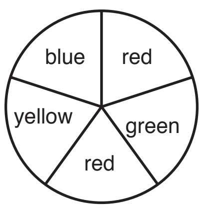
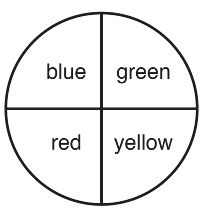
Homework Example from ACE: ***What Do You Expect?***
Investigation 1: # 5, 8, 11, 19.
Investigation 2: # 5, 11.
Investigation 3: # 1, 6.
Investigation 4: # 3, 13

ACE Question	Possible solution
Investigation 1	
5. Monita and Kyan are analyzing a game involving two different spinners. For each turn, a player spins each spinner once. They make this diagram of equally likely outcomes to find theoretical probabilities. Choose the spinner that could be spinner X. A.  B.  C.  D. None of these is correct.	5. The first stage of the counting tree indicates that there are 5 equally likely possibilities, referring to positions on the spinner: blue, red, red, green, yellow. Notice that “red” appears twice and so “red” actually has twice the probability of any of the other colors. We need a spinner that shows this. The only spinner that has this arrangement is choice B. It has a red area which is twice the size of the other colored areas. *(Note: the second stage of the tree must refer to the second spinner, which has apparently got “1’s” and “2’s” and “3’s” in some arrangement.)*

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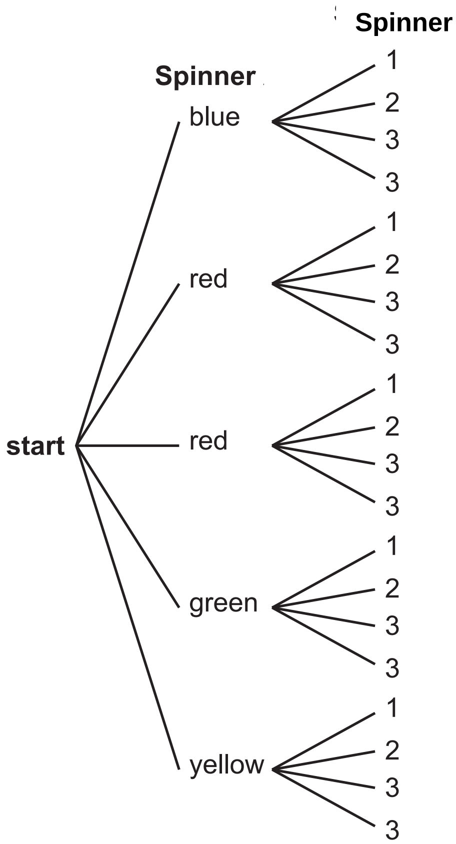
<td>
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<tr>
<td>8. What is the probability of getting red on Spinner X and 3 on Spinner Y?</td>
<td>8. A counting tree is just a way of making an organized list. From the above tree we have the list of equally likely outcomes, starting at the top of the tree: B1, B2, B3, B3, R1, R2, R3, R3, R1, R2, R3, R3, G1, G2, G3, G3, Y1, Y2, Y3, Y3. There are 20 possibilities. Four of them are R3. The probability of a red on X and a 3 on Y is $\frac{4}{20}$.</td>
</tr>
<tr>
<td>11. Raymundo invented the Prime Number Multiplication Game. In this game, two number cubes are rolled. Player A gets 10 points if the product is prime, and Player B gets 1 point if the product is not prime. Raymundo thinks this scoring system is reasonable because there are many more ways to roll a non-prime product than a prime product.</td>
<td>11.
a. One way to organize a 2 stage event like this is to make a tree or a list. Another way is to make a chart as below. The numbers on the top row are the result of rolling the first cube. The numbers in the left column are the result of rolling the second cube. The numbers in the inside of the table are the products. We could have made a long list of these outcomes: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3) etc.</td>
</tr>
</table>

a. If the cubes are rolled 100 times, how many points would you expect Player A to score? How many points would you expect Player B to score?

b. Is Raymundo's game a fair game? Explain why or why not.

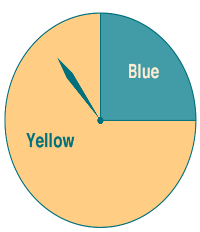
	1	2	3	4	5	6
1	1	**2**	**3**	4	**5**	6
2	**2**	4	6	8	10	12
3	**3**	6	9	12	15	18
4	4	8	12	16	20	24
5	**5**	10	15	20	25	30
6	6	12	18	24	30	36

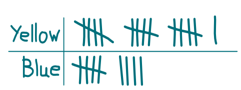
There are 36 equally likely outcomes. The prime products are 2, 3, 5. Theoretically, the probability of rolling a prime product is $\frac{6}{36}$. (Bolded in chart)

If we roll the cubes 100 times then $\frac{1}{6}$ of the time we would expect to get a prime product. Thus, we would expect about 17 prime products. Player A would score 17(10) points = 170 points. Player B would win on the other 83 games and get 83 points.

b. No, this is not a fair game, The players do not have equal chances to win.

19. Fala spins the spinner below several times and tallies the results in a table.





a. How many times did Fala spin the spinner?

b. What percent of the spins landed in the blue region? In the yellow region?

c. According to the theoretical probabilities, what percent of the spins should land in the blue region? In the yellow region?

d. Compare the experimental probability of the spinner landing in each region to the theoretical probability. If the

19.

a. Fala spun the spinner 25 times. (See tally marks.)

b. $\frac{9}{25}$ or 36% land in the blue region. $\frac{16}{25}$ or 64% land in the yellow region.

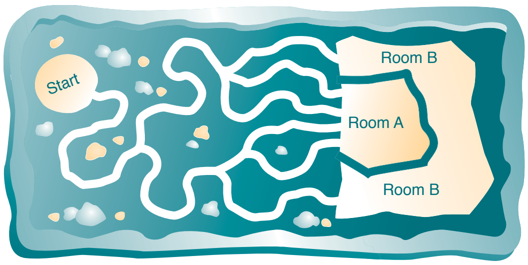
c. Theoretically 75% of the outcomes should be "yellow." This is because the spinner will land on yellow for a 270 degree rotation, out of 360 degrees. Theoretically 25% of the outcomes should be blue.

d. The theoretical probability (75%) of yellow is higher than the experimental probability (64%). However, this was a very few trials, only 25. if we did many more trials we would expect the theoretical and experimental probabilities to converge. If they did not we would have to check the fairness of the spinner's action.

probabilities are different, explain why.

Investigation 2

5. Kenisha changed the game in Problem 2.2 so it had the paths shown below.



a. If a player randomly selects a path at each fork, what is the theoretical probability that he or she will end up in Cave A? In Cave B? Show your work.

b. If you played this game 100 times, how many times would you expect to end up in Cave A? In Cave B?

5.

a. At the first fork there are 2 equally likely paths, an upper and a lower path. At the second fork the upper path splits into 3 equally likely paths, while the lower path splits into 2 equally likely paths. One of these last paths splits again into 2 equally likely paths, both leading to A. We can use an area model to represent this, where the relative sizes of the areas represents the probabilities. The top half of the diagram below represents the upper path. The bottom half represents the lower path.

]

<table>
<tr><td colspan="3">Upper, leading to B</td><td colspan="3">Upper, Leading to A</td><td colspan="3">Upper, leading to A</td></tr>
<tr><td colspan="2">Lower, leading to A</td><td colspan="2">Lower, leading to A</td><td colspan="5">Lower, leading to B</td></tr>
</table>

A simplified version below makes it easier to see what the fractions are. Probability of ending in B is shaded, A is unshaded. Notice that we have thirds in the top half of the diagram, and halves and quarters in the bottom half of the diagram. These parts have been subdivided to find a common denominator.

The theoretical probability of ending in A is represented by the fraction $\frac{14}{24}$. and the probability of ending in B is $\frac{10}{24}$.

b. If you played this game 100 times then the

	you would expect to end in A about $\frac{14}{24}$ of 100 times, or about 58 times. You would end in B about 42 times.
11. Brianna (from Problem 2.3) is given each set of marbles to distribute between the two containers. What arrangement would give Emmanuel the best chance of drawing a green marble? Three blue and two green marbles.	11. The possible arrangements (order does ot matter) are:

First container	Second Container
BBBGG	
BBBG	G
BBGG	B
BBB	GG
BBG	BG
BGG	BB

We can have 1, 2, 3, 4 or 5 marbles in each container. The above are all the possibilities without switching the containers,

We have to think about each of these arrangements separately. An area model again helps to think about this 2-stage event: choose a container, then choose a marble.

For the (BBBGG, empty) arrangement:

empty				
B	B	B	G	G

P(green) = $\frac{2}{10}$.

For the BBBG, G arrangement:

G			
B	B	B	G

P(green) = $\frac{5}{8}$

For the BBGG, B arrangement:

B			
B	B	G	G

P(green) = $\frac{1}{4}$.

For the BBB, GG arrangement:

B	B	B
G		G

P(green) = $\frac{1}{2}$.

<table>
<tr><td></td><td>The best arrangement is BBBG, G, if we are trying to maximize the probability of getting a green.</td></tr>
<tr><td>Investigation 3</td><td></td></tr>
<tr><td>1. In a one-and –one free throw situation, is the player with an 80% average most likely to score 0 points, 1 point, or 2 points. Make an area model to support your answer.</td><td>1.

First shot
<table>
<tr><td></td><td>hit</td><td>Miss</td></tr>
<tr><td>2nd shot hit</td><td>Hit both, 2 points</td><td rowspan="2">Miss so no 2nd shot. 0 points</td></tr>
<tr><td>miss</td><td>Hit then miss, 1 point</td></tr>
</table>
The largest area corresponds to the probability of 2 points. (If we subdivided this area into 100 equal square units to see what fraction this actually is, we would find that P(2 points) = 64%, P(1 point) = 16%, and P(0 points) = 20%.</td></tr>
<tr><td>6. Nishi, who has a a 60% free-throw average, is in a two-attempt free-throw situation. Remember, this means that she will attempt the second shot no matter what happens on the first shot.

a. Is Nishi most likely to score 0 points, 1 point, or 2 points? Explain your answer.

b. Nishi plans to keep track of her score on two-attempt free-throw situations. What average number of points can she expect to score per two-attempt situation?</td><td>6.

a.

First shot
<table>
<tr><td></td><td>hit</td><td>miss</td></tr>
<tr><td>2nd shot hit</td><td>Hit both, 2 points</td><td>Miss then hit, 1 point</td></tr>
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</table>
P(2 points) = 36%. P(1 point) = 24% + 24% = 48%. P(0 points) = 16%. She is most</td></tr>
</table>

	likely to score 1 point. b. If she continued as shown in the area model above, then in 100 attempts she would score 2 points 36 times, 1 point 48 times and 0 points 16 times. This would give her a total of 120 point on 100 attempts, or an average of 1.2 points per attempt.
Investigation 4	
3. Scout is about to have puppies. The vet thinks that Scout will have four puppies. Assume that each gender, male and female, are equally likely. a. List all the possible combinations of female and male puppies Scout might have. b. Is Scout more likely to have four male puppies, or two male puppies and two female puppies? Explain your reasoning.	3. a. Male and Female are equally likely. To distinguish one puppy from another we might think about birth order. So, for example, MMFM means that the first 2 were males and then a female and then a male. The list of possibilities is: MMMM, MMMF, MMFM, MFMM, FMMM, MMFF, MFMF, MFFM, FMFM. FFMM, FMMF, MFFF, FMFF, FFMF. FFFM. FFFF. (A tree diagram might be helpful.) b. There is only 1 chance out of 16 that Scout will have MMMM. There are 6 chances out of 16 that Scout will have 2 males and 2 females. Scout is more likely to have 2 males and 2 females. *(Note: MMFF is no more likely than MMMM. Each possibility has the same chance of happening, 1/16. But there are several different ways that we can order the 2 males and 2 females, so that the SUM of these probabilities is larger than P(MMMM).)*
13. In the *How Likely Is It?* unit, you learned about the genetics involved in having attached or non-attached earlobes. Every person has a combination of two tongue-curling alleles—TT, Tt, or tt—where T is the dominant tongue-curling allele, and t is the recessive non-tongue-curling allele. A person with at least one T allele will be able to curl his or her tongue. Ken found out that his tongue-curling alleles are tt and his wife Diane's alleles are Tt. He makes this table to help him determine the possible outcomes for their children.	13. a. There are 2 possible outcomes for each child born, and they are equally likely: not curl ("tt" on the chart), curl ("Tt" on the chart). These are written as "C" and "N" below.

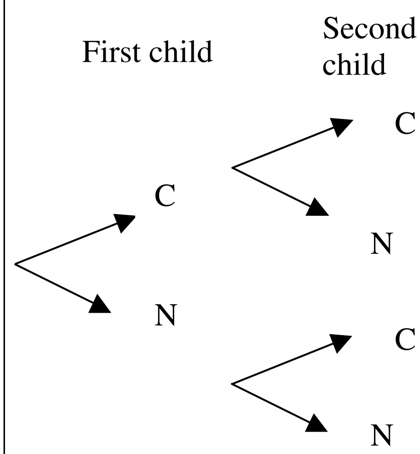
		Diane	
		T	t
Ken	t	Tt	tt
	t	Tt	tt

The table shows that the possible combinations are Tt, Tt, tt, and tt. This means that each child has a 50% chance of being able to curl his or her tongue.

a. Suppose Ken and Diane have two children, what is the probability that both of the children will be able to curl their tongues? Make a counting tree to help you answer this question.

b. If Ken and Diane have four children, what is the probability that none of the children will be able to curl their tongues?

c. Suppose Ken and Diane have four children, what is the probability that only the oldest child will be able to curl his or her tongue?



So, the possible outcomes are CC, CN, NC, NN, and they are all equally likely. The probability that they will have 2 children who can curl their tongues, CC, is $\frac{1}{4}$.

b. We need a 4-stage counting tree to answer this one. This time we want P(NNNN). There will be 16 possible outcomes, of which NNNN occurs just once. Probability of 4 children unable to curl their tongues is 1/16.

c. The same counting tree that was used in part b will work for part c. This time we want P(CNNN). 1/16.

Note: in part c the order is specified. If the question asked for the probability that only 1 child (in any birth order) would be able to curl his/her tongue then we would have to count CNNN, NCNN, NNCN, NNNC.