What Do You Expect?: Homework Examples from ACE

Investigation 1: *A First Look at Chance,* ACE #3, #4, #9, #31
Investigation 2: *Experimental and Theoretical Probability,* ACE #6, #12, #9, #37
Investigation 3: *Making Decisions with Probability,* ACE #3, #4
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Investigation 5: *Binomial Outcomes,* ACE #3

Investigation 1: *A First Look at Chance*
ACE #3

3. Kalvin tosses a coin five days in a row and gets tails every time. Do you think there is something wrong with the coin? How can you find out?

3. This question addresses the idea of probability as "what is to be expected over the long term." Kalvin should toss the coin many more times. It is unusual to get 5 tails in a row, but not impossible. If he tossed the coin 100 times and got many more tails than heads he might suspect that the coin is not fairly balanced. Theoretically, each toss of a fair coin should have a 50% chance of turning out to be a tail, but we should not be surprised if this 50% figure does not occur over a small number of tosses. (If he repeated the experiment (5 tosses of a fair coin) a hundred times and recorded how many times he got 5 tails in a row he would find that this will occur purely by chance about 3 times in a 100.)

Investigation 1: *A First Look at Chance*
ACE #4

4. Len tosses a coin three times. The coin shows heads every time. What are the chances the coin shows tails on the next toss? Explain.

4. In this case, the probability of HHH***T*** is the same as the probability of HHH***H***. Each coin toss is independent of the last toss, even though it seems that some combinations are less likely than others. In other words, the coin has no memory of what the last toss was, and so there is no change in the probability of the outcome of a single toss; each toss has a 50% chance of being H, and a 50% chance of being a T.
Note: if we had asked before any tosses had taken place whether it was more likely to get 4 heads in 4 tosses, or 3 heads and a tail, then we could say that HHHH was less likely than 3 heads and a tail. But this is because there are 4 ways to get 1 tail: HHHT, HHTH, HTHH, THHH.

Investigation 1: *A First Look at Chance*
ACE #9

9. Kalvin's sister Kate finds yet another way for him to pick his breakfast. She places one blue marble and one red marble in each of two bags. She says that each morning he can choose one marble from each bag. If the marbles are the same color, he eats Cocoa Blast. If not, he eats Health Nut Flakes. Explain how selecting one marble from each of the two bags and tossing two coins are similar.

9. In the first bag there are two equally likely outcomes: red or blue. Likewise for the second bag. Therefore, this situation is exactly like tossing a coin twice or tossing two coins; each bag is analogous to a coin toss, and "red" is analogous to "head" and "blue" to "tail." *Note:* This question foreshadows the idea of simulation. In simulations a model is chosen which has the same underlying probabilities as the situation to be investigated. The purpose in choosing the model is to set up repetitions of an experiment, using the model rather than the real situation, because the model is more convenient.

Investigation 1: *A First Look at Chance*
ACE #31

31. Yolanda watches a carnival game in which a paper cup is tossed. It costs $1 to play the game. If the cup lands upright, the player receives $5. Otherwise, the player receives nothing. The cup is tossed 50 times. It lands on its side 32 times, upside-down 13 times, and upright 5 times.

a. If Yolanda plays the game ten times, about how many times can she expect to win? About how many times can she expect to loose?
b. Do you expect her to have more or less money at the end of ten games? Explain.

31.

a. Yolanda only wins if the cup lands upright.
From the experimental data we see that the probability of winning is 5 out of 50, or 10%. Therefore, if Yolanda plays 10 times she can expect to win 10% of 10 times = 1 time. She will lose 9 times. (Note: Ten trials is a very small number of trials, so we should not be surprised if Yolanda's results are very different from the percentages produced by the longer experiment.)
b. If Yolanda wins 1 time and plays 10 times, she will have spent $10 to play and won back only $5, so she would have less money at the end of 10 games.

Investigation 2: *Experimental and Theoretical Probability*
ACE #6

6. A bag contains several marbles. Some are red, some are white, and some are blue. You count the marbles and found that the theoretical probability of drawing a red marble is 1/5 and the theoretical probability of drawing a white marble is 3/10.
   a. What is the smallest number of marbles that could be in the bag?
   b. Could the bag contain 60 marbles? If so, how many of each color must it contain?
   c. If the bag contains 4 red marbles and 6 white marbles, how many blue marbles must it contain?
   d. How can you find the probability of choosing a blue marlbe?

6.

a. The ratio of red marbles: total number of marbles must be 1:5 since the probability of choosing a red is 1:5. The actual number of red could be 1 in a total of 5, or 2 in a total of 10, or 3 in a total of 15 etc. Likewise the actual number of white could be 3 in a total of 10, or 6 in a total of 20, or 9 in a total of 30. The first ratios that use the same total number ofmarbles are2redin10and3redin10. 10 is the lowest total (or the first common denominator).
b. Red: total = 1:5 = 12:60. White: total = 3:10 = 18:60. It is possible to make correct ratios with a total of 60 marbles.
c. $\frac{Red}{Total} = \frac{1}{5}$ or $\frac{4}{?}$ We need to rename the fraction $\frac{1}{5}$ so that the numerator is 4. $\frac{1}{5} = \frac{4}{20}$. Using a total of 20 marbles we have $\frac{White}{Total} = \frac{3}{10} = \frac{6}{20}$. So there are 4 red and 6 white marbles, leaving 10 blue marbles to complete the total set of 20.
d. There are only 3 choices, so P(Red) + P(White) + P(Blue) = 1.
   So $^1/_5 + {}^3/_{10}$ + P(Blue) = 1.
   So P(Blue) = 1 - $(^1/_5 + {}^3/_{10}) = 1 - {}^5/_{10} = {}^5/_{10}$

Investigation 2: *Experimental and Theoretical Probability*
ACE #12

12. Lunch at school consists of a sandwich, a vegetable, and a fruit. Each lunch combination is equally likely to be given to a student. The students do not know what lunch they will get. Sol's favorite lunch is a chicken sandwich, carrots, and a banana.

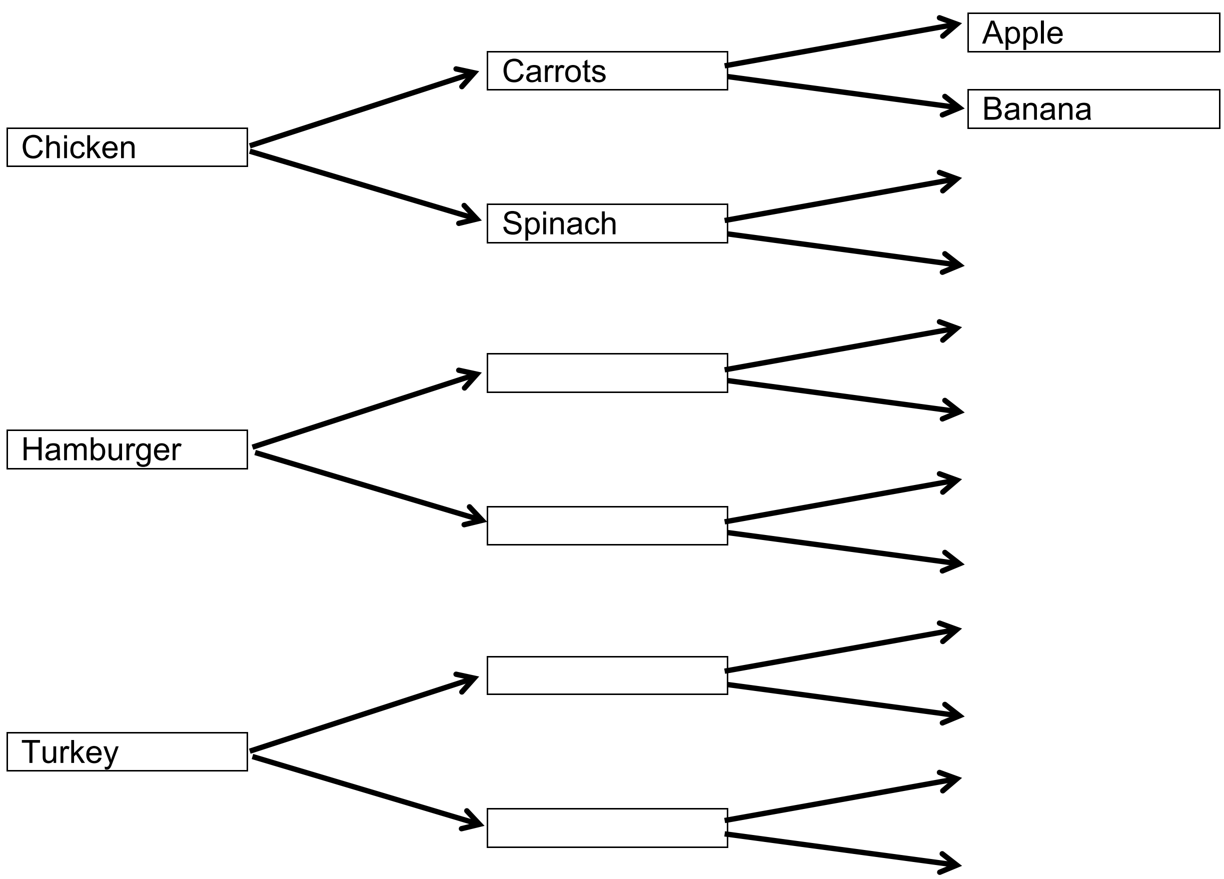
SCHOOL LUNCH MENU

SANDWICHES	VEGETABLES	FRUIT
Chicken	Carrots	Apple
Hamburger	Spinach	Banana
Turkey		

a. Make a tree diagram to determine how many different lunches are possible. List all the possible outcomes.
b. What is the probability that Sol gets his favorite lunch? Explain.
c. What is the probability that Sol gets at least one of his favorite lunch items? Explain.

12. It Is important to see the logic behind the counting tree, so that we can tell ahead of time how many possible outcomes there will be. If the tree gets large and unwieldy we can still predict total possibilities.

a.



Reading from the right we have 2 choices of cookie, for each of 2 choices of vegetable, for each of 3 choices of sandwich. The list of outcomes is CCC, CCO, CSC, CSO, HCC. HCO, HSC, HSO, TCC, TCO, TSC, TSO.

b. Only one of these, CCC, is Sage's favorite lunch so she has a 1in 12 chance of getting her favorite.
c. CCC, CCO, CSC, CSO, HCC, HCO, HSC, TCC, TCO, TSC all contain at least one of Sage's favorite items. 10/12.
   Note: it would have been easier to enumerate the times when Sage got NONE of her favorites, HSO, TSO.

Investigation 2: *Experimental and Theoretical Probability*
ACE #9

9. Pietro and Eva are playing a game involving tossing a coin three times. Isabella scores 1 point if *no* two consecutive toss results match (as in HTH). Pietro scores a point if exactly two consecutive toss results match (as in HHT). The first player to 10 points wins. Is this a fair game? Explain. If it is not a fair game, change the rules to make it fair.

9. One way to analyze this game is to list all outcomes with the accompanying winner.

OUTCOME	WINNER
HHH	No Winner
HHT	Pietro
HTH	Eva
THH	Pietro
HTT	Pietro
THT	Eva
TTH	Pietro
TTT	No Winner

There are 4 ways that Pietro can win and only 2 ways that Eva can win. This is not fair. We could change the rules so that Pietro wins on exactly 2 consecutive matches and Isabella wins otherwise, in which case there are 4 ways for Eva to win also. Or we keep the original rules but award Eva double points for a win.

Investigation 2: *Experimental and Theoretical Probability*
ACE #37

37. Suppose you are a contestant on the Gee Whiz Everyone Wins! Game show in Problem 2.4. You win a mountain bike, a vacation to Hawaii, and a one-year membership to an amusement park. You play the bonus game and lose. Then the host makes you this offer:

> You can choose from the two bags again. If the two colors match, you win $5,000. If the two colors do not match, you do not get the $5,000 and you return all the prizes that you have already won.

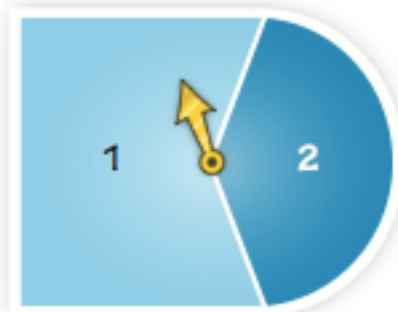
Would you accept this offer? Explain.

37. There are 9 possible outcomes from choosing 2 blocks: RR, RY, RB, YR, YY, YB, BR, BY, BB. There are 3 ways to win: RR, YY, BB. And 6 ways to lose. P(match) = 3/9=1/3, and P(no match) = 6/9=2/3. Some students will argue that because the chance of winning is less than the chance of loosing, they should keep the prizes they won and refuse the offer. Other students may argue that having a 1 in 3 chance of winning $5000 is worth the risk.

Investigation 3: *Making Decisions with Probability*
ACE #3

3. When you use each of the spinners below, the two possible outcomes are landing on 1 and landing on 2. Are the outcomes equally likely? If not, which outcome has a greater theoretical probability? Explain.

a.

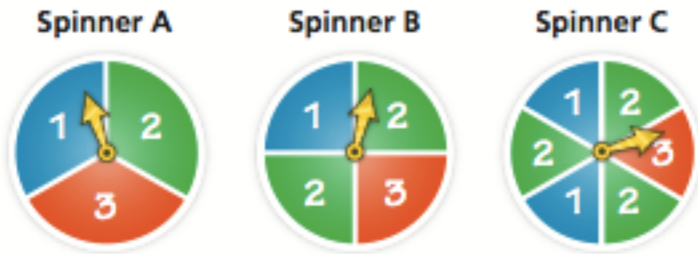


b.

3. The key is to recognize that the size of the angle associated with the pointer's fixed point, or the center of rotation, is what determines the probability, not the size of the area of the region.
   a. Not equally likely; Region 1 is more likely. Since Region 1 has a bigger central angle associated with the pointer's fixed point than Region 2 does, it has a greater theoretical probability.
   b. Equally likely; both sections have sides that make a 180° angle associated with the pointer's fixed point, or the center of rotation.

Investigation 3: *Making Decisions with Probability*
ACE #4

4. Molly designs a game for a class project. She makes the three spinners shown. She tests to see which on she likes best for her game. She spins each pointer 20 times and writes down her results, but she forgets to record which spinner gives which set of data. Match each spinner with one set of data. Explain your answer.



First data set: 1, 2, 3, 2, 1, 1, 2, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2

Second data set: 2, 3, 1, 1, 3, 3, 3, 1, 1, 2, 3, 2, 2, 2, 1, 1, 1, 3, 3, 3

Third data set: 1, 2, 3, 3, 1, 2, 2, 2, 3, 2, 1, 2, 2, 2, 3, 2, 2, 3, 2, 1

4. Students may argue that 20 trials are not enough to be certain which spinner generated which data set, and this is certainly true. Based on the spinners, it appears that the first data set is from Spinner C, the second data set is from Spinner A, and the third data set is from Spinner B. Sample explanations are provided below.

First data set:
Spinner C should produce "2" half the time in the long term, and should produce fewer "3's" than "1's. The first data set has 12 "2's" and 5 "1's" and 3 "3's."

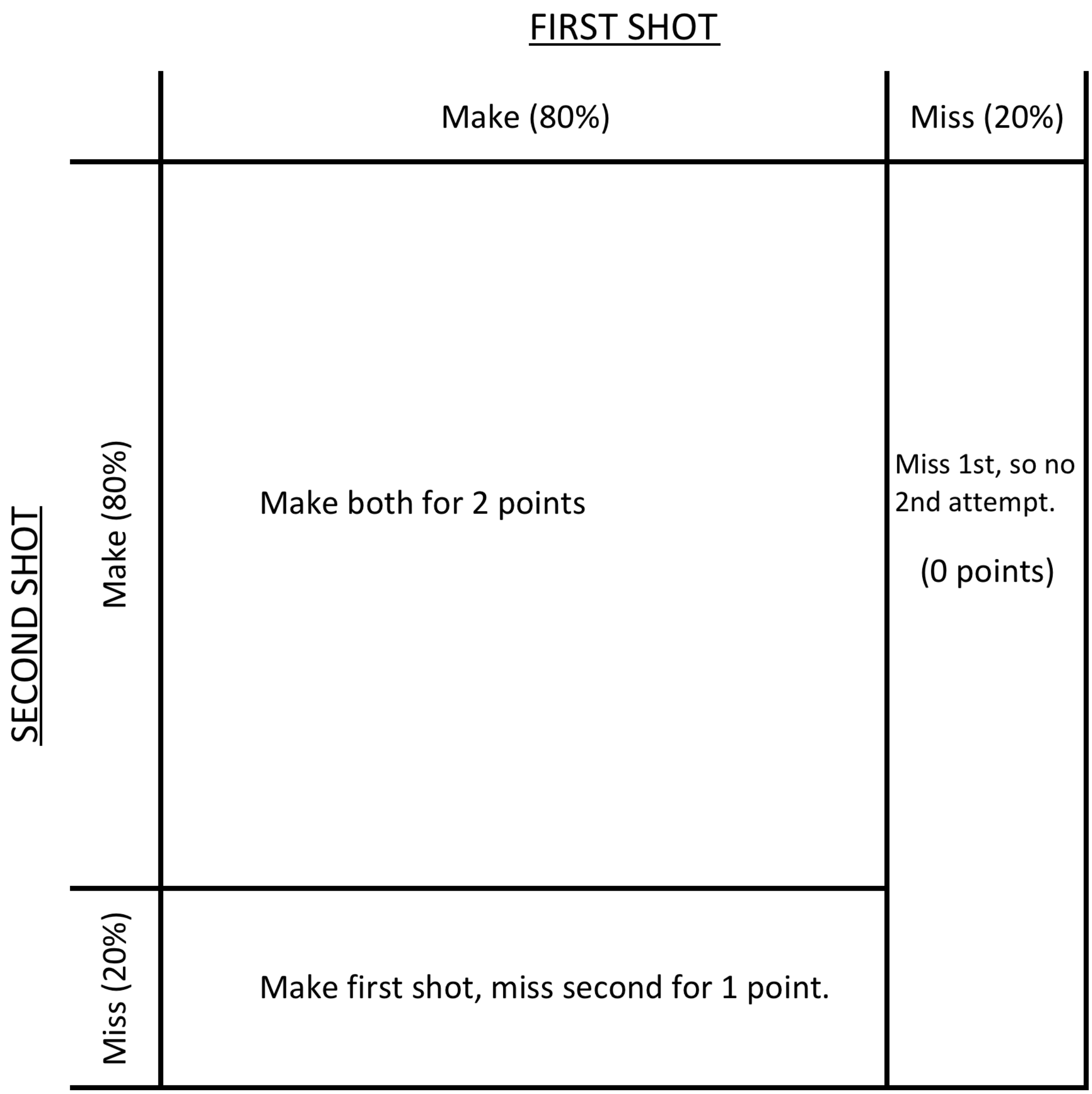
Second data set:
Spinner A has 3 equally likely outcomes. We should look for a list that reflects this, knowing that with 20 trials these theoretical probabilities will not occur. The second data set has 7 "1's" and 5"2's" and 8 "3's." This is close to the theoretically expected outcome for Spinner A.

Third data set:
Spinner B should have "2" occurring half of the time, and "1" and "3" occurring equally often. The third data set has 11 "2's" and 4 "1's" and 5 "3's."

Investigation 4: *Analyzing Compound Events Using an Area Model*
ACE #16

16. In a one-and –one free throw situation, is the player with an 80% average most likely to score 0 points, 1 point, or 2 points. Make an area model to support your answer.

16.



The largest area corresponds to the probability of 2 points. (If we subdivided this area into 100 equal square units to see what fraction this actually is, we would find that P(2 points) = 64%, P(1 point) = 16%, and P(0 points) = 20%.

Investigation 4: *Analyzing Compound Events Using an Area Model*
ACE #18

18. Nishi, who has a a 60% free-throw average, is in a two-attempt free-throw situation. Remember, this means that she will attempt the second shot no matter what happens on the first shot.
    a. Is Nishi most likely to score 0 points, 1 point, or 2 points? Explain your answer.
    b. Nishi plans to keep track of her score on two-attempt free-throw situations. What average number of points can she expect to score per two-attempt situation?

18.
a.

FIRST SHOT

SECOND SHOT		Make (60%)	Miss (40%)
	Make (60%)	Make both for 2 points (2 points)	Miss 1st, and make 2nd. (1 point)
	Miss (40%)	Make 1st and miss 2nd (1 point)	Miss 1st, and miss 2nd. (0 points)

P(2 points) = 36%. P(1 point) = 24% + 24% = 48%. P(0 points) = 16%. She is most likely to score 1 point.

b. If she continued as shown in the area model above, then in 100 attempts she would score 2 points 36 times, 1 point 48 times and 0 points 16 times. This would give her a total of 120 point on 100 attempts, or an average of 1.2 points per two-shot free throw attempts.

Investigation 5: *Binomial Outcomes*
ACE #3

3. Scout is about to have puppies. The vet thinks that Scout will have four puppies. Assume that each gender, male and female, are equally likely.
   a. List all the possible combinations of female and male puppies Scout might have.
   b. Is Scout more likely to have four male puppies, or two male puppies and two female puppies? Explain your reasoning.

3.
a. Students might list the possibilities in a tree diagram or organized list. We are assuming that Male and Female are equally likely for each puppy.

Number of Males	Number of Females	Possible Outcomes
4	0	MMMM
3	1	MMMF, MMFM, MFMM, FMMM
2	2	MMFF, MFMF, MFFM, FMMF, FMFM, FFMM
1	3	MFFF, FMFF, FFMF, FFFM
0	4	FFFF

b. There is only 1 chance out of 16 that Scout will have four male puppies (MMMM). There are 6 chances out of 16 that Scout will have 2 males and 2 females. Scout is more likely to have 2 males and 2 females.
(Note: The case of MMFF is no more likely than MMMM. Each possibility has the same chance of happening, 1/16. But there are several different ways that we can *order* the 2 males and 2 females, so that the *sum* of these probabilities is larger than P(MMMM).)