

Prime Time: Homework Examples from ACE

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Investigation 1: *Building on Factors and Multiples*
ACE #8

In the following Factor Game, some of the scores are missing on the tally sheet.

| Cathy | Keiko |
|-------|-------|
| 24 | 36 |
| 21 | ? |
| ? | 9 |
| ? | 30 |

- In Round 1, Cathy chose 24. Keiko scored 36 points. Make a game board. Mark Cathy's choice and Keiko's factors.
- In Round 2, it is Keiko's turn. Cathy scored 21 points. What number did Keiko pick?
- In Round 3, it is Cathy's turn. Keiko scored 9 points. What number might Cathy have picked?
- In Round 4, it is Keiko's turn. Keiko chose 30. What did Cathy score?
- In Round 5, it is Cathy's turn. What is Cathy's best move? Explain your choice.

In Problem 1.1, students play *The Factor Game*. The purpose of this game is to practice finding factors of whole numbers and to learn some interesting things about numbers. A copy of the board used for the factor game can be found in the student book.

- Cathy chose 24, so Keiko would mark all the proper factors (all the numbers you can evenly divide into 24, except 24. 24 is a factor of 24, but a "proper factor" are only those factors less than the original number). The proper factors of 24 are: 1, 2, 3, 4, 6, 8, and 12. Adding these up will give: $1 + 2 + 3 + 4 + 6 + 8 + 12 = 36$ (Keiko's score).

Score

| Cathy | Keiko |
|-------|-------|
| 24 | 36 |

b. Keiko picked 28. The proper factors of 28 are: 1, 2, 4, 7 and 14. The numbers 1, 2, and 4 are already marked, so Cathy's score is 21 by adding the two remaining factors 7 and 14. Here is the game board at this stage in the game:

Score

| Cathy | Keiko |
|-------|-------|
| 24 | 36 |
| 21 | 28 |

- c. Cathy picked 18 or 27. Keiko scored 9 points by circling 9 (the factor of 18 or 27 that is not already circled).
- d. Cathy scores 30 points for 5, 10, and 15, (the factors of 30 that are not already circled).
- e. The only numbers remaining with uncircled factors are 22 (11) and 26 (13). Cathy should choose 26 because she will score 13 more points than Keiko.

Investigation 1: *Building on Factors and Multiples*

ACE #28

- a. What type of number has exactly two factors? Give examples.
- b. What type of number has an odd number of factors? Give examples.
- c. Are there any prime numbers that are also square numbers? Give an example or explain why not.

- a. Prime numbers are special numbers that have only two factors, 1 and itself. Examples: 2, 3, 5, 7, 11, ... Two is the only even prime number because all other even numbers have at least three factors, 1, 2 and itself. The number 1, is not a prime number because it has only factor, 1.
- b. Square numbers have an odd numbers of factors. Examples: 1, 4, 9, 16, 25, 36, ... If you multiply a whole number by itself you will get a square number. These numbers are called square numbers, because to find the area of a square you multiply the lengths of the two sides together, which are the same in a square.
- c. No prime number is also a square number. Students might use parts a and b to reason that because prime numbers have exactly two factors (from a) and square numbers have an odd number of factors (from b), you could never have a prime number that is also square. For square numbers that are 4 or larger, you can also reason that square numbers have at least three different factors, 1, the number you multiply by itself to get the square number (e.g. $2 \times 2 = 4$), and the square number itself.

Investigation 2: *Common Multiples and Common Factors*

ACE #11

Find two pairs of numbers with the given number as their least common multiple.
36

In Problems 2.1 and 2.2, students work on Least Common Multiples, often abbreviated as LCM. The Least Common Multiple of 2 or more numbers is the lowest multiple that is common to these numbers. For example the Least Common Multiple of 5 and 6 is 30 because 30 is the lowest number that is a multiple of 5 and 6 (note: there are other common multiples, such as 60, 90, 120..., but there is only one **Least** Common Multiple).

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40...

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48...

Students might conjecture (making an educated guess) about how to find the LCM of two numbers. For example, students might think that you can just multiply the two numbers together to get the LCM (which worked for 5 and 6). However, this does not always work, such as the Least Common Multiple of 5 and 10, would be 10 not 50.

To solve #11, students might try to start with factors of 36. By doing this, it will ensure that 36 eventually shows up as a multiple.

For example $4 \times 9 = 36$, so

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36...

Multiples of 9: 9, 18, 27, 36...

So 36 is the LCM(4, 9), because 36 is a common multiple to both lists, and is the least of the common multiples.

Another possible answer is 36 and 1. Note: multiples of 1 would be all whole numbers, and multiples of 36 start with 36.

Students should be cautious to not simply choose the two numbers that multiply together to produce 36, as it may not produce the Least Common Multiple. For example:

Multiples of 18: 18, 36, 54...

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38...

Although 36 is a common multiple, 18 is actually the least common multiple.

Investigation 2: *Common Multiples and Common Factors*

ACE #16, 17

List the common factors of each pair of numbers. Then find the greatest common factor of each pair.

#16) 18 and 30

#17) 9 and 25

Similar to finding the Least Common Multiple, the Greatest Common Factor (often abbreviated as GCF) of 2 or more numbers is the largest factor in common to those numbers. For example, the Greatest Common Factor of 8 and 10 is 2 because 2 is the largest number in both lists of factors.

Factors of 8: 1, 2, 4, 8

Factors of 10: 1, 2, 5, 10

#16) The factors of 18 are: 1, 2, 3, 6, 9, 18

The factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30

Common Factors of 18 and 30 are: 1, 2, 3, 6

Because 6 is the largest number that is a common factor, this means that 6 is the Greatest Common Factor of 18 and 30.

#17) The factors of 9 are: 1, 3, 9

The factors of 25 are: 1, 5, 25

Common factor of 9 and 25 is: 1

This means that the Greatest Common Factor of 9 and 25 is 1.

(Note: Because all whole numbers have 1 as a factor, any two whole numbers will have at least one common factor, the number 1.)

Investigation 2: *Common Multiples and Common Factors*
ACE #28

The cast of the school play had a party at the drama teacher's house. There were 20 cookies and 40 carrot sticks served as refreshments. Each cast member had the same number of whole cookies and the same number of whole carrot sticks, and nothing was left over. The drama teacher did not eat. How many cast members might have been at the party? Explain your answer.

Students have to look for common factors of 20 and 40. For example, 5 is a common factor so we can divide 20 cookies into 5 servings (5 people) of 4 cookies each, and we can divide

40 carrot sticks into 5 servings of 8 carrot sticks each.

20 members – each gets 1 cookie and 2 carrot sticks, or

10 – each gets 2 cookies and 4 carrot sticks, or 5 – each gets 4 cookies and 8 carrot sticks, or 4 – each gets 5 cookies and 10 carrot sticks, or

2 – each gets 10 cookies and 20 carrot sticks, or 1 – gets it all: 20 cookies and 40 carrot sticks.

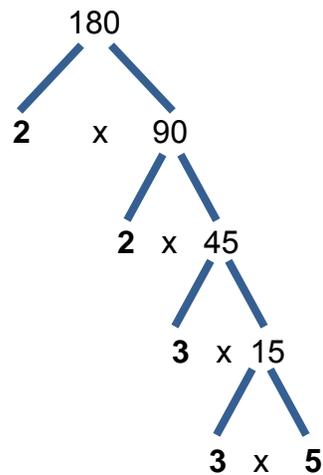
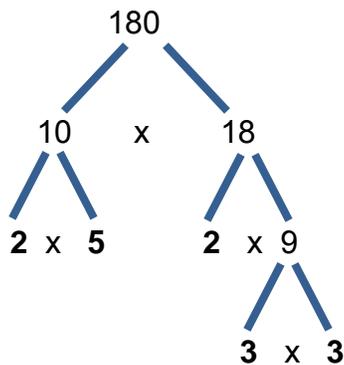
Investigation 3: Factorizations: Searching for Factor Strings
ACE #6

Write the prime factorization of each number in expanded form.
180.

In Investigation 3, students learn to find the prime factorization of a number. The prime factorization of a number is written as a product of prime numbers (those numbers that have only two factors), which is unique except for ordering. For example there is only one set of prime numbers that multiply together to give 30, which would be $2 \times 3 \times 5$. It is typical to write the prime factorization in order from least to greatest.

The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19...

One way to find the prime factorization is to create a factor tree. A “branch” stops once a prime number is found. Here’s two different ways to use factor trees to factor 180.



Prime Factorization:

$$2 \times 5 \times 2 \times 3 \times 3$$

$$2 \times 2 \times 3 \times 3 \times 5$$

Both prime factorizations are the same, except the ordering of the factors are different.

Investigation 3: *Factorizations: Searching for Factor Strings*

ACE #27

- a. 180 is the LCM of 12 and a number, N . What are possible values of N ?
 - b. 14 is the GCF of a number M and 210. What are the possible values of M ?
-
- a. One strategy to solve this problem is to start with factors of 180. The factors of 180 are: 1, 2, 3, 4, 5, 6, 9, 10, 18, 20, 30, 36, 45, 60, 90, 180. Choosing factors that are larger will get to 180 as a multiple “faster”. For example the multiples of 60 are: 60, 120, 180; but the multiples of 10 are: 10, 20, 30, 40, ..., 170, 180. The possible values for N are 45, 90 or 180. Any other factor of 180 has a common multiple with 12 that is less than 180.
 - b. One possible answer for M is 14. Other multiples of 14 would also, such as 28. However, if the multiple of 14 has a factor of 3 or 5, it will have a different GCF.

Investigation 3: *Factorizations: Searching for Factor Strings*
ACE #43

What is my number?

Clue 1 My number is a perfect square.

Clue 2 The only prime number in its prime factorization is 2.

Clue 3 My number is a factor of 32.

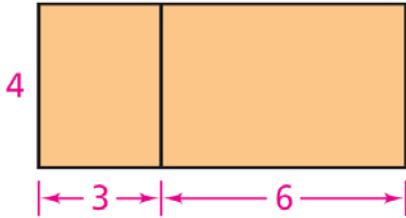
Clue 4 The sum of its digits is odd.

If we take the clues in the order we are given then clue 1 tells us to consider 1×1 , 2×2 , 3×3 , 4×4 , 5×5 , 6×6 etc. Clue 2 tells us to exclude any number that has prime factors other than 2. So we cannot use 3×3 , or 5×5 , or 6×6 (which does have 2 as a factor, but also has 3 as a factor). So now we consider only 2×2 , 4×4 , 8×8 , etc. Clue 3 says to consider only 1, 2, 4, 8, 16, 32. Taken with the other clues this leaves us to consider 4 and 16. The last clue narrows this to 16, since $1 + 6 = 7$, an odd number.

If we take the clues in a different order we will arrive at the same answer. The factors of 32 (clue 3) are 1, 2, 4, 8, and 16 and 32. Of these numbers, only 1, 16 and 32 have digits that add to odd numbers (Clue 4). One and 16 are square numbers. (Clue 1). Of these two numbers, only 16 has 2 in its prime factorization (Clue 2). The number is 16.

Investigation 4: *Linking Multiplication and Addition: The Distributive Property*
ACE #7

Write an expression for the area of the rectangle in two different ways. Then find the area using each expression.



In Investigation 4, students learn about the *Distributive Property*. This property is useful for simplifying expressions, especially when variables and numbers are involved in later units. There are two ways to write expressions to represent the area of the rectangle. These two ways are:

$4 \times (3+6)$ and $(4 \times 3) + (4 \times 6)$, which both give a total area of 36.

Investigation 4: *Linking Multiplication and Addition: The Distributive Property*
ACE #24

Replace m with a whole number to make the statement true
 $7(4 + m) = 49$

Students might solve this problem a variety of ways. If students recall that $7 \times 7 = 49$, the problem can be simplified to solving $(4 + m) = 7$. Students could also use the distributive property and simplify the equation to be:

$$7(4) + 7(m) = 49$$

$$28 + 7m = 49$$

Since $28 + 21 = 49$, this means that $7m = 21$, or in other words, that $m = 3$.