Unit Project

Introduction

The Unit Project can be used as the final assessment to *Say It With Symbols*. It allows students to apply what they have learned about writing algebraic expressions to describe patterns and verifying the equivalence to those expressions.

In *Finding the Surface Area of Rod Stacks*, students find the surface areas of stacks of rods of certain lengths by varying the number of rods n. They describe a pattern and find the relationship between the number of rods n and the surface area of the stack A. The equation for the surface area is a linear relationship in terms of the number of rods used. Students will find that different but equivalent expressions can be used to model the data. You could include questions about volume by asking how many unit rods will be needed to build a stack of n rods of length x. Then ask the students to determine the function that can model this situation.

Materials

Cuisenaire® rods for each student:

4 to 6 rods in each of three colors

3 to 4 unit rods

Assigning

Provide a set of rods so students have a physical example of the rod stack in Unit Project pages of the Student Edition. Hold up one of the colored rods and ask students:

- How long is this rod?

Providing Additional Support

You may want to have students begin the Project in class so that they are able to share their results for rods of length 2–10 (Part 1, Questions 5 and 6). If you do not begin in class, make sure that students get a chance to share their equations with each other. There will be more than one way to find an expression for, say, the green rods. Encourage students to compare their symbolic expressions with other groups. Remind students to include and discuss examples of equivalent expressions for a given rod length in their final write up of this Project.

Grading

Suggested Scoring Rubric

This rubric for scoring the Project employs a scale that runs from 0 to 4, with a 4+ for work that goes beyond what has been asked for in some unique way. You may use the rubric as presented here or modify it to fit your district's requirements for evaluating and reporting students' work and understanding.

4+ Exemplary Response

- Complete, with clear, coherent explanations
- Shows understanding of the mathematical concepts and procedures
- Satisfies all essential conditions of the problem and goes beyond what is asked for in some unique way

4 Complete Response

- Complete, with clear, coherent explanations
- Shows understanding of the mathematical concepts and procedures
- Satisfies all essential conditions of the problem

3 Reasonably Complete Response

- Reasonably complete; may lack detail in explanations
- Shows understanding of most of the mathematical concepts and procedures
- Satisfies most of the essential conditions of the problem

2 Partial Response

- Gives response; explanation may be unclear or lack detail
- Shows some understanding of some of the mathematical concepts and procedures
- Satisfies some essential conditions of the problem

1 Inadequate Response

- Incomplete; explanation is insufficient or not understandable
- Shows little understanding of the mathematical concepts and procedures
- Fails to address essential conditions of problem

0 No Attempt

- Irrelevant response
- Does not attempt a solution
- Does not address conditions of the problem

Unit Project Answers

1. & 4. Students will choose different rods, so answers will vary from 2 by 1 by 1 to 10 by 1 by 1.

2. & 4.

Length 2

Number of Rods	Surface Area
1	10
2	18
3	26
4	34
5	42

Length 3

Number of Rods	Surface Area
1	14
2	24
3	34
4	44
5	54

Length 4

Number of Rods	Surface Area
1	18
2	30
3	42
4	54
5	66

Length 5

Number of Rods	Surface Area
1	22
2	36
3	50
4	64
5	78

Length 6

Number of Rods	Surface Area
1	26
2	42
3	58
4	74
5	90

Length 7

Number of Rods	Surface Area
1	30
2	48
3	66
4	84
5	102

Length 8

Number of Rods	Surface Area
1	34
2	54
3	74
4	94
5	114

Length 9

Number of Rods	Surface Area
1	38
2	60
3	82
4	104
5	126

Length 10

Number of Rods	Surface Area
1	42
2	66
3	90
4	114
5	138

For every set of rods, each additional rod adds the same amount to the surface area.

Length of Rod Added	Surface Area Increase
2	8
3	10
4	12
5	14
6	16
7	18
8	20
9	22
10	24

3. & 4.

Length (cm)	Equation in the Form $y = mx + b$
2	$A = 8n + 2$
3	$A = 10n + 4$
4	$A = 12n + 6$
5	$A = 14n + 8$
6	$A = 16n + 10$
7	$A = 18n + 12$
8	$A = 20n + 14$
9	$A = 22n + 16$
10	$A = 24n + 18$

Some strategies students may use are given. **Note:** All of the examples use a rod of length 4.

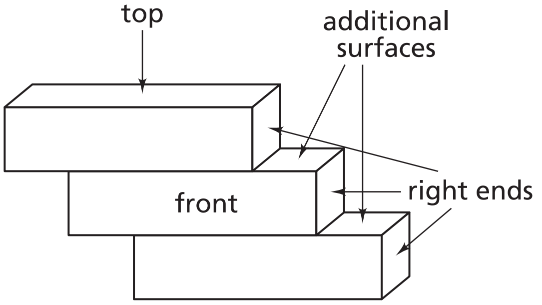
Strategy 1 Students may make a table and recognize the pattern as linear. Since an increase of 1 in the number of rods is related to an increase of 12 in the surface area, an equation is $A = 12n + 6$.

Length 4

Number of Rods	Surface Area
1	18
2	30
3	42
4	54
5	66
n	$12n + 6$

Strategy 2 Students may reason as follows: For one rod, the surface area is 18. For two rods, it is 12 + 18. For three rods, it is 18 + 12 + 12. For four rods, it is 18 + 12 + 12 + 12. Thus, the surface area is always 18 plus ($n - 1$) multiplied by 12, or $A = 18 + 12(n - 1)$.

Strategy 3 Students may analyze the number of surfaces with an area of 4 and the number of surfaces with an area of 1. The top and the front surfaces together have a surface area of $4(n + 1)$, and the right sides of the rods have a surface area of n. To account for the back, the bottom and the left sides, double the areas: $2[4(n + 1) + n]$. The number of additional surfaces with an area of 1, created by the staggering of the rods, is $2(n - 1)$. The total area is $2[4(n + 1) + n] + 2(n - 1)$.

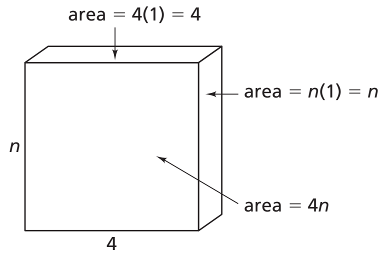


Strategy 4 Students may analyze the number of surfaces with an area of 4 and the number of surfaces with an area of 1 in a different way. The number of surfaces with an area of 4 is $2n + 2$: n in the front, n in the back, and 1 on the top and the bottom. This is a total surface area of $4(2n + 2)$. The number of surfaces with an area of 1 is $2n + 2(n - 1)$: each rod has 2 ends, for 2n surfaces, plus the $n - 1$ surfaces uncovered by the staggering on each end of the stack, for $2(n - 1)$. This expression is multiplied by 1 to get the surface area. The total surface area is thus $4(2n + 2) + 2n + 2(n - 1)$.

Strategy 5 Students might see a pattern in the number of surfaces with certain areas by making a table. Reasoning about the pattern leads to the equation $A = [2 + 4(n - 1)](1) + [4 + 2(n - 1)](4)$ or to the equation $A = (4n - 2)(1) + (2n + 2)(4)$.

Number of Rods	Faces With an Area of 1	Faces With an Area of 4	Total Surface Area
1	2	4	$2(1) + 4(4) = 18$
2	6	6	$6(1) + 6(4) = 30$
3	10	8	$10(1) + 8(4) = 42$
4	14	10	$14(1) + 10(4) = 54$
n	$2 + 4(n - 1)$ or $4n - 2$	$4 + 2(n - 1)$ or $2n + 2$	$[2 + 4(n - 1)](1) + [4 + 2(n - 1)](4)$ or $(4n - 2)(1) + (2n + 2)(4)$

Strategy 6 Some students may form the rods into a rectangular prism. For rods of length 4, this prism has dimensions n, 4, and 1. The surface area of the prism, $2(4n + n + 4)$, is then adjusted for the number of faces with a surface area of 1 that are hidden in the arrangement, a total of $2(n - 1)$.



The area of each face of the prism plus the lost area is
$2[4(n) + 1(n) + 4(1)] + 2(n - 1)$, or $2[4(n + 1) + n] + 2(n - 1)$.

Strategy 7 Some students may analyze the surface area of the figure as seen from the front, the right side, and the top; add the three numbers; and then multiply the sum by 2 to account for the back, the left side, and the bottom.

For rods of length 4, they might then produce the table below, which leads to the equation $A = 2[4n + n + 4 + (n - 1)]$ or to $A = 2(4n + n + n + 3)$, or $A = 2(6n + 3)$ (Figure 2)

Number of Rods	Surface Area From Front	Surface Area From Right Side	Surface Area From Top	Total Surface Area
1	4	1	4	$2(4 + 1 + 4) = 18$
2	8	2	5	$2(8 + 2 + 5) = 30$
3	12	3	6	$2(12 + 3 + 6) = 42$
4	16	4	7	$2(16 + 4 + 7) = 54$
n	$4n$	n	$4 + (n - 1)$ or $n + 3$	$2[4n + n + 4 + (n - 1)]$ or $2(4n + n + n + 3) = 2(6n + 3)$

5. Students' expressions should be equivalent. Explanations will vary.

6. **a.** Students are asked to compare their equations with classmates. The simplified equations are given in the table below.

Length (cm)	Equation in the Form $y = mx + b$
2	$A = 8n + 2$
3	$A = 10n + 4$
4	$A = 12n + 6$
5	$A = 14n + 8$
6	$A = 16n + 10$
7	$A = 18n + 12$
8	$A = 20n + 14$
9	$A = 22n + 16$
10	$A = 24n + 18$

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b. The expressions are all linear equations; the variable n is raised to the 1st power. Each equation has a graph which is a straight line and that has a constant rate of change. However, the slope and y-intercepts are different for all the equations. The slopes (and y-intercepts) are all multiples of 2. The slopes (and y-intercepts) increase by 2 as the rod length increase by 2.

c. Students are asked to write an equation for the surface area, A, of any stack of n rods of length ℓ. $A = 2[\ell(n + 1) + 1(n)] + 2(n - 1)$, or $(2\ell + 4)n + 2\ell - 2$.

Students do not have to start from scratch for each length of rod. They can use the strategy for the "4" rods and replace the "4" in the formula with the length of a new rod. For example:

Using Strategy 6 to make a compact rectangle that is 4 long, n high, and 1 wide, would lead eventually to the formula

$A = 2[1(n + 1) + 1(n)] + 2(n - 1)$. Replacing the "4" that represents the length of the rod we get

$A = 2[1(n + 1) + 1(n)] + 2(n - 1)$ for "1" rods,

$A = 2[3(n + 1) + 1(n)] + 2(n - 1)$ for "3" rods,

$A = 2[5(n + 1) + 1(n)] + 2(n - 1)$ for "5" rods, and so on. In general,

$A = 2[\ell(n + 1) + 1(n)] + 2(n - 1)$ or $(2\ell + 4)n + 2\ell - 2$ for any length of rod ℓ.

d. Students are asked to find the surface area of a stack of 50 rods each of length 10. Using the equation $A = 2[\ell(n + 1) + 1(n)] + 2(\ell - 1)$ for $\ell = 10$ and $n = 50$ we get that

$A = 2[10(50 + 1) + 1(50)] + 2(50 - 1) = 1{,}218$ square units.

Note: Answers are given below regarding the rectangular prism.

7. The dimensions are n, n, and 4.

8. An expression for the surface area is $2n^2 + 2(4n)$, or $2n^2 + 8n + 8n$, or $2n^2 + 16n$.

9. The surface area of a prism that is 10 rods high and 10 rods wide is $2(10)^2 + 16(10) = 360$ square units.

10. To change the expression if the rod length were something other than 4, one would replace the 4 in the expression. If the length were x, the expression would be $2n^2 + 2(xn) + 2(xn)$.

11. The relationship is quadratic. In the equation, the highest power of the variable is 2. The graph has the shape of a parabola.