Decimal Ops: Homework Examples from ACE

Investigation 1: *Decimal Operations and Estimation*, ACE #3, #20, #23, #25
Investigation 2: *Adding and Subtracting Decimals*, ACE #8, #11, #18
Investigation 3: *Multiplying and Dividing Decimals*, ACE #1 a., #3, #24 d, #30 a & d
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Investigation 1: *Decimal Operations and Estimation*
ACE #3

- Write a number sentence for the problem.
- Estimate the answer.
- Use mental arithmetic, a calculator, or some other method to find the exact answer.
- Explain how your estimate helps you check the exact answer.

3. A local farm grows strawberries, blueberries, and raspberries. Customers can pick their own fruit at the farm for $2.95 per pound.
   a. The Payne family picks 10.5 pounds of strawberries. How much do they have to pay?
   b. A week later, the Paynes pick 6.75 pounds of blueberries and 5.2 pounds of strawberries. How much do they have to pay?

3. Solution: To write the number sentence, students first need to determine which operations can be used to solve the problem (addition, subtraction, multiplication, or division). Once they have the number sentence, then they can use number benchmarks to estimate the answer. Number benchmarks are commonly the closest whole number. In *Let's Be Rational*, students also used fractional number benchmarks of ½ .
   a. Number sentence: 2.95×10.5
   The estimate is $3 \times 11 = 33$, so the Paynes have to pay approximately $33.
   The exact answer is $2.95 \times 10.5 = 30.975$, so the Paynes will pay exactly $30.98.
   Because the exact answer, $30.98 is close to $33, you can assume your calculations are correct.
   b. There are two equivalent number sentences possible.
   $2.95 (6.75 + 5.2)$ which is equivalent to $2.95 \times 6.75 + 2.95 \times 5.2$
   The estimate is $3 (7 + 5)$ or $3 \times 7 + 3 \times 5$, so the Paynes will pay approximately $36.
   The exact answer is $2.95 (6.75 + 5.2) = 35.2525$, so the Paynes will pay exactly $35.25.
   Because the exact answer, $35.25 is close to $36, you can assume your calculations are correct.

Investigation 1: *Decimal Operations and Estimation*
ACE #20 and #23

For Exercises 18-23, write a whole number, mixed number, or fraction that could replace each decimal in an estimation task.

20. 0.42

20. Solution: $0.42 \approx \frac{2}{5} or \frac{1}{2}$

23. 4.25

20. Solution: $4.25 = 4\frac{1}{4}$

Investigation 1: *Decimal Operations and Estimation*
ACE #25

25. Elliot buys a 5-pound package of ground beef for $12.50.
    a. What is the unit price of the ground beef?
    b. At the same unit price, how much would 8 pound of ground beef cost?
    c. Varna buys a package of ground beef for $7.50 at the same unit price. What is the weight of the package?

25. Solution:

    a. The unit rate at dollars per pound is $\frac{\$12.50}{5 pounds}$ = $2.50 per pound.

    Because students may not be as familiar with shopping, they may use a less common form of unit price, pounds per dollar. In this case, $\frac{5 pounds}{\$12.50}$ = 0.80 pounds per dollar.

    b. 8 pounds at $2.50 per pound costs $8 \times 2.50 = 20$, or $20.

    c. $7.50 buys 3 pounds of ground beef. There are a few ways to solve this. Students may know that 2.50 + 2.50 + 2.50 = 7.50 because this is a common fact. Each $2.50 buys 1 pound a ground beef, so three $2.50s will buy 3 pounds of ground beef.

    .

Investigation 2: *Adding and Subtracting Decimals*
ACE #8

8. Place the decimal points in 102 and 19 so that the sum of the two numbers is 1.21.

Possible Solutions

8. Since the final answer is 1.21, which is between the benchmarks 1 and 1.5, so we know that 10.2 and 1.9 or 19 are all too large to be one of the addends. We need to place the decimals in places that make each of the addends smaller than 1.21.
   1.02 + 0.19 = 1.21.

Investigation 2: *Adding and Subtracting Decimals*
ACE #11

For Exercises 10-15, find each difference without using a calculator. Show your work and explain how an estimation strategy suggests that your answer is probably correct.

11. 4.54 – 2.9

Solution

11. 4.54 – 2.9 = 1.64
    One estimate would be 4.5 – 3 = 1.5. Because 1.5 is close to 1.64, the answer 1.64 is probably correct.

Investigation 2: *Adding and Subtracting Decimals*
ACE #18

18. Find the value of N that makes the mathematical sentence correct. Use fact families to help you.
    a. 2.3 + N = 3.42
    b. N – 11.6 = 3.75

Possible Solutions

18. In the context of addition and subtraction, "fact families" record 3 ways that we can see how the same two parts relate to the whole:
    part A + part B= whole,
    whole – part A = part B,
    whole – part B = part A.

a. The relationship 2.3 + N = 3.42 can also be written as 3.42 – N = 2.3 and 3.42 – 2.3 = N. The last equation is helpful in finding a value for N. N = 3.42 – 2.3 = 1.12.

b. The relationship N – 11.6 = 3.75 can also be written as N – 3.75 = 11.6 and N = 11.6 + 3.75. This last equation is helpful in finding N = 15.35.

Investigation 3: *Multiplying and Dividing Decimals*
ACE #1 a.& b

1. For each decimal multiplication problem below:
- Write each decimal factor as an equivalent fraction.
- Find the product of the two fractions.
- Write the product in equivalent decimal form.

a. 0.3×14
b. 1.2×3.54

Note that this problem is designed to help students connect decimals to their fractional meaning.

Solution #1 a. & b.

a. $0.3 \times 14 = \frac{3}{10} \times \frac{14}{1} = \frac{42}{10} = 4.2$

b. $1.2 \times 3.54 = \frac{12}{10} \times \frac{354}{100} = \frac{4248}{1000} = 4.248$

Investigation 3: *Multiplying and Dividing Decimals*
ACE #3

For Exercises 3-6, use the given information to find each product.

3. If $34 \times 8 = 272$, what is 3.4×0.8? What is 0.34×0.08?

Solution

If $34 \times 8 = 272$, then $3.4 \times 0.8 = 2.72$ and $0.34 \times 0.08 = 0.0272$.

Note that this problem is designed to help students connect the common algorithm for multiplying decimals to that of multiplying whole numbers. Connecting back to earlier problems, students may relate the decimals to their fractional equivalents as follows:

$$3.4 \times 0.8 = \frac{34}{10} \times \frac{8}{10} = \frac{272}{100} = 2.72$$

and

$$0.34 \times 0.08 = \frac{34}{100} \times \frac{8}{100} = \frac{272}{10,000} = 0.0272$$

Investigation 3: *Multiplying and Dividing Decimals*
ACE #24 d

24. For each decimal division problem below:

- Write the dividend and divisor as fractions with common denominators.
- Estimate the quotient of the two fractions.
- Find the exact value of the quotient.

d. $7.6 \div 0.04$

Solution #24. d.

$$7.6 \div 0.04 = \frac{76}{10} \div \frac{4}{100} = \frac{760}{100} \div \frac{4}{100} = 190$$

Estimation of the quotient could be $800 \div 4$ is 200 which is fairly close to 190.

Investigation 3: *Multiplying and Dividing Decimals*
ACE #30 a & d

30. Find the value of N that makes each equation true

a. $3.2 \times N = 0.96$

d. $N \div 0.8 = 3.5$

Solutions: We can use Fact Families to rewrite the equations in equivalent forms.

a. $3.2 \times N = 0.96$ can also be written as $N \times 3.2 = 0.96$; $0.96 \div N = 3.2$; and $0.96 \div 3.2 = N$.

This last equation is most useful to find N. If we only think about the digits in the problem we have $96 \div 32$ which would is 3. Now we have to think about place value.

3.2 x 3 = 9.6

3.2 x 0.3 = 0.96.

So N = 0.3

Another way to solve this problem is to rewrite the decimals as fractions and solve:

$\frac{32}{10} \times N = \frac{96}{100}$ Using fact families, we rewrite this equation and solve:

$$N = \frac{96}{100} \div \frac{32}{10} = \frac{96}{100} \div \frac{320}{100} = 96 \div 320 = 0.3$$

Check: To check, substitute N=0.3 in the original equation. $3.2 \times 0.3 = 0.96$

$\frac{32}{10} \times \frac{3}{10} = \frac{96}{100}$. These equations are true, so the answer is correct.

d. $N \div 0.8 = 3.5$ can also be written as $N \div 3.5 = 0.8$ and $N = 3.5 \times 0.8$ and $N = 0.8 \times 3.5$.

The last two equations are the most useful to find the value of N. If we only think about the digits in the problem we have 35×8 which is 280. Now we have to think about place value.

If $35 \times 8 = 280$, then $3.5 \times 8 = 28$ and $3.5 \times 0.8 = 2.8$.

So N=2.8.

Another way to solve this problem is to rewrite the problem with fractions of common denominators:

$$N \div \frac{8}{10} = \frac{35}{10}$$

$$N = \frac{8}{10} \times \frac{35}{10} = \frac{280}{100} = 2.8$$

Check: To check, substitute N=2.8 in the original equation. $2.8 \div 0.8 = 3.5$

$\frac{28}{10} \div \frac{8}{10} = 28 \div 8 = 3.5$. These equations are true, so the answer is correct.

Investigation 4: *Using Percents*
ACE #5

5. Jen and Sarah go to lunch at the Green Grill. Their meals total $28.00. The tax is 6%.
   a. What is the total cost including tax?
   b. Jen and Sarah want to leave a 20% tip based on the cost before tax. How much tip should they leave?
   c. Describe two strategies that Marie and Sarah can use to figure the amount of the tip.

Solution

5. You can think of percents as fractions, or decimals or as rates. Different ways of thinking lead to different strategies.

   a. We might think of 6% as a rate of 6 cents on each dollar. Scaling this up to $28 we have a tax of 28 x $0.06 = $1.68. The total bill will be $29.68.

   b. We might think of 20% as a fraction. 20% is $\frac{20}{100}$ or $\frac{1}{5}$.

   Therefore we need $\frac{1}{5} \times 28 = \frac{28}{5} = 5.60$. The tip is $5.60.

   c. Instead of thinking of 20% as $\frac{1}{5}$ they might think of this tip in two stages.

   10% is $\frac{10}{100} = \frac{1}{10}$, and we know that multiplying $28 by $\frac{1}{10}$ will keep the same digits but give them place values that are $\frac{1}{10}$ of the original. So the tip would be $2.80 if we used 10% to calculate it, so 20% would be twice $2.80 or $5.60 as before.

Investigation 4: *Using Percents*
ACE #11

11. The local boutique is having a sale on hats. Lisa finds a straw hat that is already marked down. The price tag shows that the original price was $36.00. The marked down price is $27.00. What percent has the hat been marked down? Explain.

Solution:
The focus is on the amount of the discount, $9.00. If we think of percent discount as a rate then we need to compare 9 out of 36 to x out of 100. We can use equivalent fractions to solve this:

$$\frac{9}{36} = \frac{x}{100}$$

$$\frac{1}{4} = \frac{x}{100}$$

$$x = 25$$

So the percent discount on the hat is 25%.