Function Junction: Homework Examples from ACE

Investigation 1: *The Families of Functions,* ACE #5, #10
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Investigation 1: *The Families of Functions* ACE #5
Complete the sentences to give correct statements: *f(x)* = -2x + 5 **a.** f(7) = **b.** f(-3) = **c.** f(?) = 17 The notation f(-7) means that students are to substitute the number 7 into the right-hand side of the function in place of the variable *x*. In this case, substituting 7 gives -2(7) + 5 = -14 + 5 = -9 Similarly, f(-3) means substitute -3 for x. This gives -2(-3) + 5 = 6 + 5 = 11 Part **c** asks the student to determine what number must be substituted for *x* in the function to give a final answer of 17. Students may try several numbers, but the only number that works is - 6. This is because -2(-6) = 5 = 12 + 5 = 17

Investigation 1: *The Families of Functions*
ACE #10

Determine if the relationship in each tables shows that *y* is a function of *x*.

(See student text for tables).

a. In order for a table of values to show that *y* is a function of *x*, each value of x must be paired with one and only one value of y. In this table, each value of *x* (2, 3, 4, 5, and 6) is paired with one and only one value of y (4, 7, 10, 13, and 16, respectively). Therefore, this table shows that *y* is a function of *x*.

b. This table also shows that *y* is a function of *x*, because each value of *x* is paired with one and only one value of *y*.

c. This table does **not** show that y is a function of x, because the same value of x (namely, 0) is paired with *two different* values of y (namely, 1 and 9).

d. This table shows that *y* is a function of *x*, because each value of x is paired with one and only one value of *y*. Students may object that, for example, the values -3, -1, and 1 are all paired with the same value (namely, 1) of *y*. But this does not violate the rule stated above. Two (or more) values of *x* may be paired with the same value of *y*, but the same value of x may **not** be paired with *two different* values of y, as was the case in table **c**.

Investigation 2: *Arithmetic and Geometric Sequences*
ACE #4

Latrell volunteers at a local charity. The first week he works a total of 2 hours (for training). Each week after the first, he volunteers 3 ½ hours.

Suppose t(n) represents the total number of hours worked during weeks 1 through n.

a. Write an equation that represents the relationship between t(n) and t(n+1).

b. How many hours does Latrell volunteer in 1 year (52 weeks)?

Each week after the first, Latrell works 3 ½ hours. Therefore the total number of hours he has worked since starting the job goes up by 3 ½ each week. If t(n) represents the total number of hours worked during weeks 1 through n, then t(n+1) is the number of hours worked in *one more week* than t(n). Therefore, t(n+1) = t(n) + 3 ½ . And t(1) = 2, because he only worked 2 hours during the first week.

Latrell works 2 hours the first week, and 3 ½ hours each week for the next 51 weeks. To find the number of hours he works in total, t(n), we multiply 3 ½ by 51, and add 2. This gives a total of 180 ½ hours.

Investigation 2: *Arithmetic and Geometric Sequences*
ACE #17

For Exercises 17-21, students are asked to answer each question, and also:

State whether the sequence is arithmetic or geometric.
Write an equation relating s(n) and s(n+1)
Write an algebraic expression for a function s(n) that shows how to find any term in the sequence.
Explain how the equation for the *n*th term in the sequence is connected to the equation that relates term n to term (n+1)

On a game show, payoffs for correct responses in each question category increase as follows: $200, $400, $600, $800, $1,000. What is the total amount of money that can be won by correct answers for all questions of a category?

To find the total amount of money that can be won by correct answers for all questions of a category, the student needs to find the sum of the payoffs for that category.
$200+$400+$600+$800+$1000 = $3000.

This sequence is arithmetic. An arithmetic sequence is one in which each successive term is found by adding a constant amount to the previous term. In this sequence, each new term is just the previous term plus 200.

To write an equation relating s(n) and s(n+1), we notice that term s(n+1) is just 200 more than term s(n). Therefore, s(n+1) = s(n) + 200

The function s(n) represents the n*th* term in the sequence. For example, the 1^{st} term is 200. The 2^{nd} term is 400. The 3^{rd} term is 600, and so on. From this pattern, students should notice that the n*th* term is found by multiplying *n* by 200. Therefore, s(n) = 200n.

Finally, the constant rate of change in the equation relating the n*th* term to term (n+1) is the coefficient of *n* (or the slope) in the *y* = *mx* + *b* form equation.

Investigation 3: *Transforming Graphs, Equations, and Functions*
ACE #10

Match each function with its graph. Explain how you made each match. Give coordinates for the maximum or minimum points on each graph. Be prepared to explain how you can find that information from just the function rule.

(See student text for graphs)

Each equation is in vertex form, $f(x) = a(x-b)^2 + c$. In this form, the line of symmetry is simply $x = b$, and the maximum or minimum point is given by (b, c). The value of a determines the extent to which the parabola is stretched either toward or away from the x-axis.

a. $a(x) = x^2$ is Parabola 3 with line of symmetry $x = 0$ and minimum point (0, 0). This is one of three parabolas to share these characteristics, along with Parabolas 2 and 5. To determine that a(x) is represented by Parabola 3, note that when x = 2 in the equation $a(x) = x^2$, that a(2) = 4. Therefore the point (2,4) must lie on this parabola.   .

b. $b(x) = (x – 2)^2$ is Parabola 4 with line of symmetry $x = 2$ and minimum point (2, 0)

c. $c(x) = –(x – 2)^2 – 2$ is Parabola 6 with line of symmetry $x = 2$ and maximum point (2, -2)

d. $d(x) = (x + 2)^2 – 2$ is Parabola 1 with line of symmetry $x = -2$ and minimum point (-2, -2)

e. $e(x) = 0.5x^2$ is Parabola 2 with line of symmetry $x = 0$ and minimum point (0, 0). To determine that this equation is represented by Parabola 2, note that when x = 2, e(2) = 2. Therefore the point (2,2) must lie on this parabola.

f. $f(x) = 1.5x^2$ is Parabola 5 with line of symmetry $x = 0$ and minimum point (0, 0). To determine that f(x) is represented by Parabola 5, note that when x = 2, f(2) = 6. So the point (2,6) must lie on this parabola.

g. $g(x) = –(x – 3)^2$ is Parabola 7 with line of symmetry $x = 3$ and maximum point (3, 0)

h. $h(x) = –x^2 – 2$ is Parabola 8 with line of symmetry $x = 0$ and minimum point (0, -2)

Investigation 4: *Solving Quadratic Equations Algebraically*
ACE #9

For exercises 9-14, students are asked to complete the square, identify coordinates of the maximum or minimum point, identify the x-intercept(s) and y-intercept, and to state which form is more convenient to identify coordinates of the maximum/minimum point, x-intercept(s) and y-intercept.

$f(x) = x^2 + 2x - 3$

These are the basic steps to complete the square. Please refer to the student text for a thorough explanation:

Begin with the equation in standard form.
$x^2 + 2x - 3$

Re-write the equation with the constant term outside a set of parenthesis.
$(x^2 + 2x \qquad) - 3$

Take the coefficient of the second term inside the parenthesis, divide it by 2, and square that result. Add this result to the inside of the parenthesis, and subtract it outside the parenthesis.

$(x^2 + 2x + (2/2)^2) - 3 - (2/2)^2$

Simplify.

$(x + 1)^2 - 4$

Therefore, $x^2 + 2x – 3 = (x + 1)^2 – 4$. This means that the minimum point on the graph of *f(x)* is (–1, – 4), *x*-intercepts at (-3, 0) and (1, 0), *y*-intercept (0, –3).

Investigation 4: *Solving Quadratic Equations Algebraically*
ACE #24

Write the result of the indicated sum, difference, or product in the form of a single complex number **a + bi**.

(3 + 7i)(13 - 4i)

Multiplying two complex numbers is similar to the process for multiplying two binomials. The important fact to remember is that the product (i)(i) = -1.

Therefore, once the individual products have been found, it is necessary to simplify the following terms:

$39 - 12i + 91i - 28i^2$

The i^2 term becomes -1, which makes the $-28i^2$ become +28.

Combining like terms gives

$(3 + 7i)(13 – 4i) = 67 + 79i$

Investigation 5: *Polynomial Expressions, Functions, and Equations*
ACE #9

Write the sums and differences as equivalent standard polynomial expressions

$(x^3 - 7x^2 - 4x + 1) - (5x^3 + 3x^2 + 4x - 3)$

Begin this problem by distributing the negative sign over the second set of parenthesis. This results in the following sum of terms:

$x^3 - 7x^2 - 4x + 1 - 5x^3 - 3x^2 - 4x + 3$

Combining like terms gives us the following result:

$(x^3 – 7x^2 – 4x +1) – (5x^3 + 3x^2 + 4x – 3) = -4x^3 – 10x^2 – 8x + 3$

Investigation 5: *Polynomial Expressions, Functions, and Equations*
ACE #11

Write the products as equivalent standard polynomial expressions

$(3x + 4)(5x - 1)$

To compute this product, each term in the first set of parenthesis must be distributed over the second set of parenthesis. The result of that multiplication gives us the following:

$15x^2 - 3x + 20x - 4$

And adding the two terms in the middle gives us the result.

$(3x + 4)(5x – 1) = 15x^2 + 17x – 4$