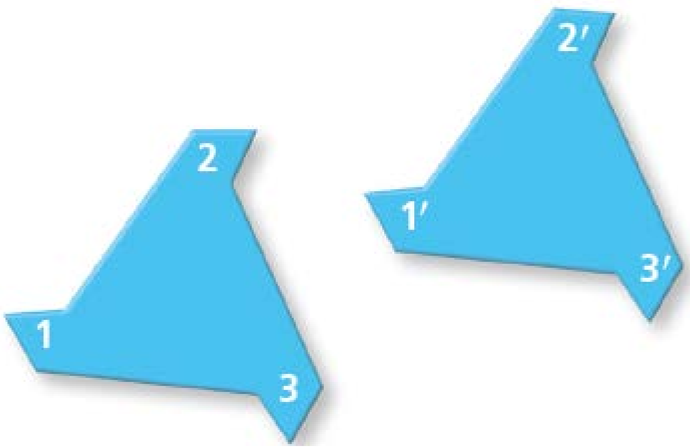
Butterflies, Pinwheels, and Wallpaper: Homework Examples from ACE

Investigation 1: *Symmetry and Transformations,* ACE #14-17
Investigation 2: *Transformations and Congruence,* ACE #19-22
Investigation 3: *Transforming Coordinates, ACE #8*
Investigation 4: *Dilations and Similar Figures,* ACE #21-22

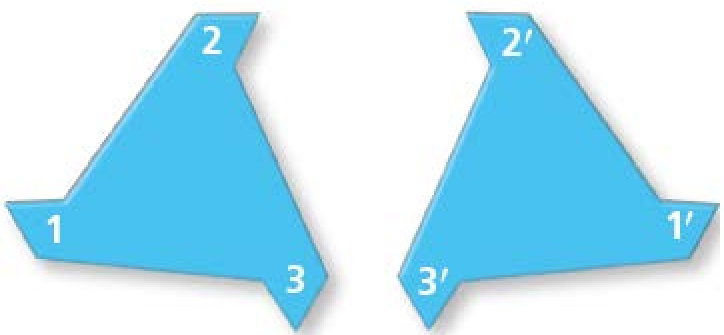
Investigation 1: *Symmetry and Transformations*
ACE #14-17

Exercises 14–17 each give a figure and its image under a flip, turn, or slide. In each case, name the type of transformation used. For a flip, sketch the line of reflection. For a turn, locate the center and find the angle of rotation. For a slide, draw a line showing the direction and distance of the translation.

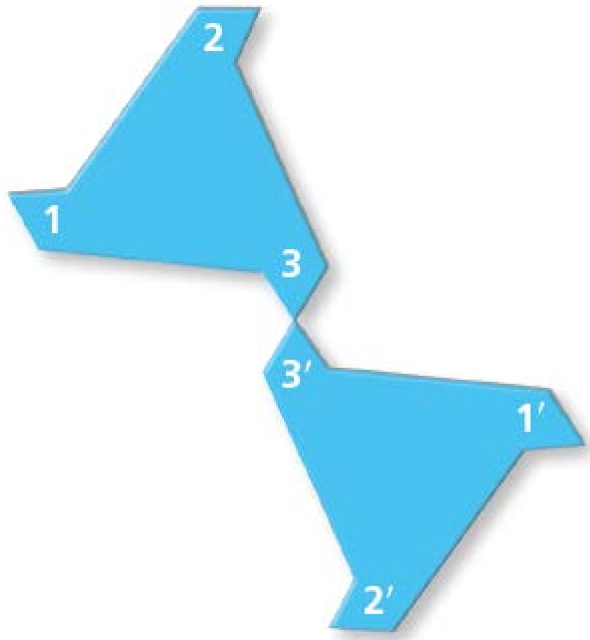
14.



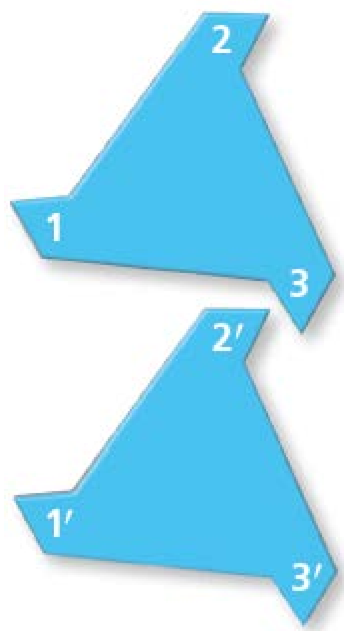
15.



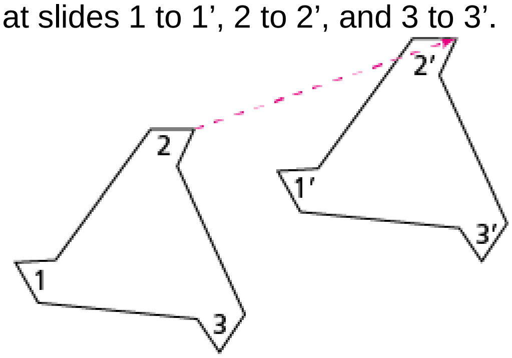
16.



17.-

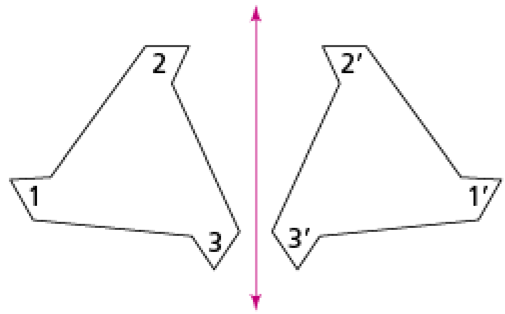


14. Translation on a slant that slides 1 to 1’, 2 to 2’, and 3 to 3’.

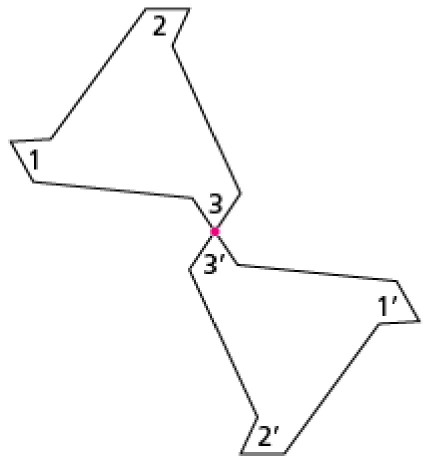


15. Line reflection about the perpendicular bisector of any segment joining corresponding

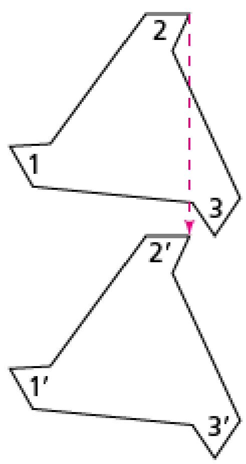
points.



16. Half-turn about the point where 3 and 3′ touch.



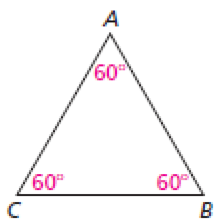
17. Translation downward that slides 1 to 1’, 2 to 2’, and 3 to 3’.



Investigation 2: *Transformations and Congruence*
ACE #19-22

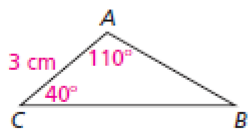
In Exercises 19–22, you are given a triangle *ABC* and information about another triangle, *DEF*. Tell whether triangle *DEF* is *definitely congruent* to triangle *ABC, possibly congruent* to triangle *ABC*, or *definitely not congruent* to triangle *ABC*. Explain your reasoning in each case.

19.



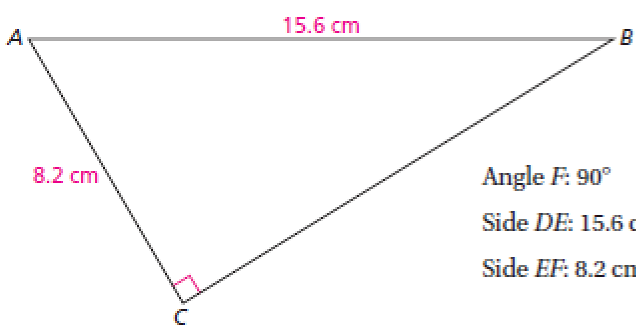
Angle D: 60°
Angle E: 60°
Angle F: 60°

20.



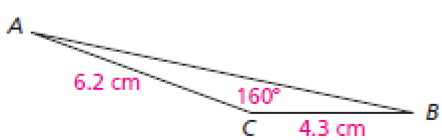
Angle D: 110°
Angle E: 40°
Side DF: 3 cm

21.



Angle F: 90°
Side DE: 15.6 cm
Side EF: 8.2 cm

22.



Angle F: 160°
Side DF: 4.3 cm
Side EF: 6.2 cm

19. It is possible, but not certain, that the two triangles are congruent. One could be larger than the other. Three corresponding angles do not guarantee congruence.

20. It is not possible for the two triangles to be congruent. If they were, then *DE* = 3 cm and Δ*DEF* would have to be an isosceles triangle with congruent base angles. Δ*ABC* cannot be isosceles, because ∠*B* has measure of 30°.

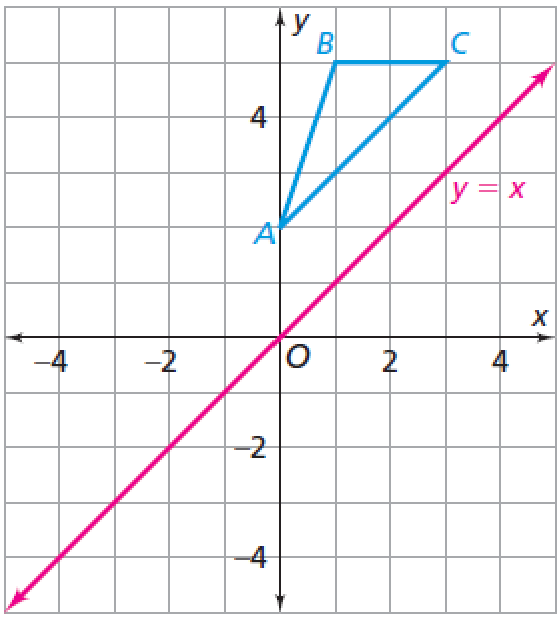
21. The two triangles must be congruent because the Pythagorean Theorem implies that the third sides of each must be $\sqrt{15.6^2 - 8.2^2} \approx 13.3$ cm.

22. These two triangles are congruent by the Side-Angle-Side criterion.

Investigation 3: *Transforming Coordinates*
ACE #8

8.

a. Use triangle *ABC* shown in the diagram.



Copy and complete the table showing the coordinates of points *A*–*C* and their images after a reflection in the line *y* = *x*.

Point	*A*	*B*	*C*
Original Coordinates	■	■	■
Coordinates After a Reflection in *y* = *x*	■	■	■

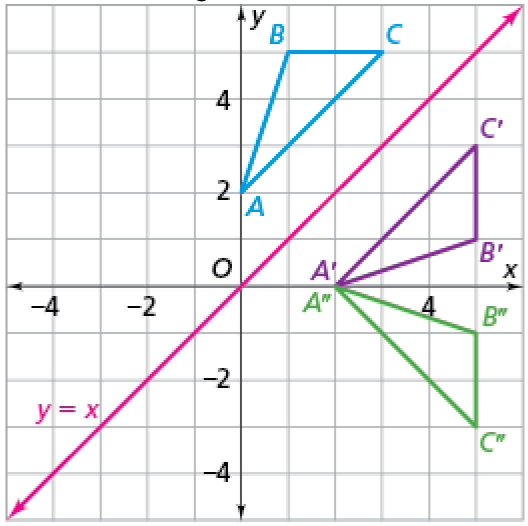
b. Draw the image and label the vertices *A*', *B*', and *C*'.
c. Add a row to your table to show the coordinates of points *A*–*C* and their images after a reflection of triangle *A*'*B*'*C*' in the *x*-axis.
d. Draw the image and label the vertices *A*'', *B*'', and *C*''.
e. Draw the image of triangle *ABC* after the same two reflections, but in the reverse order. That is, reflect triangle *ABC* in the *x*-axis and then reflect its image, triangle *A*'*B*'*C*', in the line *y* = *x*. What does the result suggest about the commutativity of a sequence of line reflections?

8. Composites of line reflections.

a. The table (after both reflections) will look like this:

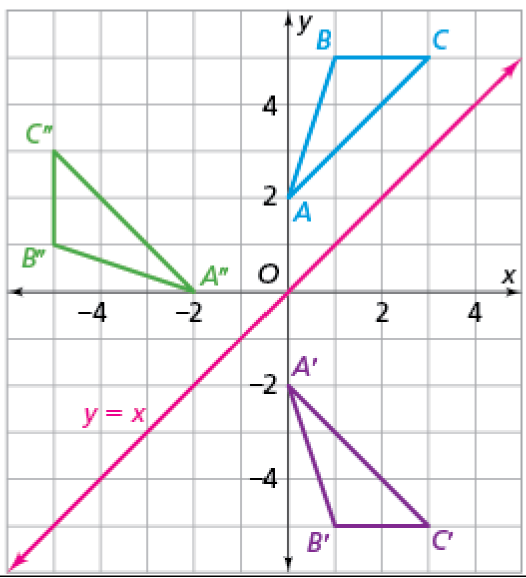
Point	*A*	*B*	*C*
Original Coordinates	(0, 2)	(1, 5)	(3, 5)
Coordinates After a Reflection in *y* = *x*	(2, 0)	(5, 1)	(5, 3)
Coordinates After a Reflection in *x*-axis	(2, 0)	(5, –1)	(5, –3)

b. –c. The image of the triangle after reflection in *y*=*x* is purple; the image after a subsequent reflection in the *x* -axis is green.



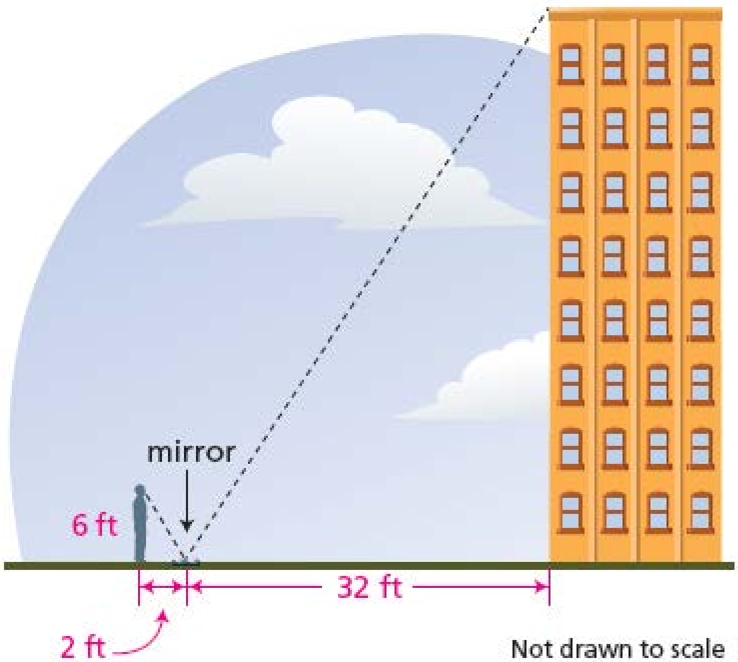
d. –e. The image after first reflecting in the *x* -axis is green and then after reflecting in the line *y*=*x* is purple. Comparing this drawing to that in parts (a)–(c) shows that composition of line reflections is not commutative. It is a general property of such compositions that the result is a rotation about the point of intersection of the lines through an angle that is double the angle between the two lines (from first line to second line).

Note: That is more than we expect students to get out of this Exercise. It is revisited in Extensions.



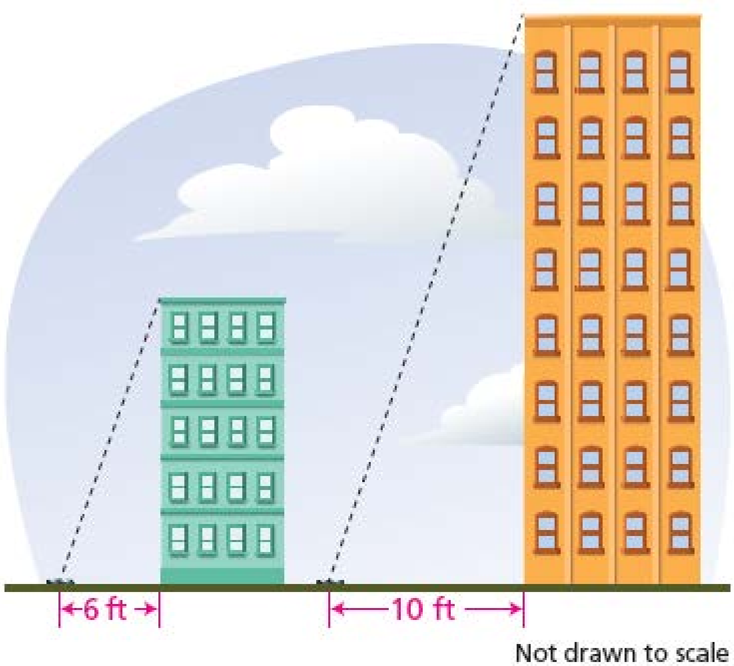
Investigation 4: *Dilations and Similar Figures*
ACE #21-22

21. Stan uses the mirror method to estimate the height of a building. His measurements are shown in the diagram below.



a. How tall is the building?
b. How do you know that your calculation is correct?

22. One afternoon, the building in Exercise 21 casts a shadow that is 10 feet long, while a nearby building casts a shadow that is 6 feet long.



a. How tall is the shorter building?
b. How do you know that your calculation is correct?

21. Measuring height of a tall building

a. 96 feet.

b. The triangles pictured are similar by the same reasoning applied in Problem 4.4 and the scale factor relating corresponding side lengths is 32/2 = 16.

22. Using shadows to form similar triangles

a. The shorter building must be 57.6 feet tall.

b. The triangles are similar because at any specific time of day the sun rays strike the earth at essentially the same angle when one is looking in a small geographic region. The buildings are assumed to meet the ground at right angles, so the two triangles have two corresponding angles. The taller building is 96 feet high and the scale factor from larger to smaller is 0.6. We have 0.6(96) = 57.6.