Look for and Make Use of Design Structure: A Powerful Practice for Teachers of CMP

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Standard for Mathematical Practice #7

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure.
 - Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have.
 - Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
 - Older students, in the expression $x^2 + 9x + 14$, can see the 14 as 2 x 7 and the 9 as 2 + 7.
 - They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- step back for an overview and shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.
 (e.g., They see 5 3(x y)² as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.)

Why focus on mathematical structure?

Structure names the very foundation of mathematical thinking, namely that noticing structure is noticing patterns and relationships in the world and more abstractly.

- Mathematical structures act as a foundation on which students can connect related ideas efficiently
- A well connected network of ideas is likely to persist longer in memory and can be applied flexibly in unfamiliar contexts
- Recognizing and using structure allows us to manage complexity

What does it mean for teachers to "Look for and make use of *design* structure" using a curriculum such as CMP?









Where does a CMP teacher look to find:

- Opportunities and support for formative assessment?
- Support for engaging all students in the learning?
- Opportunities to differentiate?
- Support for struggling students?
- Help with pacing?



- Task Design
- ACE Problems
- Lesson Design Launch, Explore, Summary
- Mathematical Reflections
- Mathematical Goals
- Arc of Learning











Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. CCSS p. 8

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Task Design

An Introduction task designed to Set the Scene

- ACE Problems
- Lesson Design Launch, Explore, Summary
- Mathematical Reflections
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Task Design

- An Introduction task designed to Set the Scene
- Opportunities to formatively assess students' knowledge
- Opportunities to activate prior knowledge
- Opportunities to introduce the mathematical topic
- Strategies to engage students in the mathematics
- Opportunities to enable productive discourse
- ACE Problems
- Lesson Design
- Mathematical Reflections
- Mathematical Goals
- Arc of Learning





- Task Design
 - An Introduction task designed to Set the Scene
 - Opportunities to formatively assess students' knowledge
 - Opportunities to activate to prior knowledge
 - Opportunities to introduce the mathematical topic and vocabulary
 - Strategies to engage students in the mathematics
 - Opportunities to enable productive student discourse

Task Design: An Introduction task designed to Set the Scene



- Explore Problem 1.1 What opportunities do the tasks offer for each of the following:
 - Opportunities to formatively assess students' knowledge (ways of representing comparisons – fractions, ratios, rates, percents)
 - Opportunities to activate to prior knowledge
 - Opportunities to introduce the mathematical topic and vocabulary
 - Strategies to engage students in the mathematics
 - Opportunities to enable productive student discourse

Comparing and Scaling - Lesson 1.1 Surveying Opinions: Analyzing Comparison Statements





Surveying Opinions: Analyzing Comparison Statements

Problem 1.1

Here are four statements about the cola taste-test results.

 In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola-Nola by a ratio of 17,139 to 11,426.

 In a taste test, 5,713 more people preferred Bolda Cola.

 In a taste test, 60% of the people preferred Bolda Cola.

 In a taste test, people who preferred Bolda Cola outnumbered those who preferred Cola-Nola by a ratio of 3 to 2.

A

1. Describe what you think each statement above means.

- Which of the above statements do you think would be best in an advertisement for Bolda Cola? Why?
- Is it possible that all four statements are based on the same survey data? Explain your reasoning.
- In what other ways could you express the claims in the four statements? Explain your reasoning.
- Suppose you surveyed 1,000 cola drinkers. What numbers of Bolda Cola and Cola-Nola drinkers would you expect? Explain your reasoning.



Surveying Opinions: Analyzing Comparison Statements

Problem 1.1 continued

- Students at Neilson Middle School are planning an end-of-year event. Of the 150 students in the school, 100 would like an athletic event and 50 would like a concert. Decide whether each statement below accurately reports the results of the Neilson Middle School survey.
 - At Neilson Middle School, ¹/₃ of the students prefer a concert to an athletic event.
 - 2. Students prefer an athletic event to a concert by a ratio of 2 to 1.
 - The ratio of students who prefer a concert to students who prefer an athletic event is 1 to 2.
 - The number of students who prefer an athletic event is 50 more than the number who prefer a concert.
 - The number of students who prefer an athletic event is two times the number who prefer a concert.
 - At Neilson Middle School, 50% of the students prefer a concert to an athletic event.
- C 1. Study each correct comparison statement from Question B. What information does each statement give you about the situation? What information is left out?
 - 2. Use the Neilson Middle School survey results above. Suppose you consider a sample of students at a larger school. How might you predict the number of students who prefer an athletic event to a concert?





Task Design

An Introduction task designed to Set the Scene

- Opportunities to formatively assess students' knowledge
- Opportunities to activate to prior knowledge
- Opportunities to introduce the mathematical topic and vocabulary
- Strategies to engage students in the mathematics
- Opportunities to enable productive student discourse

Question: Where does a teacher look? Answer: Inside the tasks!

- Opportunities and support for formative assessment?
- Support for engaging all students in the learning?
- Opportunities to differentiate?
- Support for struggling students?
- Help with pacing?



- Task Design
- ACE Problems
- Lesson Design Launch, Explore, Summary
- Mathematical Reflections
- Mathematical Goals
 - TE and ST
 - Focus Questions (TE)
 - Scope and Sequence
- Arc of Learning



As a table group - choose a question that interests you.

- Opportunities and support for formative assessment?
- Support for engaging all students in the learning?
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- Help with pacing?



Comparing & Scaling Lesson 1.2



- Investigate each of the following components looking for opportunities to answer your question:
 - Task Design
 - ACE Problems
 - Lesson Design Launch, Explore, Summary
 - Mathematical Reflections
 - Mathematical Goals
 - TE and ST
 - Focus Question
 - Scope and Sequence
- Appoint a recorder for your table and be prepared to share your thinking!





Which mix will make juice that is the most "orangey?" Explain.



Mixing Juice: Comparing Ratios

Problem 1.2

- 1. Which mix will make juice that is the most "orangey"? Explain your reasoning.
 - 2. Which mix will make juice that is the least "orangey"? Explain your reasoning.
- **B** 1. Isabelle and Doug used fractions to express their reasoning.

Isabelle:Doug:
$$\frac{5}{9}$$
 of Mix B is concentrate. $\frac{5}{14}$ of Mix B is concentrate.

Do you agree with either of them? Explain.

- 2. Max thinks that Mix A and Mix C are the same. Max says "They are both the most 'orangey' since the difference between the number of cups of water and the number of cups of concentrate is 1." Is Max's thinking correct? Explain.
- Assume that each camper will get ¹/₂ cup of juice. Answer Questions (1) and (2) below for *each* of the four recipes.
 - 1. How many batches are needed to make juice for 240 campers?
 - 2. How much concentrate and how much water are needed to make juice for 240 campers?
- For each recipe, how much concentrate is needed to make 1 cup of juice? How much water is needed?



Ways of Comparing: Ratios and **Proportions**

E)

Applications | Connections | Extensions

Applications

- In a comparison taste test of two juice drinks, 780 people preferred Cranberry Blast, Only 220 people preferred Melon Splash, Complete each statement.
 - a. There were more people who preferred Cranberry Blast.
 - b. In the taste test, \$\overline{1}\%\$ of the people preferred Cranberry Blast.
 - c. People who preferred Cranberry Blast outnumbered those who preferred Melon Splash by a ratio of to .
- In a taste test of new ice creams invented at Moo University. 750 freshmen preferred Cranberry Bog ice cream, while 1,250 freshmen preferred Coconut Orange ice cream. Complete each statement.
 - a. The fraction of freshmen who preferred Cranberry Bog is .
 - b. The percent of freshmen who preferred Coconut Orange is \$\]%.
 - c. The ratio of freshmen preferring Coconut Orange to those who preferred Cranberry Bog was I to I.
- A town is debating whether to put in curbs along the streets. The ratio of town residents who support putting in curbs to those who oppose it is 2 to 5.
 - a. What fraction of the residents oppose putting in curbs?
 - b. If 210 people in the town are surveyed, how many do you expect to favor putting in curbs?
 - c. What percent of the residents oppose putting in curbs?





Ways of Comparing: Ratios and Proportions

Mathematical Reflections

In this Investigation, you used ratios to make comparisons. You also used ratios and developed proportions to solve mixture problems. You used scaling techniques to solve proportions and determine relationships between known and unknown quantities. The questions below will help you summarize what you learned.

Think about your answers to these questions, and discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

- a. In this Investigation you have used ratios, percents, fractions, and differences to make comparison statements. How have you found these ideas helpful?
 - b. Give examples to explain how part-to-part ratios are different from, but related to, part-to-whole ratios.
- 2. How can you use scaling or equivalent ratios
 - a. to solve a proportion? Give an example.
 - **b.** to make a decision? Give an example.



Ways of Comparing: Ratios and Proportions

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems and how you used Mathematical Practices.

Jayden described his thoughts in the following way:

In our class there were many different strategies for solving Problem 1.2. Our group scaled down each recipe to find how much water is needed for 1 cup of concentrate.

For A, we need 1 cup concentrate for $\frac{3}{2}$ cups of water.

For B, we need 1 cup concentrate for $\frac{9}{5}$ cups of water.

For C, we need 1 cup concentrate for 2 cups of water.

For D, we need 1 cup concentrate for $\frac{5}{3}$ cups of water.

The recipe with the least amount of water for 1 cup of concentrate is the most orangey. It is recipe A.

Common Core Standards for Mathematical Practice MP2 Reason abstractly and quantitatively

- What other Mathematical Practices can you identify in Jayden's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Goals to Support Planning and Teaching Scope and Sequence

	Comparing and Scaling						
	Content Connections to Other Units						
Goals of the Unit	Prior Work	Future Work					
Ratios, Rates, and Percents: Understand ratios, rates, and percents.	 Exploring and applying rational number concepts (<i>Comparing Bits and Pieces;</i> <i>Let's Be Rational; Decimal</i> <i>Ops; Accentuate the Negative</i>) Percent defined as a ratio to 100 and connected to fractions and decimals (<i>Comparing Bits and Pieces;</i> <i>Let's Be Rational; Decimal</i> <i>Ops</i>) 	 Calculating and applying slope with equations in y = mx + b form (Moving Straight Ahead; Thinking With Mathematical Models; Say It With Symbols) Making comparisons between groups of different sizes (Samples and Populations; Growing, Growing, Growing) 					

Goals to Support Planning and Teaching Focus Questions & Mathematical Reflections

Investigation 1: Ways of Comparing: Ratios and Proportions

Problem 1.1

Surveying Opinions: Analyzing Comparison Statements

FQ: What do different comparisons of quantities tell you about their relationship?

Problem 1.2 Mixing Juice: Comparing Ratios

FQ: What strategies do you use to determine which mix is the most orangey?

Problem 1.3 Time to Concentrate: Scaling Ratios

FQ: When you scale up a recipe and change the units, like from cups to ounces, what are some of the issues you have to deal with?

Problem 1.4 Keeping Things in Proportion: Scaling to Solve Proportions

FQ: What strategies can you use to find a missing value in a proportion? What is your preferred strategy and why?

Goals to Support Planning and Teaching Focus Questions & Mathematical Reflections

Investigation 1: Ways of Comparing: Ratios and Proportions

Mathematical Reflections

- 1a. In this Investigation you have used ratios, percents, fractions, and differences to make comparison statements. How have you found these ideas helpful?
- 1b. Give examples to explain how part-to-part ratios are different from, but related to, part-to-whole ratios.
- 2a. How can you use scaling or equivalent rations to solve a proportion? Give an example.
- 2b. To make a decision? Give an example.
- 3. You learned about scaling in *Stretching and Shrinking*. You learned about proportions and rates in *Comparing and Scaling*. How are the ideas in these two Units the same? How are they different?
- 4. Describe the connections you have found among unit rates,

As a table group - choose a question that interests you.

- Opportunities and support for formative assessment?
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- Help with pacing?



Comparing & Scaling Lesson 1.2



- Investigate each of the following components looking for opportunities to answer your question:
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Where does a teacher look to find: Opportunities and support formative assessment?

- Questions built into the tasks, explain and judge reasoning (A1 and A2 state explain reasoning)
- Look at different strategies launch, explore, summarize questions in the teacher edition
- Teacher reflection questions (e.g., what evidence do I have that my students understand the focus question)



Where does a teacher look to find: Support for engaging all students in the learning?

- B1 and B2 wording of the task support engagement
- TE (launch, explore) suggestion to bring in actual concentrate to compose one mixture – help obstacles about prior knowledge
- Move students to different sections and ask them to justify their reasoning and to convince others (helps build student identities)



Where does a teacher look to find: Opportunities to differentiate?

- Limiting the numbers of mixtures at start, then build towards them later
- Use visuals and charts of work
- Look at the mathematical reflections
- Have students explain their reasoning in multiple ways
- ACE structure builds in and across the applications, connections, and extensions
- Knowing your mathematical goal important for preparation



Where does a teacher look to find: Support for struggling students?



Where does a teacher look to find: Help with pacing?



What does it mean to for teachers to "Look for and make use of *design* structure" using a curriculum such as CMP?



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CMP - Arc of Learning Unit Design

Arc of Learning for Connected Mathematics (revised 2-5-16)										
Introducing Setting the Scene	Exploring Mucking About	Analyzing Going Deeper	Synthesizing Looking Across	Abstracting Going Beyond						
This problem provides an opportunity to										
 reveal the mathematical theme for the unit informally highlight the key mathematical concepts in the unit assess what students bring to the lesson in terms of the goals of the unit 	 establish a platform for developing key aspects of the understanding of the concepts and strategies explore (consider) a context that students can use to build, connect, and retrieve mathematical understandings 	 make connections between concepts and representations examine nuances in key aspects of the core mathematical ideas often with a variety of contextual situations 	 recognize core ideas across multiple contextual or problem situations begin to consolidate and refine emerging mathematical understanding(s) into a coherent structure 	 make judgments about which representations, operations, rules, or relationships are useful across various contexts look back on prior learning to generalize, extend, and abstract the underlying mathematical structure assess understandings at a more general level 						
*		Connecting Reflecting Assessing Mathematical Practices								

Arc of Learning for Comparing and Scaling

Comparing and Scaling: Ratios, Rates, Percents, and Proportions									
Reasoning Proportionally	Introduction Setting the Scene	Exploration Mucking About	Analysis Going Deeper	Synthesis Looking Across	Abstraction Going Beyond				
Investigation 1: Ways of Comparing: Ratios a	nd Proportions	5		_					
1.1 Surveying Opinions: Analyzing Comparison Statements	1.1	1.1							
1.2 Mixing Juice: Comparing Ratios		1.2	1.2						
1.3 Time to Concentrate: Scaling Ratios			1.3						
1.4 Keeping Things in Proportion: Scaling to Solve Proportions			1.4	1.4					
Mathematical Reflections				MR					
Investigation 2: Comparing and Scaling Rates									
2.1 Sharing Pizza: Comparison Strategies			2.1						
2.2 Comparing Pizza Prices: Scaling Rates			2.2	2.2					
2.3 Finding Costs: Unit Rate and Constant of Proportionality				2.3					
Mathematical Reflections				MR	MR				
Investigation 3: Markups, Markdowns, and Me	asures: Using	Ratios, Percent	s, and Proporti	ons					
3.1 Commissions, Markups, and Discounts: Proportions With Percents			3.1	3.1					
3.2 Measuring to the Unit: Measurement Conversions				3.2	3.2				
3.3 Mixing It Up: Connecting Ratios, Rates, Percents, and Proportions				3.3	3.3				
Mathematical Reflections					MR				
Looking Back					LB				

What does it mean to for teachers to "Look for and make use of *design* structure" using a curriculum such as CMP?



Why focus on mathematical structure?

It names the very foundation of mathematical thinking, namely that noticing structure is noticing patterns and relationships in the world and in our minds.

- Mathematical structures act as a foundation on which students can connect related ideas efficiently
- Understanding a well connected network of ideas is likely to persist longer in memory and can be applied flexibly in unfamiliar contexts
- Recognizing and using structure allows us to manage complexity



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Why focus on mathematical structure? understand and use

It names the very foundation of mathematical thinking, namely that noticing structure is noticing patterns and relationships in the world and in our minds.

- Design structures act as a foundation on which teachers can plan and implement lessons efficiently
- Understanding a well connected network of ideas is likely to persist longer in memory and can be applied flexibly in unfamiliar contexts

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It names the very foundation of mathematical thinking, namely that noticing structure is noticing patterns and relationships in the world and in our minds.

- Design structures act as a foundation on which teachers can plan and implement lessons efficiently
- Understanding a well connected network of textbook features is likely to persist longer in memory and can be applied flexibly in unfamiliar contexts
- Recognizing and using structure allows us to manage complexity

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Like Ragu'.....



Like Ragu', the needed resources are in there!



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Thank you!

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Is it like....?



Standards for Mathematical Practice

Mathematical facts and procedures, the content part of what we teach, are the results of the application of mathematical habits of mind reflected in the Practices. For that reason, fidelity to the way mathematics is made and used, a big part of the intent of the Mathematical Practices. requires that the Content be taught through the Practices. That way, the connections are real, integrated rather than interspersed.

Make sense of problems and persevere in solving them

Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

http://thinkmath.edc.org/index.php/Differences_between%2C_and_ connections_between%2C_Content_and_Practice_standards

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Structure

"the teaching and learning of structure, rather than the simple mastery of facts and techniques, is at the center of the classic problem of transfer...if learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible." (Bruner, 1960:12)

- What evidence do I have that students understand the focus question?
- Where did they get stuck?
- What strategies did they use?
- What break-throughts did my students have today?
- How will I use this to plan for tomorrow's lesson, for the next time I teach this lesson?
- Where will I have the opportunity to reinforce these ideas as I continue through this unit? The next unit?

Exploring CMP Design Structures

