Unit Project

Introduction

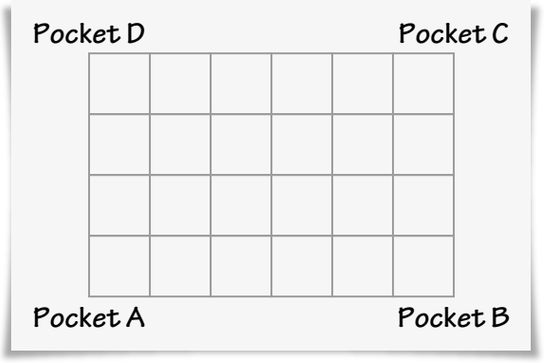
The suggested Unit Project will ask students to investigate the pattern of bounces for a pool ball as it makes its way around pool tables of various dimensions. As you might expect, ratio and scaling are involved in this Project. There is also an online version of **Paper Pool**.

Assigning

The Project is formally assigned near the end of the Unit. We recommend that students work on the Project with a partner.

Preparing for the Unit Project

For this project, students are asked to play a game called Paper Pool. This game is played on rectangular square-grid tables, such as the one shown below.



An imaginary ball is hit from the lower left-hand corner, Pocket A, at a 45° angle. A ball hit in this way will bounce off each side it hits at a 45° angle. The ball continues to roll until it hits a pocket. Pockets are located at each corner of the table. Students play Paper Pool on different-sized rectangular tables.

Students are to predict the pocket in which the ball will stop and how many hits (anything making contact with the ball–the sides of the table, the imaginary cue, the pocket) will occur by the time the ball comes to a stop (reaches a pocket).

To do the task, students will need to investigate several sizes of Paper Pool tables. They will need to gather and organize data and search for patterns. Finding a solution will require students to recognize relationships between rectangles whose sides have the same ratio.

Materials

- **Labsheet:** Student's Guide, Part 1
- **Labsheet:** Student's Guide, Part 2
- **Labsheet:** Student's Guide, Part 3
- **Labsheet:** Paper Pool A
- **Labsheet:** Paper Pool B
- **Labsheet:** Paper Pool C
- centimeter grid paper
- colored pencils or markers

Using Technology

Students can use the online **Paper Pool** as an alternative to, or to supplement, the Labsheets provided for Paper Pool.

Goals for Students

- gather and organize data
- search for patterns
- recognize rectangles whose sides have the same ratio (similar rectangles)
- use the simplest ratio to predict the stopping pocket and number of hits

Launch

Distribute the Unit Project labsheets, and review the task introduction with your students. Make sure they understand how the ball travels on the Paper Pool tables and how to count the number of hits that occur on any table. Check that the students have drawn the paths correctly for the two sample tables.

An extension question is offered with this task. You may want to assign it to everyone, or you may use it as an extra challenge for those groups that want to investigate patterns further.

Explore

We recommend that students work on this project with a partner. One class period will be needed for students to collect their data. They can continue to investigate the task and draft their reports outside of class. Part of a second class period may be used for comparing results and finalizing reports.

Summarize

You may want to have students share their results. If the extension question was given as an extra challenge, be sure to ask any students who attempted it to share their results.

Grading

The Paper Pool project is an open-ended investigation with a wide range of possible observations that students could make. In the project, students are asked to find patterns and write rules about how the ball travels on Paper Pool tables.

Possible Student Responses

Some students may give rules that are related to a specific example, while others may give rules that generalize across several different rectangles. In the examples that follow, we call a rule that works for all tables with a specific characteristic in common (such as horizontal length of 1 and a vertical line that is an even number) a "specific" rule. Rules that look beyond a single common characteristic and apply to a more general category, we call "sophisticated" rules.

Possible Rules for Predicting the Dropping Pocket

Specific Rules Specific rules usually involve only one variable.

- If the table is a square, the ball will stop at Pocket C.
- If the table has a horizontal dimension of 1 unit and a vertical dimension that is an odd number, the ball will stop at Pocket C.
- If the table has a horizontal dimension of 1 unit and a vertical dimension that is an even number, the ball will stop at Pocket D.
- If the table has a vertical dimension of 1 unit and a horizontal dimension that is an even number, the ball will stop at Pocket B.
- The ball will never stop at Pocket A.
- In all similar rectangles of the same orientation, the ball will stop at the same pocket.

Sophisticated Rules Sophisticated rules compare more than one variable.

In all similar rectangles of the same orientation, the ball will stop at the same pocket. The pocket is determined by the ratio of the table's sides. If the ratio of the horizontal dimension to the vertical dimension is

- an odd number to an odd number, the ball will stop at Pocket C.
- an odd number to an even number, the ball will stop at Pocket D.
- an even number to an odd number, the ball will stop at Pocket B.

Possible Rules for Predicting the Number of Hits

Specific Rules Specific rules usually involve only one variable.

- If the table is a square, there will be two hits.
- If the table has a side length of 1, the number of hits will be 1 greater than the length of the other side (or, the number of hits will be the sum of the two sides).
- For all similar rectangles, the number of hits is the same.

Sophisticated Rules Sophisticated rules compare more than one variable.

- For all similar rectangles, the number of hits is the same. The number of hits is the sum of the numbers of the simplified ratio (to the least whole-number amounts) of the table's sides.

Suggested Scoring Rubric

A scoring rubric and samples of student work are given in the Assessment Resources section.

This rubric for scoring the Project employs a scale for four different areas of assessment for a total of 21 possible points. The rubric does ***not*** assess students' work on the optional extension question.

Mathematics (0 to 8 points total)

Rules/Patterns for predicting the ending corner (0 to 4 points)

0. Student did not engage; no patterns or rules are given.

1. Student shows evidence of searching for a pattern but states no original pattern or rule, *OR* student states one specific rule.

2. Student states at least two correct specific rules.

3. Student states a correct sophisticated rule and/or several specific rules, which address several possible situations for where the ball will stop.

4. Student states at least one correct sophisticated rule and addresses all possible situations for which corner the ball will stop.

Rules/Patterns for predicting the total number of hits (0 to 4 points)

0. Student did not engage; no patterns or rules are given.

1. Student shows evidence of searching for a pattern but states no original pattern or rule, *OR* student states one specific rule.

2. Student states at least two correct specific rules.

3. Student states a correct sophisticated rule and/or several specific rules, which address several possible situations for the number of hits that will occur.

4. Student states at least one correct sophisticated rule and addresses all possible situations for the number of hits that will occur.

Problem Solving and Reasoning (0 to 4 points total)

0. Student did not engage in the given task.

1. Student shows reasoning about rules through words or organizational instruments used, but it may be faulty—incorrect logic or nonsensible statements in the context of the problem *OR* only reasons through one specific rule.

2. Student shows reasoning about rules through words or organizational instruments, but reasoning is weak—tests an inadequate number or variety of situations, drawing conclusions that would require testing more cases or examining more varied arrangements *OR* has only one or two specific rules and does not address both situations.
3. Student shows adequate reasoning to support given sophisticated rule(s) or gives complete reasoning to support specific rules for both situations.
4. Student shows complete reasoning to support sophisticated rules for both situations.

Communication (0 to 4 points)

0. Student does not communicate in any form.
1. Student does not address the task presented.
2. Significant effort is needed to follow the student's report.
3. With some extra effort, the reader can follow the student's report.
4. Report is clearly stated and easy to follow.

Checklist (5 points possible)

2. Student gives a correct new table for each rule given. Response must have at least two rules. (One rule and one correct new table that fit the rule is worth 1 point.)
1. Student completes the labsheets for Paper Pool.
2. Student uses organizational tool(s) to search for patterns and rules for the task. Quality is the determining factor for giving a paper 0, 1, or 2 points.

Sample Student Work

A Teacher's Comments on Sample 1

Mary Beth Student Work #1

Mary Beth's project received 8 of the 8 points for the mathematics that she presented in her report. A 4 was given for her sophisticated rule on which corner the ball would stop. Her rule for the stopping corner covers all possible cases. She notes it makes a difference if the table is odd by even or even by odd but does not tell in her description if she is giving the horizontal dimensions first or the side length first. Her drawings and organizational tools (she made tables to organize her information) made it possible to determine what she meant, so full credit for her rules was given. A 4 was given for her sophisticated rule for the number of hits that would occur. Her rule identifies the sum of the dimensions of the table as the needed relationship. This is the correct relationship that allows you to find out the total number of hits that will occur, but her written report does not state that it is the sum of the dimensions when expressed as a ratio in simplest form (or what she calls earlier "basic" form). I might have taken a point off for this if it were not for the fact that her labsheets, new drawings, and organizational tools showed that she understood that it was the sum of the simplified ratio.

A score of 4 was given to Mary Beth's work for problem solving and reasoning and a 3 was given for communication. Her reasoning for her rules was complete when you took into account her written summary, new drawings, and organizational tools. The 3 was given for communication because of the effort required to sort out which side she was referring to for her odd by even and even by odd rules. Also, this decision was made because of the lack of clarity and completeness in her written description of her rule for the ending corner.

Mary Beth's new table, labsheet, and organizational tool were included in her report, and she was given all the points for this section due to their quality and completeness. Mary Beth has 20 of the 21 points for this project and was given an *A* for the project.

Mathematics—Ending Corner Rule	4 out of 4
Total Hits Rule	4 out of 4
Problem Solving Reasoning	4 out of 4
Communication	3 out of 4
Checklist—New Tables	2 out of 2
Completed Labsheets	1 out of 1
Organizational Tools	2 out of 2
TOTAL POINTS	20 out of 21

A Teacher's Comments on Sample 2

Heather Student Work #2

Heather's project received 5 of the 8 points for the mathematics that she presented in her report. A 3 was given for six basic rules and one sophisticated rule that identified at which corner the ball would stop. A 2 was given for her three basic rules for the number of hits that would occur (a fourth rule—"two hits"—is started but not finished). Her rules for the ending corner cover several possibilities and her rule "On an odd by odd it will always end up at corner C" is considered a sophisticated rule. She does not address the orientation of the rectangles and the reader can only make sense of her rules by examining her drawings and organizational tools. Heather's rules for ending corner suggest that she looked for patterns. Her rules for the number of hits are part of some of her rules for ending corners. The count she gives for the number of hits is incorrect and suggests that she does not understand what counts as a hit. It seems that she has not counted the hit from the imaginary cue nor the hit at the last pocket in her count.

A score of 2 was given to Heather's work for both communication and problem solving and reasoning. Because she shows no evidence of being able to reason about how many hits will occur and because the reader's only evidence of Heather's reasoning is through her labsheet and single organization chart, she was given a 2 for problem solving and reasoning. A 2 was given for communication,

because the reader needs to make a significant effort to follow the student's report. Because Heather does not deal with orientation of the rectangles, one must make an effort to sort through her work and make sense of the rules she has given.

Heather's new tables to demonstrate her rules were complete and she was given 2 point for her work on this. Her labsheet was also complete; thus, she received the one point for including these papers in her report. She received a 1 for her organizational tools. Her tools included only a table/chart that organized the information as to at which pocket the ball would come to a stop. She did not include any organizational tool that addressed the number of hits that occurred.

Heather's work was given 13 of 21 points and a grade of a C in my class. Heather's labsheet shows that she is not counting the number of hits correctly. I'm not sure why this is, because when we launched the project in class, both she and her partner were able to get the correct number of hits for the Paper Pool tables in the Launch. Further instruction will probably be needed to help the two of them address the issue of hits. I will also want to talk to Heather and all other students about how to look for patterns and how to organize information to help in looking for patterns.

Mathematics—Ending Corner Rule	3 out of 4
Total Hits Rule	2 out of 4
Problem Solving Reasoning	2 out of 4
Communication	2 out of 4
Checklist—New Tables	2 out of 2
Completed Labsheets	1 out of 1
Organizational Tools	1 out of 2
TOTAL POINTS	13 out of 21