Extended Response for Grade 6 EdReports Evaluation of *Connected Mathematics* Alignment with Common Core State Standards for Mathematics

The EdReports evaluation of instructional materials for Grades 6 – 8 developed by the Connected Mathematics Project and published by Pearson concludes that those materials, "do not meet the requirements for alignment to the Common Core State Standards." We challenge many specific critical statements about alignment of the instructional materials and the Common Core State Standards for Mathematics. In general, the three major themes and our clarifying responses are summarized as follows:

- Not enough time attending to the major clusters. Only four of the Grade 6 units were consider attending to major work. Let's be Rational, Covering and Surrounding, and Data About Us need to be re-examined and identified as attending to major work.
- Some unit test items target content above grade level. The four of the total 63 unit test items need to be re-examined and identified as meeting the Grade 6 CCSSM.
- *Limited support for differentiation*. Differentiation is a strong component of Connected Mathematics and that its curriculum and pedagogical model is designed to enhance the learning experience of *all* students. This claim needs to be re-examined as fully supporting differentiation.

To provide clarification for the specific critical statements, the following section is organized by the *Focus* and *Coherence* indicators for Gateway One. This is the only Gateway of three reviewed for Connected Mathematics 3. The *Indicators* and *Claims* provided below are direct quotes from the Report released to Connected Mathematics on February 18, 2015.

We believe the proven record of Connected Mathematics and the concerns elaborated for each grade level review raise serious doubts about validity of the 'does not meet expectations' judgment in the EdReports evaluation of Connected Mathematics 3 and that reconsideration of the evaluation is in order.

Gateway I Major Work (Focus) and Coherence

FOCUS

Focus: Indicator I

The instructional materials assesses* the grade-level content indicated on the reference sheet and, if applicable, content from earlier grades. Content from future grades may be introduced but students should not be held accountable for future content.

Claim

The instructional materials reviewed for Grade 6 do not meet the expectations for assessing material at the grade level. The materials assess topics that are in future grades. For example,

- On the Variables and Patterns unit test, question 6 is on functions using two variables, which is a Grade 8 standard 8.F.B.4.
- On the Decimal Ops Unit, tests 3-5 are on application of percents, which is a Grade 7 standard 7.RP.A.3.

Response

We provide clarification that the four identified unit assessment items (6.4% of the total 63 assessment items for Grade 6) do meet the 6th grade CCSSM, and therefore, the instructional materials for Grade 6 do meet the expectations for assessing material at the Grade 6 level.

Unit Test for Variables and Patterns

For unit assessment item 6 in *Variable and Patterns: Focus on Algebra*, students consider a situation where Dee checks the price of game systems from two different stores (see figure below for the entire assessment item). Students are asked in Part a to write an equation that relates the dependent variable (cost c) with the independent variable (the number of games g). There is no mention of 'function.' We argue that Item 6 is aligned with the Standard 6.EE.C.9:

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation (p. 44).

Therefore, Item 6 is appropriate for unit assessment at the Grade 6 level.

Assessment Item as Referenced Above:

- **6.** Dee owns a game system. He checked some stores to find the least expensive place to buy games.
 - Taylor's Department Store sells games for \$15.50 each.
 - Buyer's Warehouse has an initial \$25 membership fee. Then, each game is \$12.
 - **a.** Write an equation relating the cost *c* for any number of games *g* at each store.

Taylor's Department Store: Buyer's Warehouse:

- **b.** How many games would Dee have to purchase for Buyer's Warehouse to be the best place to buy games? Show your work.
- **c.** How many games would Dee have to purchase for Taylor's Department Store to be the best place to buy games? Show your work.
- **d.** How many games can Dee buy from Taylor's Department Store if he wants to keep his total cost to no more than \$155? Use a number line to represent your answer.

Unit Test for Decimal Ops

For unit assessment items 3-5 in *Decimal Ops: Computing with Decimals and Percents*, students consider discounts and sales tax situations where they find percents of prices before tax as rates per 100 (see figure below for the entire assessment items). For example, in Item 4, students need to find the 5% sales tax of a total bill of \$14.90 before tax as 5/100 times \$14.90. Students also apply their understanding of rational numbers by taking the numerical tax and adding it to the price before tax to determine the total price (see 6.NS.C). Items 3-5 are aligned with the Standard 6.RP.A.3:

Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity) (p. 42).

Therefore, Items 3-5 are appropriate for unit assessments at the Grade 6 level.

Unit Assessment Items as Referenced Above:

3. A toy store offers an additional 25% discount on board games that have already been reduced by 30%. Will the final cost be the same as a discount of 55% on the original price? Use an example with an original cost of your choice to help explain your answer.

- 4. The total bill for drinks and a pizza for three people is \$14.90 before tax.
 - a. The sales tax is 5%. What will be the amount of sales tax added to the bill? Show your work.
 - **b.** The group wants to leave a 15% tip on the cost of the food, not including tax. What will be the amount of the tip? Show your work.
 - **c.** How much should each person pay if they are to share the bill equally? Show your work.

5. Suppose that your mom gives you \$20 to go to the school supplies store. There is a 6% sales tax on supplies. You need to buy the following items:

Item	Cost
pencils	\$1.29
notebook binder	\$5.99
paper	\$3.49
highlighter pens	\$2.35

a. How much will the supplies cost? Show your work.

b. Your favorite lunch costs \$5.39. Will you have enough money left to buy the lunch? Explain.

Focus: Indicator II

Instructional material spends the majority of class time on the major cluster of each grade.

Claim

The instructional materials reviewed for Grade 6 do not meet the expectations for spending the majority of class time on the major cluster for Grade 6. The instructional materials reviewed for Grade 6 had only approximately 50% of the work on the major clusters for 6.RP.A, 6.NS.A, 6.NS.C, 6.EE.A, 6.EE.B and 6.EE.C.

- The units Prime Time, Comparing Bits and Pieces, Decimal Operations and Variables and Patterns have lessons specifically addressing this major work.
- There are also lessons in many units that are supporting cluster concepts but they are used to enhance the major work.
- Two examples of this are in Prime Time where 6.NS.B is found throughout

- investigations 2-4 but it is used to enhance the work with 6.RP.A.
- In Covering and Surrounding in investigation 3, 6.G.A is used to enhance the understanding of 6.NS.C.

Response

The units Prime Time: Factors and Multiples, Comparing Bits and Pieces: Ratios, Rational Numbers, and Equivalence, Decimal Ops: Computing with Decimals and Percents, and Variables and Patterns: Focus on Algebra specifically addresses the major work. We provide clarification that the Let's Be Rational: Understanding Fraction Operations **should be** designated as "major work" as it attends to the major clusters of 6.NS.A, 6.EE.A, and 6.EE.B. Moreover, references made later in the Report further support this statement.

In addition, we provide clarification that the units, Covering and Surrounding: Two-Dimensional Measurement and Data About Us: Statistics and Data Analysis, designated as "supporting cluster concepts" or "used to enhance the major work" should be reexamined and identified as attending to the "major work" or "major cluster of each grade." Therefore, all the Grade 6 units of Connected Mathematics address the major Grade 6 work.

Let's Be Rational: Understanding Fraction Operations

The *Let's Be Rational: Understanding Fraction Operation* unit supports students in developing meaning for and skill with computations involving fractions. This work involves (a) using knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying, and dividing fractions, (b) understanding that estimation is a tool used in a variety of situations for approximating results of arithmetic operations, and (c) using variables, equations, and fact families to represent unknown real-world values and relationships. Therefore, the work in this unit addresses the major work clusters of 6.NS.A, 6.EE.A, and 6.EE.B.

In terms of the major cluster 6.NS.A, problems in Investigation 2 and 3 provide opportunities for students to address Standard 6.NS.A.1:

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem (p. 42).

For example, problems in Investigation 3 emphasize division of fractions. In particular, Problem 3.1 begins with a visual model that is provided to represent division of fractions. Students are asked to provide visual models in solving problems involving divisions, as shown below.

Examples as Referenced Above:

Dividing With Fractions pp. 47-49

When you do the division $12 \div 5$, what does the answer mean?

The answer should tell you how many fives are in 12 wholes. Because a whole number of fives will not fit into 12, you might write

$$12 \div 5 = 2\frac{2}{5}$$

Then, what does the fractional part of the answer mean?

The answer means you can make 2 fives and $\frac{2}{5}$ of *another* five.



You can check your work by seeing that the related number sentence is true.

$$5 \times 2\frac{2}{5} = 12$$

In the division problem above, the divisor and dividend are both whole numbers. In Problem 3.1, you will explore division problems in which the divisor and dividend are both fractions. You will answer questions such as the following.

- How many $\frac{1}{4}$'s are in $\frac{1}{2}$?
- · How can you draw a model to show this?



In terms of the major cluster 6.EE.A, problems in Investigation 1, 3, and 4 provide opportunities for students to address 6.EE.A.2:

Write, read, and evaluate expressions in which letters stand for numbers (p. 43)

For example, problems in Investigation 4 provide instances where students need to write, read, and evaluate expressions in which letters stand for numbers (see Problem 4.1 shown below). Practice questions at the end of the investigation provide additional examples for students to practice with this work (shown below).

Examples as Referenced Above:

Problem 4.1 For each number sentence, write a complete fact family and find the value of N. 1. ⁵/₁₀ - ²/₅ = N 2. 3³/₅ + 1²/₃ = N 3. Describe the relationship between addition and subtraction. Use the fact families in parts (1) and (2) as examples. For each number sentence, find the value of N. 1. N + 1²/₃ = 5⁵/₆ 2. ³/₄ + N = ¹⁷/₁₂ 3. N - ¹/₂ = ³/₈ 4. How can fact families help you find the value of N in parts (1)-(3)?

Applications

For each of Exercises 1–4, write a complete fact family.

1. $\frac{1}{16} + \frac{1}{12} = N$	2.	$\frac{5}{4} - \frac{4}{5} = N$
3. $N-1\frac{1}{3}=2\frac{2}{3}$	4.	$N + \frac{4}{3} = \frac{1}{3}$

For Exercises 5–10, find the value for *N* that makes each number sentence true.

5.
$$\frac{2}{3} + \frac{3}{4} = N$$

6. $\frac{3}{4} + N = \frac{4}{5}$
7. $N - \frac{3}{5} = \frac{1}{4}$
8. $\frac{2}{2} - \frac{2}{4} = N$
9. $\frac{3}{8} - N = \frac{1}{4}$

Problems in Investigation 2 provide opportunities for students to address Standard 6.EE.A.3:

Apply the properties of operations to generate equivalent expressions" (p. 44).

Problem 2.3 provides an example where students consider different strategies for generating equivalent expressions that aligns with Standard 6.EE.A.3 (shown below). In Problem 4.3, students need to apply their understanding of operations in order to decide whether equations they are provided with describe the same situation or not.

Examples as Referenced Above:



In terms of the major cluster 6.EE.B, problems in Investigation 1 provide opportunities for students to address Standard 6.EE.B.5:

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (p. 44).

For example, Problem 4.1 provides students an opportunity to find values that make an equation hold true (shown below).

Example as Referenced Above:



Problems in Investigation 1 and 4 provide opportunities for students to address Standard 6.EE.B.6:

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set (p. 44).

For example, Problem 4.3 provides an opportunity for students to write number sentences relating to real-world problem situations (shown below).

Example as Referenced Above:



Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers (p. 44).

For example, Problem 1.4 provides an opportunity for students to solve real-world problems by solving equations of the form x + p = q for cases in which p, q and x are all nonnegative rational numbers (shown below).

Example as Referenced Above:

Ms. Garza buys spices to make one batch of Garam Masala. When she weighs her spices at home, she only has 10¹¹/₁₂ ounces of spice. Which spice did Ms. Garza forget?

Renuka has two pounds of pepper in her cupboard. She knows that there are 16 ounces in one pound. After Renuka makes one batch of Garam Masala, how many ounces of pepper does Renuka have left in her cupboard?

For each number sentence below, write a spice story. Then find the value for N that makes the sentence true.

1. $3\frac{1}{6} - 1\frac{3}{4} = N$ **2.** $N + \frac{3}{4} = 1\frac{1}{2}$ **3.** $2\frac{2}{3} - N = 1\frac{1}{4}$

 Describe a strategy for estimating sums and differences of fractions, including mixed numbers.

Further, there are several references within the Report indicating that the unit *Let's Be Rational: Understanding Fraction Operations* should be designated as major work.

- For the third coherence indicator, the Report states, "All three grades levels have major work on equations, EE.A and EE.B...Grade 6: Reason about and solve one-variable equations and inequalities found in several units (e.g., *Let's Be Rational, Variables & Patterns*) using informal methods of solving."
- For the third coherence indicator, the Report also states, "All three grades have major work on the number system (6.NS.A, 6.NS.B, 6.NS.C, 7.NS.A, 8.NS.A)...This leads to finding the least common multiple in order to find common denominators for fractions in *Comparing Bits and Pieces, Let's be Rational*, and *Decimal Ops* in Grade 6 and extends to ratios in Comparing and Scaling in Grade 7... *Let's be Rational* begins 6.NS.A with student dividing fractions. This continues in Grade 7 with 7.NS.A in *Accentuate the Negative*."
- For the fourth coherence indicator, the Report also states, "There are many links between major clusters in this curriculum...In *Let's Be Rational* students apply and extend previous understanding of multiplication and division to divide fractions by fractions (6.NS.A), and identify parts of an expression using mathematical terms (6.EE.A)."

Covering and Surrounding: Two-Dimensional Measurement

The Unit *Covering and Surrounding: Two-Dimensional Measurement* provides students the opportunity to strengthen their understanding of number by examining measurement situations by exploring area and perimeter provides a context in which students explore additive (perimeter) and multiplicative situations (area) and the relationship between them (distributive property). In addition, measurement becomes another context that provides opportunity for students to recognize variables, the relationship among the variables expressed in terms of the variables, test the equivalence of expressions for area or volume, and solve equations given information about one or more of the variables. Therefore, the work in this unit addresses the major work clusters of 6.NS.C, 6.EE.A, and 6.EE.B.

In terms of the major cluster 6.NS.C, problems in Investigations 1 and 3 provide opportunities for students to address Standard 6.NS.C.8:

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate (p. 43).

For example, Problem 3.4 provides an opportunity for students to solve problems by graphing points in the coordinate plane (shown below).





In terms of the major cluster 6.EE.A, problems in Investigations 1, 2, 3, and 4 provide opportunities for students to address Standard 6.EE.A.2:

Write, read, and evaluate expressions in which letters stand for numbers (p. 43).

For example, ACE exercise 3 provides an opportunity for students to use words to describe a formula for finding the perimeter of a rectangle (shown below).

Example as Referenced Above:

3. The dimensions of a rectangle are called **length** ℓ and **width** w. Look for patterns throughout Problem 1.1 to help you answer the questions below.



- **a.** Use words to describe a formula for finding the perimeter of a rectangle. Write the formula using symbols. Explain why it works.
- **b.** Use words to describe a formula for finding the area of a rectangle. Write the formula using symbols. Explain why it works.
- c. Find the perimeter and area of a rectangle with a width of 6 centimeters and a length of 15 centimeters.

Problems in Investigations 1, 2, and 4 provide opportunities for students to address Standard 6.EE.A.3:

Apply the properties of operations to generate equivalent expressions (*p. 44*).

For example, ACE exercise 61 provides an opportunity for students to examine different expressions representing area and perimeter and asks students to verify whether those are equivalent expressions or not (shown below).

Example as Referenced Above:

61. Stella, Gia, and Richard wrote different formulas for finding the perimeter of a rectangle with length ℓ and width w.



Who is correct? Explain your reasoning.

Problems in Investigations 2 and 4 provide opportunities for students to address Standard 6.EE.A.4:

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them) (p. 44).

For example, Problem 3.1 provides an opportunity for students to formulate different expressions for finding perimeter and area of parallelograms and to describe their strategies (shown below).

Example as Referenced Above:



In terms of the major cluster 6.EE.B, problems in Investigations 1, 2, 3, and 4 provide opportunities for students to address Standard 6.EE.B.6:

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set (p. 44).

For example, Problem 1.2 provides an opportunity for students to use variables to describe designs with the least and greatest perimeter for a fixed area (shown below).

Example as Referenced Above:

- Suppose you build a storm shelter with 36 square meters of rectangular floor space. Which design has the least perimeter? Which has the greatest perimeter? Explain your reasoning.
 - **2.** In general, describe the rectangle that has the greatest perimeter for a *fixed*, or unchanging, area. Describe the rectangle that has the least perimeter for a fixed area.

Data About Us: Statistics and Data Analysis

The unit *Data About Us: Statistics and Data Analysis* provides students the opportunity to learn about the process of statistical investigations. This work involves: (a)

understanding and using the process of statistical investigation, (b) distinguishing data and data types, (c) displaying data with multiple representations, and (d) recognizing that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread). This unit provides another context, namely data, to explore understandings of the rational numbers. Therefore, the work in this unit addresses the major work clusters of 6.RP.A and 6.NS.C.

In terms of the major cluster 6.RP.A, problems in Investigation 3 provide opportunities for students to address Standard 6.RP.A.3:

> *Use ratio and rate reasoning to solve real-world and mathematical* problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations (p. 42).

For example, Problem 3.1 provides an opportunity for students to use ratio and rate reasoning to solve problems involving a table (shown below).

Example as Refere	enced Abo	ve:											
	Problem 3	.1	Con	tinu	ed								
	The stu Raisin I	dents Iakes	also p	ooure	d esti	mate	d servi	ngs (1 <u>1</u> cuj	o or 3	3 gran	ns) of	
	1. Or the	n a co ey gat	py of t hered	he ta l.	ble be	low,	write t	he se	rving	sizes	of the	data	
				F	ours	of R	laisin	Flak	es				
	Grams Poured	44	33	31	24	42	31	28	24	15	36	30	41
	Serving Size	1.33	1.00	jii	j.		0.94				1.09	ш	
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In terms of the major cluster 6.NS.C, problems in Investigations 2, 3, and 4 provide opportunities for students to address Standard 6.NS.C.7:

Understand ordering and absolute value of rational numbers (p. 43).

Using frequency tables and graphs (line plots, histograms, and box plots), finding and using measures of center and spread are all about ordering numbers. For example, Problem 2.3 provides an opportunity for students to compute the median and mean of the data from the two stores presented in the dot plots (shown below).

Example as Referenced Above:



Further, there is a reference within the Report indicating that the unit *Data About Us* should be designated as major work. For the fourth coherence indicator, the Report states, "There are many links between major clusters in this curriculum...*Data About Us* uses data to compute with numbers, measure and graph data connects 6.SP.A, 6.SP.B and 6.RP.A."

COHERENCE

Coherence: Indicator I

Supporting content enhances focus and coherence simultaneously by engaging students in the major work of the grade.

Claim

The instructional materials reviewed for Grade 6 partially meets the expectations for the supporting content enhancing the major work. There are areas where the materials have strong connections and areas that could be stronger.

- In *Comparing Bits and Pieces* there are problems in investigations 1 and 2 that use supporting work of fraction operations to enhance the major work 6.NS.C and 6.RP.A.
- In *Prime Time, Covering and Surrounding* and *Decimal Ops,* there is work with equations while working on area, and greatest common factor and lowest

common multiple decimal operations that supports the major work of 6.EE.A.

- There are lessons asking students to determine equivalence between fractions, decimals, and percents that is not part of Grade 6 work. This section was not completed alongside 6.RP.A or 6.NS.C, which would have enhanced the major work.
- In *Prime Time* there was a heavy emphasis on greatest common factor and lowest common multiple that was not connected to more work with integers for 6.NS.C.
- There are problems in *Data About Us* that could have tied into integers and enhance the major work of 6.NS.C.

Response

We agree with the first two claims in this indicator for coherence:

In Comparing Bits and Pieces: Ratios, Rational Numbers, and Equivalence (designated as a unit addressing major work), "there are problems in investigations 1 and 2 that use supporting work of fraction operations to enhance the major work 6.NS.C and 6.RP.A."

In Prime Time: Factors and Multiples (designated as a unit addressing major work); in Covering and Surrounding: Two-Dimensional Measurement (a unit argued earlier as addressing major work); and in Decimal Ops: Computing with Decimals and Percents (designated as a unit addressing major work), "there is work with equations while working on area, and greatest common factor and lowest common multiple decimal operations that supports the major work of 6.EE.A.

Comparing Bits & Pieces: Ratios, Rational Numbers, and Equivalence

We provide clarification that determining equivalence between fractions, decimals, and percents is work found in the Grade 6 unit *Comparing Bits & Pieces: Ratios, Rational Numbers, and Equivalence*. The equivalence work in this unit, designated in the report as focusing on major work, attends to and occurs alongside 6.RP.A and 6.NS.C.

In *Comparing Bits & Pieces: Ratios, Rational Numbers, and Equivalence*, students are provided opportunities to understand equivalence of fractions and ratios, and use equivalence to solve problems. Specifically for equivalence, students explore problems where they may: (a) recognize that equivalent fractions represent the same amount, distance, or location; develop strategies for finding and using equivalent fractions, (b) recognize that comparing situations with different sized wholes is difficult without some common basis of comparison, (c) use partitioning and scaling strategies to generate equivalent fractions and ratios and to solve problems, (d) develop meaningful strategies

for representing fraction amounts greater than 1 or less than -1 as both mixed numbers and improper fractions, (e) recognize that equivalent ratios represent the same relationship between two quantities; develop strategies for finding and using equivalent ratios, and (f) build and use rate tables of equivalent ratios to solve problems. This work in this unit attends to and occurs alongside 6.RP.A and 6.NS.C and therefore is Grade 6 work.

In terms of the major cluster 6.RP.A, problems in Investigations 1, 2, and 4 provide opportunities for students to address Standard 6.RP.A.1:

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities (p. 42).

For example, Problem 1.5, students are explicitly wrestling for the first time with both fractions and ratios in the same Problem. Students explore the fundamental idea of equivalence as it applies to both ratios and fractions (shown below).

Example as Referenced Above:



Problems in Investigations 1, 2, and 4 provide opportunities for students to address Standard 6.RP.A.2:

Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship (p. 42).

For example, in Problem 2.2, students use the ratios idea and are expected to relate unit rate to ratio (shown below).

Example as Referenced Above:



Problems in Investigations 1, 2 and 4 provide opportunities for students to address Standard 6.RP.A.3:

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations (p. 42).

For example, in Problem 2.3, students need to create and use a rate table to find many equivalent comparisons (shown below).

Example as Referenced Above:

4

		C	ers of c hewy F	hewy fr Fruit W	uit wor orm Pr	ms. icing			
Numb	er of Worms	1	5	10	15	30	90	150	180
Reduc	ed Price	11	1	Ξ.		\$3	111		11
2. 3. 4. The of p	How much do How many che What is the un e student coun oppcorn (unpo g of popcorn th	3 chev ewy fru it price cil also pped) at hold	vy fruit uit worr e of one decide kernels ls 2 cup	worms ns can y chewy es to sell yields s of pop	cost? 3 /ou buy fruit w l popco 4 cups o ocorn.	00 chev for \$50 orm? W rn to ra of popce	vy fruit)? For \$ 'hat is tl ise mor orn. On	worms? 10? he unit ney. On e servir	rate? e ounce ng is a

In terms of the major cluster 6.NS.C, problems in Investigation 3 provide opportunities for students to address Standard 6.NS.C.5:

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation (p. 43).

For example, in Problem 3.1 students are introduced to fractions greater than 1 and less than 0. These numbers are also explained with real life examples within the same problem (shown below).

Examples as Referenced Above:

The number line can be extended in both directions, as shown below. Numbers to the left of zero are marked with a "-" sign and are read as *negative one, negative two,* etc.



In this Problem, you will use fractions, mixed numbers, and improper fractions. You can represent positive and negative fractions and mixed numbers as points on the number line.



- Betty says that the mark between 2 and 3 should be labeled ¹/₂. Do you agree?
- Judi says that the mark between 2 and 3 should be labeled ⁵/₂. Do you agree?
- 1. a. Griffin visited her grandfather in Canada twice in the same year. During those visits, her grandmother took pictures of Griffin with her grandfather. Griffin says the absolute value of the temperature each day was 10. Is this possible? Explain. What is the difference between the two temperatures in degrees?
 - **b.** Griffin says the bird's height above and the fish's depth below sea level are opposites. Is this possible? Explain.



Problems in Investigation 3 provide opportunities for students to address Standard 6.NS.C.6:

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates (p. 43).

For example, Problem 3.1 introduces rational numbers as points on the number line and extends the number line by including rational numbers greater than 1 and less than zero (shown below).

Example as Referenced Above:



Problems in Investigation 3 provide opportunities for students to address Standard 6.NS.C.7:

Understand ordering and absolute value of rational numbers (p. 43).

For example, in Problem 3.1, students are introduced to the concept of absolute value (shown below).

Example as Referenced Above:

The **absolute value** of a number is its distance from 0 on the number line. Numbers that are the same distance from 0 have the same absolute value. The absolute value of $2\frac{1}{2}$ and the absolute value of $-2\frac{1}{2}$ are both $2\frac{1}{2}$.

You can express the absolute value of a number two ways without words.



- What is the opposite of $-\frac{2}{3}$? What is the opposite of $\frac{2}{3}$?
- What is the absolute value of $-\frac{2}{3}$? What is the absolute value of $\frac{2}{3}$?

Zero, whole numbers, fractions, and their opposites are **rational numbers**. The numbers $-\frac{9}{5}$, -3, 0, $\frac{2}{3}$, and $2\frac{1}{3}$ are all rational numbers.

Prime Time: Factors and Multiples

We provide clarification that greatest common factor and lowest common multiple in *Prime Time: Factors and Multiples* are used to develop equivalent fractions, ratios, and rates (6.RP.A) and address Standard 6.NS.B.4. Further, we fail to see how greatest common factor and lowest common multiple (involving whole numbers) are connected to the 6.NS.C cluster, which relate only to positive and negative numbers represented as opposites, points on a number line, absolute values, or coordinate pairs in all four quadrants.

In *Prime Time: Factors and Multiples*, students are provided opportunities to understand relationships among factors, multiples, divisors, and products and understand why two expressions are equivalent. Greatest common factor and lowest common multiple are essential to proportional reasoning, which is foundational to work in equivalent fractions and ratios. Problems in Investigations 2, 3, and 4 provide opportunities for students to address Standard 6.NS.B.4:

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor (p. 42).

For example, in Problem 2.1 students develop strategies for finding common multiples. When they are asked to find the first revolutions in which this happens, they are finding the least common multiple (shown below). It should be noted that the development of the Distributive Property with variables is continued in *Variables and Patterns: Focus on Algebra*.

Example as Referenced Above:

Problem 2.1

A The large Ferris wheel makes one revolution in 60 seconds. The small Ferris wheel makes one revolution in 20 seconds.

The large Ferris wheel makes one revolution in 50 seconds. The small Ferris wheel makes one revolution in 30 seconds.

- The large Ferris wheel makes one revolution in 20 seconds. The small Ferris wheel makes one revolution in 11 seconds.
- For Questions A-C, find the number of times each Ferris wheel goes around before Jeremy and his sister are both at the bottom again.



Further, greatest common factor and lowest common multiple do not apply to negative numbers and so it is unclear how the Report indicates that they should be connected to the 6.NS.C cluster, which relate only to positive and negative numbers represented as opposites, points on a number line, absolute values, or coordinate pairs in all four quadrants.

Data About Us: Statistics and Data Analysis

We provide clarification that in the unit, Data About Us: Statistics and Data Analysis, there are problems that are tied into the cluster of 6.NS.C.

The unit *Data About Us: Statistics and Data Analysis* provides students the opportunity to learn about the process of statistical investigations. This work involves: (a) understanding and using the process of statistical investigation, (b) distinguishing data and data types, (c) displaying data with multiple representations, and (d) recognizing that a single number may be used to characterize the center of a distribution of data and the degree of variability (or spread). This unit provides another context, namely data, to explore understandings of the rational numbers. These problems focus mostly on positive rational numbers to understand a rational number as a point on the number line and as an ordered pair in graphing situations. Doing significant work with negative rational numbers would require attending to Grade 7 standards around work on operations involving negative numbers. Therefore, in this Grade 6 unit, we used a geometric interpretation of distance to determine the Mean Absolute Deviation. This is

revisited in Grade 7 where students use operation work involving positive and negative integers. Thus, the work in the unit addresses the major work clusters of 6.NS.C.

In terms of the major cluster 6.NS.C, problems in Investigations 2, 3, and 4 provide opportunities for students to address Standard 6.NS.C.7:

Understand ordering and absolute value of rational numbers (p. 43).

Understanding the order of numbers is fundamental to effectively using frequency tables and graphs (line plots, histograms, and box plots), finding and using measures of center and spread. For example, Problem 2.3 provides an opportunity for students to compute the median and mean of the data from the two stores presented in the dot plots (shown below).

Example as Referenced Above:



Further, there is a reference within the Report indicating that the unit *Data About Us* should be designated as major work. For the fourth coherence indicator, the Report states, "There are many links between major clusters in this curriculum...*Data About Us* uses data to compute with numbers, measure and graph data connects 6.SP.A, 6.SP.B and 6.RP.A."

Coherence: Indicator II

The amount of content designated for one grade level is viable for one school year in order to foster coherence between grades.

Claim

The instructional materials reviewed for Grade 6 partially meet the expectations for being able to be taught in one school year.

- The Grade 6 materials could be completed within the timeline of 180 days.
- This includes all lessons, mathematical reflections, *Looking Back* and *Looking Ahead* and all assessments.
- While overall it is viable for a school year, the amount of time on the major work for Grade 6 is less than 65% of the year, which means that teachers would need to find supplemental materials in order to effectively cover the standards.

Response

We address the second indicator for coherence in the following way. The Grade 6 materials could be completed within the timeline of 180 days. This includes all lessons, mathematical reflections, *Looking Back* and *Looking Ahead*, and all assessments.

We provide clarification that the Connected Mathematics materials supports nearly 100% of the time on the major work for Grade 6. As described in the second focus indicator, all seven of the Grade 6 units of Connected Mathematics address the major work in Grade 6. Therefore, nearly 100% of the time addresses the major work in Grade 6.

Coherence: Indicator III

Materials are consistent with the progressions in the Standards i. Materials develop according to the grade-by-grade progressions in the Standards. If there is content from prior or future grades, that content is clearly identified and related to grade-level work ii. Materials give all students extensive work with grade-level problems iii. Materials relate grade level concepts explicitly to prior knowledge from earlier grades.

Claim

The materials reviewed for Grade 6 partially meet the expectations for being consistent with the progressions in the standards. The connections between standards to build student understanding are strong. There are some off grade level topics that could be identified to help teachers and students know that these are topics that are beyond the CCSSM necessary for that grade.

All three grade levels have major work on equations, EE.A and EE.B:

• Grade 6: Reason about and solve one-variable equations and inequalities – found in several units (e.g., *Let's Be Rational, Variables & Patterns*) using informal methods of solving.

- Grade 7: Solve real-life and mathematical problems using numerical and algebraic expressions and equations-primarily in *Moving Straight Ahead* where they start using symbolic equations and properties of equality.
- Grade 8: Analyze and solve linear equations and pairs of simultaneous linear equations-found in *It's in the System* where various methods of solving systems are explored.

All three grade levels have major work on ratio and proportional reasoning (6.RP, 7.RP)

- Grade 6: *Comparing Bits and Pieces* begins work with ratios/rates and proportions then continues the major work of Grade 6 ratio and proportion into *Variables and Patterns*.
- Grade 7: *Stretching and Shrinking* works with ratios using scale factors and *Comparing and Scaling* continues the work by solving proportions using many strategies learned from Grades 6 and 7.
- Grade 8: *Butterflies, Pinwheels and Wallpaper* use the concepts of proportional reasoning in transformational geometry work.

All three grades have major work on the number system (6.NS.A, 6.NS.B, 6.NS.C, 7.NS.A, 8.NS.A):

- Finding the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor begins the work with 6.NS.B.4 in *Prime Time*.
- This leads to finding the least common multiple in order to find common denominators for fractions in *Comparing Bits and Pieces, Let's be Rational,* and *Decimal Ops* in Grade 6 and extends to ratios in Comparing and Scaling in Grade 7. This continues into Accentuate the Negative in Grade 7 with performing arithmetic operations with integers and rational numbers (7.NS.A).
- *Comparing Bits and Pieces* begins developing the ideas of positive and negative numbers on a number lines and absolute value (6.NS.C). This leads to operations on rational numbers in *Accentuate the Negative* (7.NS.A). This also leads into 8.NS.A on approximating rational numbers.
- *Let's be Rational* begins 6.NS.A with student dividing fractions. This continues in Grade 7 with 7.NS.A in *Accentuate the Negative*.

There is limited support for differentiation of instruction.

- There is guidance for the teacher in the book titled A Guide to Connected Mathematics 3 that discusses differentiation. This gives best practices from research to be used while working on the problem with all students.
- Differentiation is embedded within the instructional model for Connected Mathematics 3 that all students experience the problem launch and summary in the same way and that the differentiation comes during the explore phase of the

problem.

- There are specific strategies and guidance for English language learners.
- To help make differentiation more explicit, strategies need to be discussed in the teacher's unit planning pages and it needs to be tied into the specific problems so the teachers have guidance.
- The guide has general best practices but what to use with specific parts of a unit would make it more accessible for teachers and students.

There are many places where the materials relate grade level concepts explicitly to prior knowledge from earlier grades. These can be found in the student editions in the problems and in the teacher editions in charts and in a narrative called Mathematics Background.

- Let's Be Rational in Grade 6: Page 3, "These situations require addition, subtraction, multiplication division of fractions, including mixed numbers. You will decide which operation makes sense in each situation: "You may already know shortcuts for working with fractions..."
- *Comparing and Scaling* in Grade 7: Problem 2.3 refers to work in unit rates in prior Grade 6 unit *Comparing Bits and Pieces*.
- Accentuate the Negative in Grade 7: Problem 4.2, refers to work with the distributive property in Grade 6.
- Accentuate the Negative in Grade 7: Page 3, "Most of the numbers you have worked with in math class have been greater than or equal to zero. However..." "You will also learn more about the properties of operations on numbers." Page 4, "You will extend your knowledge of negative numbers." Page 8, "You have worked with whole numbers, fractions, decimals in earlier units." Page 58, "You have already examined patterns in ..."
- *Thinking With Mathematical Models* in Grade 8: Page 3, "In earlier Connected Mathematics units, you explored relationships between two variables. You learned how to find linear relationships from tables and graphs and then write their equations. Using the equations, you solved problems."
- *Looking for Pythagoras* in Grade 8: Problem 4.2, refers to earlier work with rational numbers.
- *It's in the System* in Grade 8: Problem 2.1 refers to the previous Grade 8 unit *Say it With Symbols* and the Grade 7 unit Moving Straight Ahead that work with solving equations.
- *Thinking with Mathematical Models* in Grade 8: Problem 1.1 refers to several previous units in Connected Mathematics 3 using tables, graphs and equations to explore variables.
- *Butterflies, Pinwheels Wallpaper* in Grade 8: Problem 4.1 refers to previous work in the Grade 7 unit *Stretching and Shrinking* with enlargements and reductions.

Response

While some of the statements are fairly positive, the reviewers still fail to recognize the focus of the "major work" in all of the seven units listed for Grade 7. See earlier discussions about focus of "major work." We also challenge the following claim:

There is limited support for differentiation:

We provide clarification that differentiation is a strong component of Connected Mathematics and that its curriculum and pedagogical model is designed to enhance the learning experience of *all* students.

For over 25 years of field-testing, revision, and evaluation, differentiation is and has been an important concern of the authors of Connected Mathematics. We take seriously our overarching goal:

The overarching goal of CMP is to help students and teachers develop mathematical knowledge, understanding, and skill along with an awareness of and appreciation for the rich connections among mathematical strands and between mathematics and other disciplines. The CMP curriculum development has been guided by our single mathematical standard:

All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics, including the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.

Connected Mathematics is a problem-centered curriculum. The problems were carefully selected to provide multiple access points and time for students to acquire the understanding of important mathematical understandings embedded in the Problems. The development of a concept moves gradually from informal to formal mathematics through a sequence of problems within Investigations in a Unit and continues when students revisit the concept to build understandings of related ideas in other units throughout Grades 6, 7, an 8.

The reviewers note that the Explore is a significant time to attend to differentiation, which we agree with. But we also contend that differentiation starts with the teacher's planning and is implemented throughout the lesson, ending with the teacher's reflection. The Launch, Explore, Summarize phases of the lesson were developed to provide guidance on differentiation at each stage. For example, the first part of a Launch connects the challenge of the problem to prior knowledge. This is an important step in scaffolding and hence, differentiation. Throughout the Launch the teacher is asking

questions and taking note of strengths and weaknesses, which are used to guide the rest of the lesson during the Explore and Summarize. Each phase, Launch, Explore and Summarize, contain numerous questions, possible student responses and suggestions for follow-up to each response. *Going Further* and *Check for Understanding* are also important features that occur in the Explore or Summarize, which the teacher can use to attend to the individual needs of students. The homework sections, Applications, Connections, and Extensions , are also designed to provide for individual needs.

Further, the authors have provided numerous examples of possible student responses, stumbling blocks, and misconceptions with suggestions that teachers can use in these situations throughout the Teacher Guide, the Math Background, Unit, Investigation and Problem Overviews, Mathematical Reflections, Looking Back, Labsheets, Assessments, Self Assessments and in the Guide to CMP3: Understanding, Implementing, and Teaching.

Comment

It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. CCSSM, P. 4

Finally, we are curious as to why the comments on Differentiation occur with Gateway I. According to the EdReports.org *Quality Instructional Materials Tool: Grades K–8 Mathematics* differentiation, is not mentioned until Gateway III. p. 19

Rating Sheet 4: Differentiated Instruction

For 'Differentiated Instruction' to attain a score of 'Meets Expectations,' material must earn at least 10 points.

CRITERION	INDICATORS	RATING	EVIDENCE
Differentiated instruction: Materials support teachers in differentiating instruction for diverse learners within and across grades.	3r. Materials provide strategies to help teachers sequence or scaffold lessons so that the content is accessible to all learners.	0 1 2	
Earned: of 12 points	3s. Materials provide teachers with strategies for meeting the needs of a range of learners.	0 1 2	
	3t. Materials embed tasks with multiple entry- points that can be solved using a variety of solution strategies or representations.	0 1 2	
Meets expectations (10-12 points)	3u. Materials suggest support, accommodations, and modifications for English Language Learners and other special populations that will support the support of the suppor	0 1 2	
	mathematics (e.g., modifying vocabulary words within word problems).		
Partially meets expectations (8-9 points)	3v. Materials provide opportunities for advanced students to investigate mathematics content at greater depth.	0 1 2	
	3w. Materials provide a balanced portrayal of various demographic and personal characteristics.	0 1 2	
Does not meet expectations	3x. Materials provide opportunities for teachers to use a variety of grouping strategies.		
(<8 points)	3y. Materials encourage teachers to draw upon home language and culture to facilitate learning.		

Coherence: Indicator IV

Materials foster coherence through connections at a single grade, where appropriate and required by the Standards i. Materials include learning objectives that are visibly shaped by CCSSM cluster headings. ii. Materials include problems and activities that serve to connect two or more clusters in a domain, or two or more domains in a grade, in cases where these connections are natural and important.

Claim

The materials reviewed for Grade 6 meet the expectation for coherence. Each investigation within each unit lists the CCSSM that are taught. The mathematical highlights for each unit stress the clusters from CCSSM. All investigations in the student books contain the standards included in that lesson. Every investigation includes activities that connect two or more clusters in a domain, or two or more domains.

- An example of this is in *Prime Time*, where the mathematical highlights state "...Develop strategies for finding factors and multiples, least common multiples and greatest common factors; and use exponential notation to write repeated factors." This follows one strand, Expressions and Equations, which says to use properties of operations to generate equivalent expressions. (6.EE.A.3-4, 7.EE.A, 8.EE.A.1)
- *Prime Time*: Equivalent Expressions: Understand why two expressions are equivalent.
- *Comparing Bits and Pieces*: Equivalence: Understand equivalence of fractions and ratios. Use equivalence to solve problems.
- Another example is in *Comparing Bits and Pieces: Ratios, Rational Numbers Equivalence*. This unit refers to relationships that exist between ratios, ratio reasoning proportions and using them in real-world problems while working on

fraction and decimal operations (6.RP.A).

There are many links between major clusters in this curriculum.

- In *Variables and Patterns* students solve unit rate problems including those involving unit pricing and constant speed (6.RP.A) and use variables to represent two quantities in a real-world problem that change in relationship to one another (6.EE.C).
- In *Let's Be Rational* students apply and extend previous understanding of multiplication and division to divide fractions by fractions (6.NS.A), and identify parts of an expression using mathematical terms (6.EE.A).
- In *Decimal Ops* students interpret and compute quotients of fractions to solve word problems involving division of fractions by fractions (6.NS.A) and solve real world and mathematical problems by writing and solving equations in the form p+x=q and px=q (6.EE.B).
- Comparing Bits and Pieces connects 6.NS.C with 6.RP.A.
- *Decimal Ops* connects multiplying and dividing decimals with 6.RP.A, 6.EE.A and 6.EE.B.
- *Data About Us* uses data to compute with numbers, measure and graph data connects 6.SP.A, 6.SP.B and 6.RP.A.
- Variables and Patterns analyzes relationships among variables 6.EE.A, 6.EE.B, 6.EE.C, and 6.RP.A.
- There is no unit or investigation that only focuses on one aspect of the CCSSM. Connections are evident in all grade levels and in all units. This is a very strong aspect of Connected Mathematics 3.

Response

We agree with the claims made in the fourth indicator for coherence in the following ways. The materials reviewed for Grade 6 meet the expectations for coherence. Each investigation within each unit lists the CCSSM that are taught. The mathematical highlights for each unit stress the clusters from CCSSM. All investigations in the student books contain the standards included in that lesson. Every investigation includes activities that connect two or more clusters in a domain, or two or more domains. There exists many links between major clusters in the curriculum. There is no unit or investigation that only focuses on one aspect of the CCSSM. Connections are evident in all grade levels and in all units.