Shapes and Designs: Homework Examples from ACE

Investigation 1: *The Family of Polygons,* ACE #10
Investigation 2: *Designing Polygons: The Angles Connection,* ACE #2, 20
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Investigation 1: *The Family of Polygons*
ACE #10

In parts (a) – (h), decide whether each angle is closest to 30 degrees, 60 degrees, 90 degrees, 120 degrees, 150 degrees, 180 degrees, 270 degrees, or 360 degrees without measuring. Explain your reasoning.
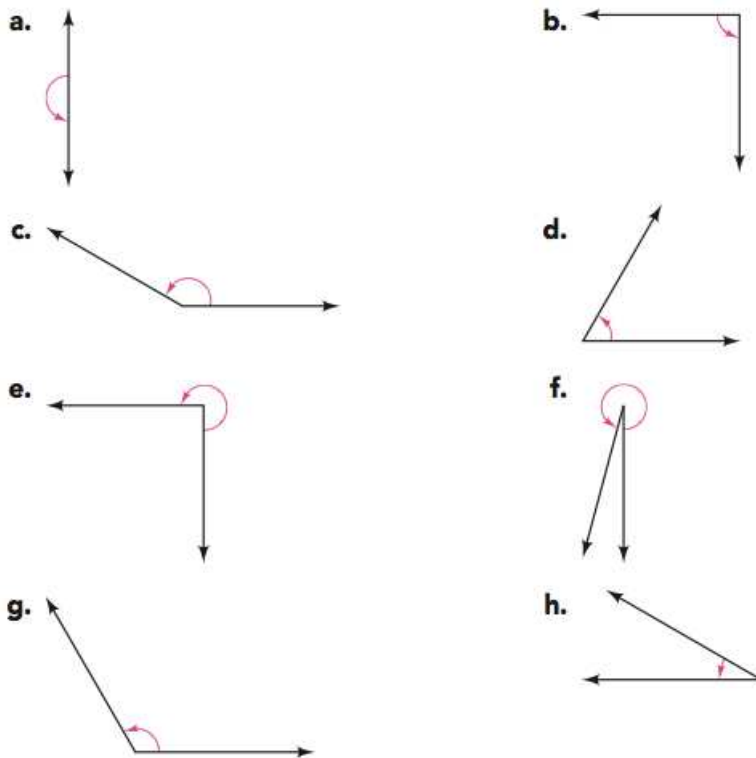
i. For each angle in parts (a) – (h), classify them as right, acute, or obtuse.

Students can use a **right angle**, 90 degrees, a **straight angle**, 180 degrees, etc. as a **benchmark**. Notice that neither of the arms of the angle has to be vertical or horizontal. Students need to be able to identify the vertex of the angle no matter the orientation.

a. 180 degrees – this does not fall into one of the categories for (i). It is actually called a straight angle

b. 90 degrees – right angle

c. This is more than 90 and less than 180 degrees. It looks closer to 180 degrees, so 150 would be a good estimate – obtuse angle

d. This is less than 90, but closer to 90 than 0 degrees. So 60 would be a good estimate – acute angle

e. This looks like 3 right angles placed adjacent to each other. So one estimate would be 270 degrees – some students might say this is obtuse, but the definition of obtuse is actually an angle between 90 and 180. So this angle does not fall into one of the possible categories.

f. This is almost a complete rotation, so 350 would be a good estimate – does not fall into one of the possible categories.

g. This is more than 90 but less than 180 degrees. It looks closer to 90 than to 180. So 120 degrees would be a good estimate – obtuse

h. This is between 0 and 90 degrees, but less than half of 90 degrees. So 30 degrees would be a good estimate – acute

Below are sets of regular polygons of different sizes. Does the length of a side of a regular polygon affect the sum of the interior angle measures? Explain.
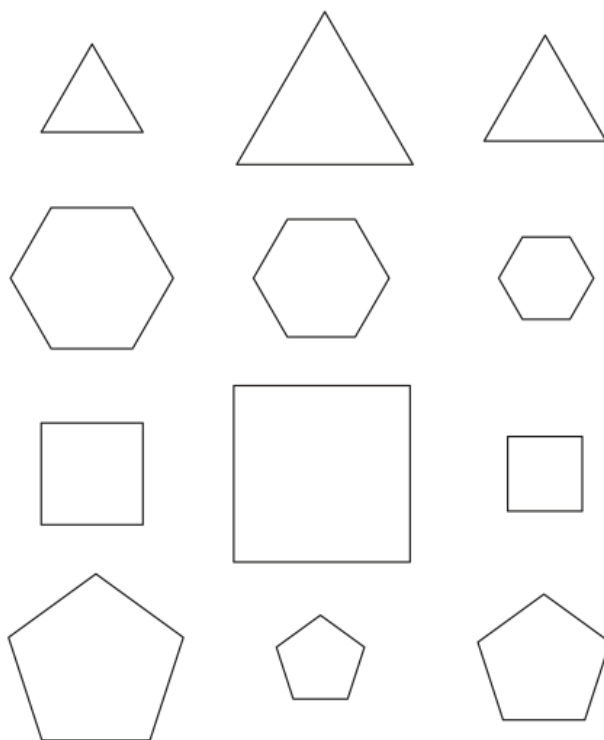
Students might reason about this in different ways. They might use the formula they have found in class work to say that the sum of the interior angles of a polygon is
$(n-2)180$ degrees. Therefore, the angle sum is dependent only on the number of sides n, not the length of the sides. For example, for a pentagon the angle sum is $3(180) = 540$ degrees.

Or they might reason more visually: The smaller pentagon in the figure below, for example, fits exactly into the corner of the larger pentagon. The size of the common angle does not depend on the length of the side.
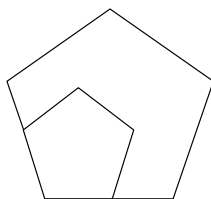
Investigation 2:   *Designing Polygons: The Angles Connection*
ACE #20

Choose a scalene triangle (all three sides of different lengths) from your Shapes Set or draw one of your own. Using copies of your scalene triangle, can you make a tiling pattern? Sketch a picture to help explain what you found.

Any scalene triangle can also be used as a tile. First, make a parallelogram by reflecting the triangle over one of its sides. Then, tile using the parallelogram!

Example of a reflected scalene triangle:
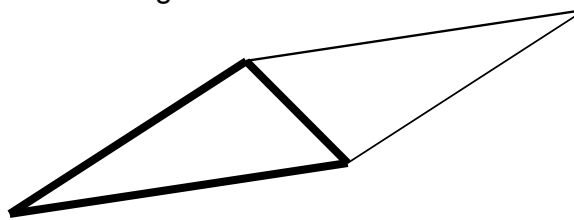
Investigation 3: *Designing Triangles and Quadrilaterals*
ACE #10, 11, 12, 13

- If possible, build a quadrilateral with the side lengths. Sketch your quadrilateral.
- Tell whether your quadrilateral is the only one that is possible. Explain.
- If a quadrilateral is not possible, explain why.

10. 5, 5, 8, 8

First we should determine that the lengths will make a quadrilateral. That is, is the sum of the shortest three sides longer than the fourth side? With 5, 5, 8 and 8 we know a quadrilateral is possible. In fact, there are two types of shapes possible; if we place the equal sides next to each other we get a **kite** shape (on the left), and if we place the equal sides opposite each other we get **a parallelogram**. Since the angle sizes are not fixed just because the side lengths are fixed, we actually have an infinite number of kites and parallelograms.
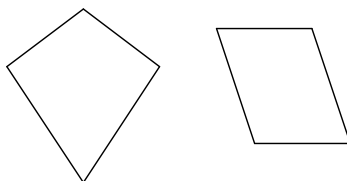
11. 5, 5, 6, 14

This time we could have the equal sides opposite each other, in which case we might get a **trapezoid** (but not necessarily a trapezoid since the angles are not fixed). Again, there are many different possibilities, because the figure is "flexible" and not rigid. We could have the equal sides adjacent to each other. In this situation, an infinite number of shapes is possible, *none* of which is a parallelogram. A trapezoid is possible again. Here are some examples, with the equal sides bolded:
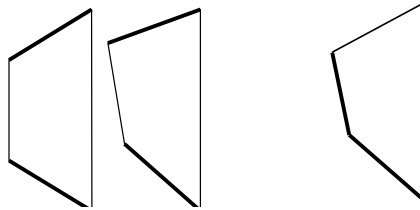
12. 8, 8, 8, 8

Since all 4 sides are equal we can have a **rhombus** or a **square** (which is a particular kind of rhombus). Since the angles are not fixed, the following sketch shows only one of the possible rhombuses (on the left).
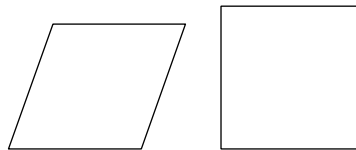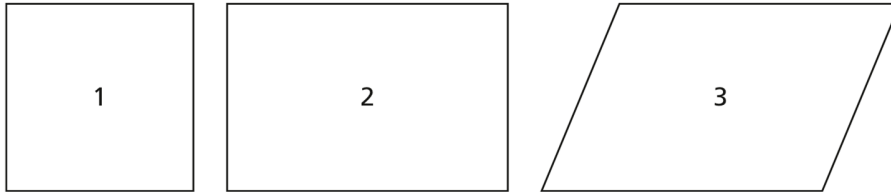
13. 4, 3, 5, 14

This quadrilateral is not possible, because the sum of the smallest three sides (4 + 3 + 5) is 12, which is less than the length of the fourth side (14).

Investigation 3: *Designing Triangles and Quadrilaterals*
ACE #29

Compare the three quadrilaterals below.

    a. How are all three quadrilaterals alike?
    b. How does each quadrilateral differ from the other two?

This problem asks students to look for **properties** of each quadrilateral, such as angle sizes, side lengths, parallel sides, etc. Some possible descriptions include:

    a. All 3 quadrilaterals have opposite sides equal and parallel—they are all parallelograms. All 3 quadrilaterals have congruent opposite angles and consecutive angles are supplementary (they add to 180 degrees). All 3 quadrilaterals also have the same height, though student may not notice this since they have not focused on height at this time.

b. Quadrilateral 1 has all four sides equal; none of the others has this property. Quadrilateral 3 has no right angles; both of the others have this property. Quadrilateral 3 also has unequal diagonals, while the other two have equal diagonals, but students might not spot this since the diagonals are not drawn.