Moving Straight Ahead: Homework Examples from ACE

Investigation 1: *Walking Rates,* ACE #4
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Investigation 1: *Walking Rates*
ACE #4

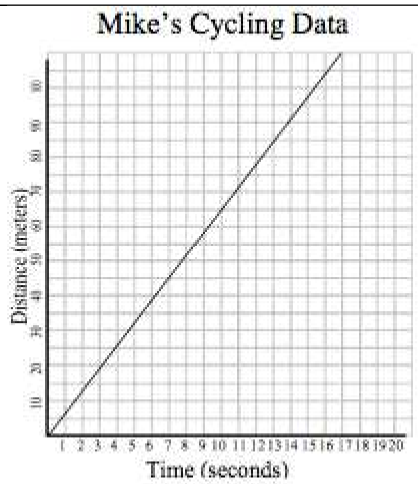
Mike makes the following table of the distances he travels during the first day of the trip.

Cycling Distance

Time (hours)	Distance (miles)
0	0
1	6.5
2	13
3	19.5
4	26
5	32.5
6	39

a. Suppose Mike continues riding at this rate. Write an equation for the distance Mike travels after *t* hours.
b. Sketch a graph of the equation. How did you choose the range of values for the time axis? For the distance axis?
c. How can you find the distances Mike travels in 7 hours and in $9\frac{1}{2}$ hours, using the table? Using the Cycling Distance graph? Using the equation?
d. How can you find the numbers of hours it takes Mike to travel 100 miles and 237 miles, using the table? Using the graph? Using the equation?
e. For parts (c) and (d), what are the advantages and disadvantages of using each model—a table, a graph, and an equation—to find the answers?
f. Compare the rate at which Mike rides with the rates at which Jose, Mario, and Melanie ride. Who rides the fastest? How can you determine this from the tables? From the graphs? From the equations? [You will need to refer back to #3 in the ACE for this part.]

a. Students should see the rate of change in distance is constantly 6.5 miles for each one hour change. $d = 6.5t$
b. Students should look at the range of numbers needed for time and for distance in the table, and then decide on a reasonable scale. Since the distance values reach about 40, it is convenient to mark the vertical axis in increments of 5 or 10. The time axis only has to cover times to 6 hours, therefore scaling by 1 would be be appropriate, though students may want to mark the scale on the time axis in half hours.



The graph above has a larger range of values for both axes because parts (c) and (d) ask for times greater than 6 hours and distances greater than 40 miles.

c. The table can be extended to show 7 and $9\frac{1}{2}$ *hours.* On the graph, the distances at these points may be approximated. In the equation, the values of 7 and 9 12 can be substituted for *t*, which gives the answers of 45.5 and 61.75 miles.

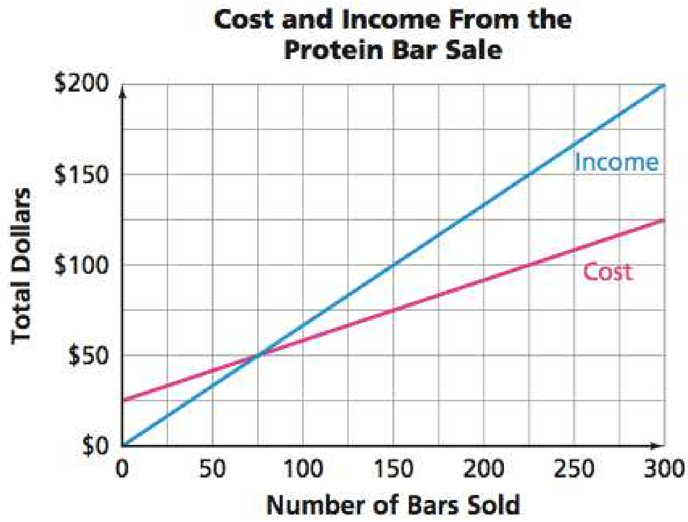
d. The table can be extended by adding increments of 6.5 miles to show values that are close to 100 miles and 237 miles. On the graph, the times at these points may be approximated after the graph has been extended. In the equation, the values of 100 miles and 237 miles can be substituted for *d*, which gives the approximate answers 15.4 hours and 36.5 hours.

e. Possible answer: If the given value for one variable is already showing, the table or graph would be easy to use to find the corresponding value for the other variable. If the values are far from those shown on the table or graph, or if you need an exact quantity, it is easier to use equations to get the answer.

f. See question 3.

Investigation 2: *Exploring Linear Relationships With Graphs and Tables*
ACE #6

A band decides to sell protein bars to raise money for an upcoming trip. The cost (the amount the band pays for the protein bars) and the income the band receives for the protein bars are represented on the graph.



a. How many protein bars must be sold for the cost to equal the income?
b. What is the income from selling 50 protein bars? 125 bars?
c. Suppose the income is $200. How many protein bars were sold? How much of this income is profit?

Notice that this problem is still about constant rate, though this time it is "per protein bar" not "per hour."

a. About 75 bars (Note: Students are reading answers from the graph, so some inaccuracy is expected.) One line (the pink line) is a representation of the relationship between cost and number of protein bars. The other (blue line) represents the relationship between income and number of protein bars. So the *intersection point* gives the number of protein bars for an equal cost and income.
b. The income from selling 50 bars is $33.50 because 0.67(50)=33.5. (Note: The 0.67 was derived from points (0, 0) and (300, 200), which shows that 300 bars cost $200, so each bar would sell for 67¢. That means the slope of the "income" line is 0.67.) Students could also answer this from the graph. The income from selling 125 bars is $83.75 because 0.67(125) = 83.75.
c. Using the graph, we see that for an income of $200, the band would have to sell about 300 protein bars. Again using the graph, we see that the cost for 300 bars would be $125, leaving a profit of about $75. Another approach is to recall that in part (b), we saw that the income per bar is $0.67. Using the "cost" line we see that it costs $125 for 300 bars, so each bar costs about $0.42. So the profit per bar is $0.25. For 300 bars, the profit is 300(0.25) = $75.

Investigation 3: *Solving Equations*
ACE #12

Use properties of equality and numbers to solve each equation for *x*. Check your answers.

a. $7 + 3x = 5x + 13$
b. $3x – 7 = 5x + 13$
c. $7 – 3x = 5x + 13$
d. $3x + 7 = 5x – 13$

A few notes: Students already have table and graph strategies to solve these equations. For example, to solve $7 + 3x = 5x + 13$, students could graph $y = 7 + 3x$ and $y = 5x + 13$. The intersection point will give the value of *x* that makes $7 + 3x = 5x + 13$. The goal in MSA Investigation 3 is not to suggest that other strategies aren't useful, but rather that *symbolic* methods using properties of equality are accurate and efficient.

Upcoming units will introduce other types of equations, such as exponential (*Growing, Growing*) and quadratic (*Frogs and Fleas*). For some complex equations, *graphical* methods are more efficient, though perhaps less accurate, than symbolic methods. In fact, for some complex equations there are *no* symbolic methods, and graphical solutions are the *only* solutions possible!

a. $7 + 3x = 5x + 13$. Subtracting 7 from each side, we get $3x = 5x + 6$. Subtracting $5x$ from each side, we get $-2x = 6$; so $x = -3$. (Check: Is $7 + 3(-3) = 5(-3) + 13$? Yes.)

b. $3x – 7 = 5x + 13$. Add 7 to each side to get $3x = 5x + 20$. Subtract $5x$ from each side for $-2x = 20$. Divide by -2 to get $x = -10$. (Check: Is $3(-10) – 7 = 5(-10) + 13$? Yes.)

c. $7 – 3x = 5x + 13$. Add $3x$ to both sides $7 = 8x + 13$. Subtract 13 from both sides for $-6 = 8x$. Divide by 8 on both sides to get $-0.75 = x$. (Check: Is $7 – 3(-0.75) = 5(-0.75) + 13$? Yes.)

d. $3x + 7 = 5x – 13$. Subtract $3x$ from both sides to get $7 = 2x – 13$. Add 13 to both sides for $20 = 2x$. Divide by 2 to get $10 = x$. (Check: Is $3(10) + 7 = 5(10) – 13$? Yes.

Investigation 4: *Exploring Slope: Connecting Rates and Ratios*
ACE #15

In parts (a) and (b), the equations represent linear relationships. Use the given information to find the value of b.

a. The point (1, 5) lies on the line representing $y = b - 3.5x$.
b. The point (0, -2) lies on the line representing $y = 5x - b$.
c. What are the y-intercepts in parts (a) and (b)? What are the patterns of change in parts (a) and (b)?
d. Find the x-intercepts for the linear relationships in parts (a) and (b). (The x-intercept is the point where the graph intersects the x-axis.)

a. If a point lies on a line then it must make the equation of the line true. Thus, $5 = b - 3.5(1)$, so $5 = b - 3.5$, so $b = 8.5$. Thus, the only way that (1, 5) can lie on the line $y = b - 3.5x$ is if the value of b is 8.5.

b. $-2 = 5(0) - b$, so $b = 2$.

c. The linear relationship in part (a) is $y = 8.5 - 3.5x$; this shows y decreasing at a rate of -3.5 for each unit change in x, with a y-intercept of (0, 8.5). The linear relationship in part b is $y = 5x - 2$; this shows y increasing at a rate of 5 for each unit change in x, with a y intercept of (0, -2).

d. Students will probably use what they know about the meaning of m and b in the linear equation $y = mx + b$ to answer part (c). But there is no shortcut for finding the x-intercept just by inspecting the equation. Students need to know that because the x-intercept is the point where the line crosses the x-axis, it has coordinates (x, 0).

Thus, substituting $y = 0$ into the equation in part (a), $y = 8.5 - 3.5x$, we have $0 = 8.5 - 3.5x$, so $3.5x = 8.5$, and $x = 8.5/3.5 = \frac{17}{7}$. The x-intercept is ($\frac{17}{7}$, 0), or about (2.4, 0). For the linear relationship in part (b), $y = 5x - 2$, we have $0 = 5x - 2$, so $x = 2/5 = 0.4$. Thus the x-intercept is (0.4, 0).