Extended Response for Grade 7

EdReports Evaluation of *Connected Mathematics* Alignment with Common Core State Standards for Mathematics

The EdReports evaluation of instructional materials for Grades 6 – 8 developed by the Connected Mathematics Project and published by Pearson concludes that those materials, "do not meet the requirements for alignment to the Common Core State Standards." We challenge many specific critical statements about alignment of the instructional materials and the Common Core State Standards for Mathematics. In general, the three major themes and our clarifying responses are summarized as follows:

- *Not enough time attending to the major clusters. Only two of the eight units, Accentuate the Negative and Comparing and Scaling, were consider attending to major work.* The units, Stretching and Shrinking, Moving Straight Ahead, Shapes and Designs, Filling and Wrapping, What Do You Expect?, and Samples and Populations, need to be re-examined and identified as attending to major work.

- *Some unit test items target content above grade level.* These items need to be re-examined and identified as meeting the Grade 7 CCSSM.

- *Limited support for differentiation.* Differentiation is a strong component of Connected Mathematics and its curriculum and pedagogical modelare designed to enhance the learning experience of all students. This claim needs to be re-examined as fully supporting differentiation.

To provide clarification for the specific critical statements, the following section is organized by the *Focus* and *Coherence* indicators for Gateway One. This is the only Gateway of three reviewed for Connected Mathematics 3. The *Indicators* and *Claims* provided below are direct quotes from the Report released to Connected Mathematics on February 18, 2015.

We believe the proven record of Connected Mathematics and the concerns elaborated for each grade level review raise serious doubts about validity of the 'does not meet expectations' judgment in the EdReports evaluation of Connected Mathematics 3 and that reconsideration of the evaluation is in order.

Gateway I Major Work (Focus) and Coherence

FOCUS

Focus: Indicator I

The instructional materials assesses the grade-level content indicated on the reference sheet and, if applicable, content from earlier grades. Content from future grades may be introduced but students should not be held accountable for future content.

Claim

The majority of the concepts reviewed for Grade 7 do meet the expectations assessing material at the Grade 7 level.

The instructional materials reviewed for Grade 7 do not meet the expectations for assessing material at the Grade 7 level. The materials assess topics that are in future grades.

- On the *Shapes and Designs* unit test, questions 1 and 6 are on angles and transversals which are the Grade 8 standard 8.G.A.5.
- On the *Stretching and Shrinking* unit test, questions 5, 6 and 7 are the Grade 8 standard 8.G.A.4 on similarity.
- On the *Moving Straight Ahead* unit test it uses the terminology of slope and function, which are Grade 8 CCSSM terms. Lines in Grade 7 should go through the origin and there are many items with y-intercepts that are not at 0. Question 6 is on systems, which is the Grade 8 standard 8.EE.C.8.
- On the *Filling and Wrapping* unit test, questions 8 and 9 are on volume and surface area of cones and cylinders, which is the Grade 8 standard 8.G.C.9.

Response

We provide clarification that the assessment items do meet the 7th grade CCSSM. Assessment items need to be interpreted in conjunction with the unit, which we provide in the following paragraphs.

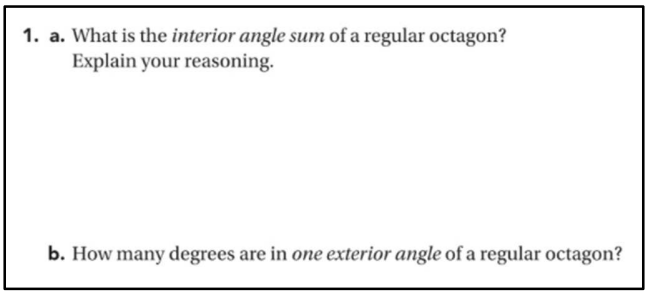
Unit Test for Shapes and Designs

The goal of this unit is to study the properties of shapes, which are determined by their angle measure and side length. To accomplish this goal, it is necessary to study each of these properties and how they affect shape. Since angle is last mentioned in Grade 4 in the CCSSM, angle is revisited in the context of studying properties of shape. In so doing, understanding of angle is deepened as it is used to investigate properties of shape. Students find a relationship between angle sums of a polygon and its shape. This directly aligns with standard, 7.G.A, *Draw. Construct, and describe geometrical figures and describe the relationships between them* and in particular standard, 7.G.A.2, *Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides,*

noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Angles and Transversals-Question 1

The concepts tested in Question 1 are necessary to develop the understanding of the standard, 7.G.A. Earlier in the unit students found the angle sum of a triangle and used these findings to investigate angles sums of polygons. Knowing the angle sum of a triangle and the relationship among the sides of a triangle are necessary to determine whether given conditions of a triangle determine a unique triangle, more than one triangle, or no triangle. An important part of understanding a concept is the ability to apply the basic understandings to a new situation, in this case, polygons. This question assesses whether students understand the concept, not just remember a particular fact.



1. a. What is the *interior angle sum* of a regular octagon? Explain your reasoning.

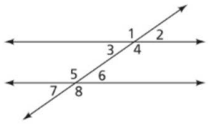
b. How many degrees are in *one exterior angle* of a regular octagon?

Angles and transversals-Question 6

This question addresses standard, 7.G.B, *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume***,** *and in particular, standard 7.G.B.5***,** *Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure*. The development of transversals is revisited in a unit on transformations in the 8th grade of Connected Mathematics.

Revisiting and extending the work of angles is also needed to support the development of scale drawings or similarity in *Stretching and Shrinking*. (7.G.A and 7.RP.A)

6. Use the figure below which shows parallel lines and a transversal. The measure of $\angle 2$ is 45°. Find the measures of angles 1, 3, and 6. Explain how you found each measure.



a. measure of $\angle 1$ = _____

b. measure of $\angle 3$ = _____

c. measure of $\angle 6$ = _____

Unit Test for Stretching and Shrinking

We provide clarification that Questions, 5, 6, and 7 directly assess standard, 7.G.A, *Draw, construct, and describe geometrical figures and describe the relationships between them* and in particular, standard, 7.G.A.1*, Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale*. We choose to use the mathematical term, similar figures for scale drawings as well as scale drawings. So any mention of similarity can be read as scale drawing and vice versa. This explicitly explained in Investigation 1.

Similarity-Question 5

This question asks students to create similar figures (scale drawings) using coordinates, which directly assesses standard, G.A.

5. The following rules for drawing backpacks for the Wumps are given below:

Backpack 1: (x, y)

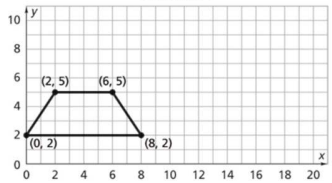
Backpack 2: $(2x, 2y)$

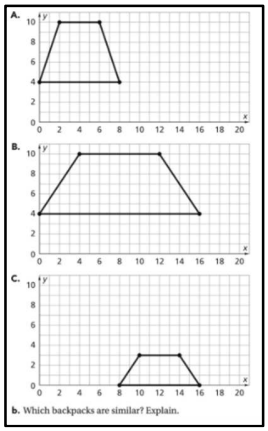
Backpack 3: $(x + 8, y - 2)$

Backpack 4: $(x, 2y)$

a. Backpack 1 is plotted on the grid below. Match the remaining Backpacks 2–4 with graphs A–C on the next page. Explain your reasoning.

Backpack 1

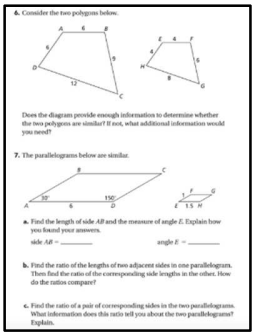




b. Which backpacks are similar? Explain.

Similarity-Questions 6 and 7

Questions 6 and 7 also assess standard, G.A. It asks if the two figures are similar (scale drawings), what is the length of one side given the length of a corresponding side. In addition, this question also assesses, standard 7.RP.A, *Analyze proportional relationships and use them to solve real-word and mathematical problems.*


6. Consider the two polygons below.

Does the diagram provide enough information to determine whether the two polygons are similar? If not, what additional information would you need?

7. The parallelograms below are similar.

a. Find the length of side AB and the measure of angle E. Explain how you found your answers.

side AB = _____ angle E = _____

b. Find the ratio of the lengths of two adjacent sides in one parallelogram. Then find the ratio of the corresponding side lengths in the other. How do the ratios compare?

c. Find the ratio of a pair of corresponding sides in the two parallelograms. What information does this ratio tell you about the two parallelograms? Explain.

Comment:

The development of similarity (scale drawings) in the *Stretching and Shrinking* unit directly attends to the geometry content as prescribed by CCSS grade 7. It also provides a strong base to continue a more sophisticated development of similarity using transformations in 8th grade. Equally important is that this unit provides a deep development of proportional reasoning a major focus of 7th Grade CCSSM.

Using similar geometric shapes to introduce proportional reasoning provides an alternative visual entry point to this idea before the numerical proportional contexts. This visual representation provides an entry point for some students that may not be available in a numeric context. It also enriches understanding of proportionality for students who are inclined to use numeric representations.

Unit Test for Moving Straight Ahead

Comment:

The development of algebra and functions in Connected Mathematics is treated as one strand, which is developed across grades 6, 7 and 8. The development of algebra and functions in Connected Mathematics provides a strong coherent development of algebra, relationships and functions, and proportional reasoning. The following is a brief excerpt from *A Guide to CMP3: Understanding, Implementing, and Teaching:*

> *CMP is a problem-centered curriculum in which quantities or variables naturally arise in the context of a problem. In this curriculum, it makes sense to think about how variables are related, how they can be represented, and the information we can get from the way they are represented. CMP's approach intertwines the CCSSM algebra and functions strands into one coherent algebra and functions, strand across three years.*

The CMP algebra and functions Units challenge students, from the very first Investigation in sixth grade to the very last Investigation in eighth grade, to represent algebraic relationships in words, tables, graphs, and symbols. Writing and reasoning with symbolic expressions, an important goal of the CCSSM, is incorporated throughout the program. Equations are first understood as symbolic rules that relate independent and dependent variables. One-variable equations are, therefore, specific instances, or snapshots, of function relationships, in which one of the variables is a known quantity. A natural outcome of combining functions and algebra through problem situations is that solving one-variable equations is first done using graphs, tables, or numeric reasoning and then by symbolic methods. Manipulating symbols to write equivalent expressions, another important CCSSM goal, reveals new insights into the relationship that represents a problem situation, and opens up other solution strategies.

As students continue to develop their understanding of linear and non linear, they also develop deeper understandings of expressions and equations that are required to represent more complicated situations and vice versa more complicated situations push the understanding of linear relationships. Because of CMP's emphasis on reasoning about and with relationships during grades 6–8, students are well prepared for the Common Core Functions standards. P.160

See also: https://connectedmath.msu.edu/the-math/development/algebra-and-functions/

Lines that do not go through the origin

To understand proportional reasoning situations that can be represented as a line that goes through the origin, it is important to consider non-proportional situations or lines that do not go through the origin. The CCSSM document claims:

Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called slope. They distinguish proportional relationships from other relationships. P. 46

Slope

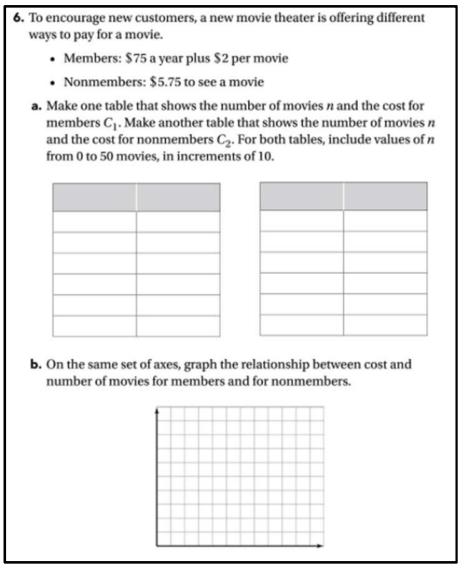
The use of slope is mentioned in the preceding comment in CCSSM for 7th grade and slope is a natural extension of grade standard, 6.EE. C, *Represent and analyze quantitative relationships between dependent and independent variables* and in particular, 6.EE.C.9, *Use variables to represent two quantities in a real-world problem that*

change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

In addition, the use of slope, a mathematical term for ratio of vertical distance to horizontal distance between two points on a line, continues and strengthens understanding of rate of change between two variables in a linear relationship. It supports standard, 7.EE.A and it provides a foundation for further study of linear functions in grade 8.

Systems of Equations, Question 6

Question 6 on the unit test could be *interpreted* as solving a system of equations, but at this point students treat it as solving one equation. They find an expression for the cost of two movie plans and then set the two expressions equal which leads to an equation of the form, 2n + 75 = 5.75n, where n is the number of people attending the movie. Writing expressions and solving this equation directly assesses standard, 7.EE.A, *Use properties of operations to generate equivalent expressions* and standard, 7.EE.B, *Solve real-life and mathematical problems using numerical and algebraic expressions and equations.*

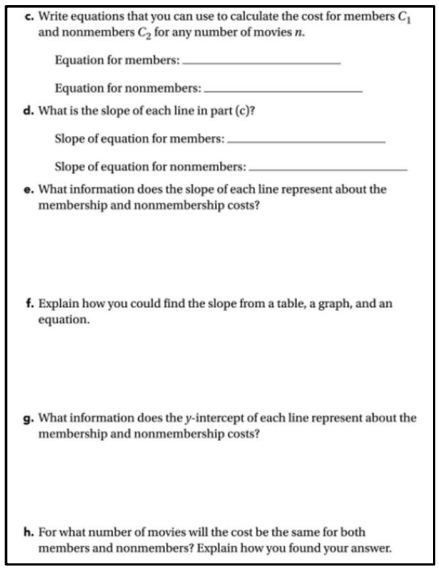


6. To encourage new customers, a new movie theater is offering different ways to pay for a movie.

- Members: $75 a year plus $2 per movie
- Nonmembers: $5.75 to see a movie

a. Make one table that shows the number of movies n and the cost for members C_1. Make another table that shows the number of movies n and the cost for nonmembers C_2. For both tables, include values of n from 0 to 50 movies, in increments of 10.

b. On the same set of axes, graph the relationship between cost and number of movies for members and for nonmembers.



c. Write equations that you can use to calculate the cost for members C_1 and nonmembers C_2 for any number of movies n.

Equation for members: _____

Equation for nonmembers: _____

d. What is the slope of each line in part (c)?

Slope of equation for members: _____

Slope of equation for nonmembers: _____

e. What information does the slope of each line represent about the membership and nonmembership costs?

f. Explain how you could find the slope from a table, a graph, and an equation.

g. What information does the y-intercept of each line represent about the membership and nonmembership costs?

h. For what number of movies will the cost be the same for both members and nonmembers? Explain how you found your answer.

Functions

Finally, we could not find any mention of the word function on the assessments or student book for *Moving Straight Ahead*.

In summary, the study of linear relationships and their representations in *Moving Straight Ahead* provide a context for developing linear expressions and equations (7.EE.A, 7.EE.B) and proportional reasoning (7.RP.A) and thus are appropriate for grade 7.

Unit Test for Filling and Wrapping

This unit addresses standard, 7.G. B, *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume*. Specifically, this standard calls for volume of right prisms. However, to strengthen the development of volume of a prism, cylinders, spheres and cones are used as non –examples. Studying the relationship among these solids strengthens the understanding of each solid. In this unit, the formulas for the volumes are developed using an experimental approach. i.e. filling one solid with rice and pouring it into another. In 8^{th} grade, these concepts are approached at a more sophisticated level using algebraic expressions to express the relationship among the volumes with algebraic expressions and then proving that the expressions are equivalent. Furthermore, the inclusion of cylinders provides a natural connection to the area and circumference of circles, which is called for in standard, 7.G.A, *Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.* The connection between the area of a circle and the volume of a cylinder strengthens the understanding of circles and cylinders and reinforces understanding of polygons and prisms. That is, the volume of a cylinder is the area of the base times the height of the cylinder. The area of a prism is area of the base x the height of a prism.

Comment: Support for including topics not mentioned for a grade are provided by the following quotes from *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*:

> *Giving all students extensive work with grade-level problems. Differentiation is sometimes necessary, but materials often manage unfinished learning from earlier grades inside grade-level work, rather than setting aside grade-level work to reteach earlier content. Unfinished learning from earlier grades is normal and prevalent; it should not be ignored nor used as an excuse for cancelling grade level work and retreating to below-grade work. (For example, the development of fluency with division using the standard algorithm in grade 6 is the occasion to surface and deal with unfinished learning about place value; this is more productive than setting aside division and backing up.) Likewise, students who are "ready for more" can be provided with problems that take grade-level work in deeper directions, not just exposed to later grades' topics. p.12*

In summary, the inclusion of volume of cylinders, cones, and spheres, provide support for volume of prisms and a natural connection to circles and is appropriate for grade 7.

Focus: Indicator II

Instructional material spends the majority of class time on the major cluster of each grade.

Claim

- The instructional materials reviewed for Grade 7 do not meet the expectations for spending the majority of class time on the major cluster for Grade 7.
- Grade 7 has less than 50% of the work on the major clusters of 7.RP.A, 7.NS.A, 7.EE.A and 7.EE.B.
- Out of the 8 units for Grade 7, the only two units focused on this work were Accentuate the Negative and Comparing and Scaling.
- The unit Moving Straight Ahead has the majority of its work on linear relationships that is Grade 8 CCSSM.
- The unit What Do You Expect is work on the additional cluster of 7.SP.C and this unit is more than twice as long as the unit Comparing and Scaling which is a unit on the major work of 7.RP.A.
- The unit Filling and Wrapping has work with volume and surface area of cylinders and cones, which are Grade 8 CCSSM.

Response

We provide clarification that the units, *Accentuate the Negative*, *Stretching and Shrinking*, *Comparing and Scaling*, and *Moving Straight Ahead*, are part of the "major work" as they address the major clusters of 7. NS.A, RP.A, and EE. A and EE.B. We also provide evidence that the units, *Filling and Wrapping* and *What Do You Expect?* designated as "supporting cluster concepts" are "used to enhance the major work" and thus, do attend to the "major work" or "major cluster" supporting 7.RP.A and 7.EE.A and 7.EE.B.

Comment
As stated earlier in the 6th grade cluster, to develop conceptual and procedural fluency, concepts need to be addressed in several different contexts after their initial

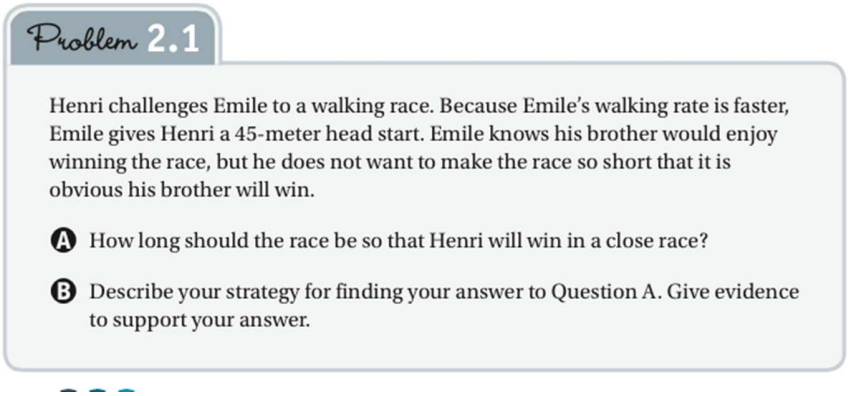
development. This provides students an opportunity to recognize a concept in a various contexts, apply it, and hence strengthen their understanding of the concept.

Moving Straight Ahead

As discussed in the preceding Indicator 2 for Focus, this unit is compliant with

- 7.EE.A, *Use properties of operations to generate equivalent expressions*, and
- 7.EE. B, *Solve real-life and mathematical problems using numerical and algebraic expressions and equations*, and
- 7.RP. A, *Analyze proportional relationships and use them to solve real-world and mathematical problems***.**

Since quantitative relationships were introduced in standard 6.EE.C, *Represent and analyze quantitative relationships between dependent and independent variables*, this unit revisits quantitative relationships in 7th grade as a context to develop understanding of expressions and equations. The following problems are a sample from the unit that demonstrates how expressions and equations are generated. In Problem 2.1 students use a variety of strategies to solve the problem. In Problem 2.2 they write expressions and equations to solve the problem. They use graphs and tables to provide insights into the meaning that the equations and expressions represent.



Problem 2.1

Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.

A How long should the race be so that Henri will win in a close race?

B Describe your strategy for finding your answer to Question A. Give evidence to support your answer.

Problem 2.2

A For each brother in Problem 2.1:

1. Make a table showing the distance from the starting line at several different times during the first 40 seconds. How can the table be used to find the length of the race?
2. Graph the time and the distance from the starting line on the same set of axes. How can the graph be used to find the length of the race?
3. Write an equation representing the relationship between time and distance. Explain what information each variable and number represents.
4. How does the walking rate of each brother show up in the graph, the table, and the equation?

B

1. How far does Emile walk in 20 seconds?
2. After 20 seconds, how far apart are the brothers? How is this distance represented in the table and on the graph?
3. Is the point (26, 70) on either graph?
4. When will Emile overtake Henri? Explain.

C How can you determine which of two lines will be steeper from

1. a table of the data?
2. an equation?

D

1. At what points do Emile's and Henri's graphs cross the *y*-axis?
2. What information do these points represent in terms of the race?
3. How can these points be found in a table? In an equation?

At this point in the unit, students may use a graph, table or numeric reasoning to find the distance that the race ends in a tie. In Investigation 3, properties of equality are used to solve equations symbolically. Each Investigation and Problem in *Moving Straight Ahead* involves writing expressions and equations and solving equations.

In Problem 2.2, the constant of proportionality is foreshadowed as students explore the steepness of a line. In Problem 4.2 students directly address 7.EE.A.2.d, *Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to (0,0) and (1,r) where r is the unit rate.* This problem also reinforces rates and ratios.

Problem 4.2 *continued*

B The points (3, 5) and (–2, 10) lie on a line.

1. What is the slope of the line?
2. Find two more points that lie on this line. Explain your method.
3. Eun Mi observed that any two points on a line can be used to find the slope. How is Eun Mi's observation related to the idea of "linearity?"

C

1. John noticed that for lines represented by equations of the form $y = mx$, the points (0, 0) and (1, m) are always on the line. Is he correct? Explain.
2. What is the slope of a horizontal line? A vertical line? Explain your reasoning.

D

1. Compare your methods for finding the slope of a line from a graph, a table, and an equation.
2. In previous Investigations, you learned that linear relationships have a constant rate of change. As the independent variable changes by a constant amount, the dependent variable also changes by a constant amount. How is the constant rate of change of a linear relationship related to the slope of the line that represents that relationship?

The development of the depth of understanding of linear relationships continues in 8th grade as linear functions, which are contrasted with non-linear functions.

What Do You Expect?

This unit is long as it is the *only* unit in grades 6, 7 or 8 that develops understanding of probability. Additionally, it strongly supports standard, 7.RP.A. Probability is an important context to study proportionality. On p. 12 of the student edition of *What Do You Expect?* probability is defined as follows:

1.3 One More Try
Finding Experimental Probabilities

In the last two Problems, you conducted experiments to find the chances of particular results. You represented those chances as fractions or percents. The mathematical word for chance is **probability.** A probability that you find by conducting an experiment and collecting data is called an **experimental probability.**

Suppose you toss a paper cup 50 times, and it lands on its side 31 times. Each toss of the cup is a **trial.** In this experiment, there are 50 trials. **Favorable outcomes** are the trials in which a desired result occurs.

In this case, a favorable result, *landed on side,* occurred 31 times. To find the experimental probability, use the ratio below.

$$\frac{\text{number of favorable outcomes}}{\text{total number of trials}}$$

You can write "the probability of the cup landing on its side" as P(side). The equation below gives the results of the experiment just described.

$$P(\text{side}) = \frac{\text{number of times cup landed on its side}}{\text{number of times cup was tossed}} = \frac{31}{50}$$

The ratio of number of desired results to the total number of trials is also called **relative frequency.**

The following paragraph in the Unit Overview of the Teacher Guide highlights the opportunities that the unit provides to continue the development of proportional reasoning and for differentiation.

> *It is important to note that the Problems in this Unit can usually be taught quickly. For example, Problems 1.4, 2.3, 3.1, and 4.1 most likely will not take a class period to complete. This allows teachers many chances to provide students with additional in-class help or opportunities to move quickly through problems so more time will be available for more challenging Problems. Students will most likely have time to work on ACE Exercises during class time, which enables them to work together or to ask questions of their teacher before leaving the classroom. Teachers may also find that they have more time for interventions with students who require additional guidance with proportional reasoning or other concepts. P. 2*

Filling and Wrapping

This unit supports standards, 7.EE. A and 7.EE.B. Students develop formulas for area and circumference of circles and surface area and volume for prisms. It further strengthens 7.RP.A. as students explore questions in Problem 1.4, p. 13-14. *Filling and Wrapping*.

Recipe for a 1-2-3 Compost Box

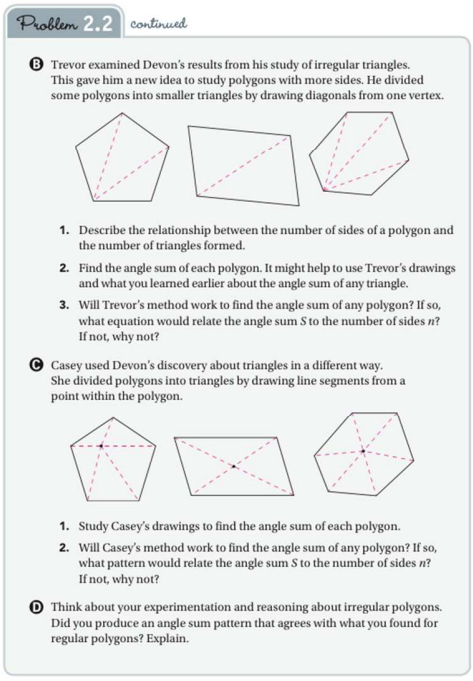
- Start with an open rectangular wood box that is 1 foot high, 2 feet wide, and 3 feet long. This is a 1-2-3 box.
- Mix 10 pounds of shredded newspaper with 15 quarts of water. Put the mixture in the 1-2-3 box.
- Add a few handfuls of soil.
- Add about 1,000 redworms (about 1 pound).

Every day, mix collected kitchen waste with the soil in the box. The worms will do the rest of the work, turning the waste into new soil. A 1-2-3 box will decompose about 0.5 pound of garbage a day.

- If you want to double the amount of garbage you can compost, what would be the dimensions of the new box?
- How many worms would you need?

Shapes and Designs

This unit develops understanding of standards, 7.EE. A and 7.EE.B. Students generate expressions and equations to represent the angle sum of a polygon. Furthermore, they use these equations to find missing information about a polygon.



Problem 2.2 continued

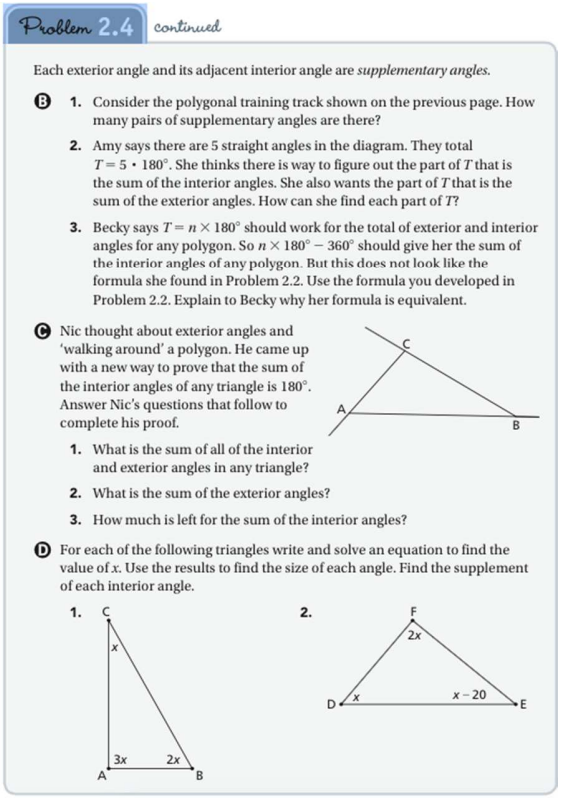
B Trevor examined Devon's results from his study of irregular triangles. This gave him a new idea to study polygons with more sides. He divided some polygons into smaller triangles by drawing diagonals from one vertex.

1. Describe the relationship between the number of sides of a polygon and the number of triangles formed.
2. Find the angle sum of each polygon. It might help to use Trevor's drawings and what you learned earlier about the angle sum of any triangle.
3. Will Trevor's method work to find the angle sum of any polygon? If so, what equation would relate the angle sum S to the number of sides n? If not, why not?

C Casey used Devon's discovery about triangles in a different way. She divided polygons into triangles by drawing line segments from a point within the polygon.

1. Study Casey's drawings to find the angle sum of each polygon.
2. Will Casey's method work to find the angle sum of any polygon? If so, what pattern would relate the angle sum S to the number of sides n? If not, why not?

D Think about your experimentation and reasoning about irregular polygons. Did you produce an angle sum pattern that agrees with what you found for regular polygons? Explain.



Problem 2.4 continued

Each exterior angle and its adjacent interior angle are *supplementary angles*.

B

1. Consider the polygonal training track shown on the previous page. How many pairs of supplementary angles are there?
2. Amy says there are 5 straight angles in the diagram. They total $T = 5 \cdot 180°$. She thinks there is way to figure out the part of T that is the sum of the interior angles. She also wants the part of T that is the sum of the exterior angles. How can she find each part of T?
3. Becky says $T = n \times 180°$ should work for the total of exterior and interior angles for any polygon. So $n \times 180° - 360°$ should give her the sum of the interior angles of any polygon. But this does not look like the formula she found in Problem 2.2. Use the formula you developed in Problem 2.2. Explain to Becky why her formula is equivalent.

C Nic thought about exterior angles and 'walking around' a polygon. He came up with a new way to prove that the sum of the interior angles of any triangle is 180°. Answer Nic's questions that follow to complete his proof.

1. What is the sum of all of the interior and exterior angles in any triangle?
2. What is the sum of the exterior angles?
3. How much is left for the sum of the interior angles?

D For each of the following triangles write and solve an equation to find the value of x. Use the results to find the size of each angle. Find the supplement of each interior angle.

1. 2.

Samples and Populations

This unit uses number and proportionality to explore the important topics of samples and populations. It also supports standards, 7.NS.A and 7.RP.A.

Conclusion:

"Major Work" can and should arise in "Supporting Clusters." Students' understandings are developed by building on and connecting to their prior knowledge. In the process, students deepen their understandings of the concepts and procedures and gain insight into new concepts and procedures. Thus, the focus on the major work in Grade 7 is substantially higher than the recorded percent. Including these additional units, all of Grade 7 materials focus on major work.

COHERENCE

Coherence: Indicator I

Supporting content enhances focus and coherence simultaneously by engaging students in the major work of the grade.

Claim

The instructional materials reviewed for Grade 7 partially meet the expectations for the supporting content enhancing the major work. There are areas where the materials have strong connections and areas that could be stronger.

- In *What Do You Expect*, *Samples and Populations* and *Stretching and Shrinking* there are problems that use supporting work to enhance 7.RP.A.
- In all the units, there is evidence of the supporting work enhancing 7.NS.A. Much of the work in Grade 7 is geometry-based and tied to similarity which is a Grade 8 standard

Response

We provide clarification that *Stretching and Shrinking* using a geometric context, scale drawings (similarity), and *What Do You Expect*? using the context of probability, develop proportional reasoning

Comment
Geometry is a strong context in the 7th grade in Connected Mathematics. It provides opportunities for students to explore critical content through the use of visual representations. Visualization provides some learners access to mathematics that may not be totally accessible through number or algebraic situations. It allows students to reason quantitatively not just numerically or symbolically. See earlier arguments in Indicator 1 for Focus that provides evidence that *Stretching and Shrinking* directly supports the development of standard, 7.RP.A.

See Indicator II for Focus that provides evidence that *What Do You Expect*? directly supports the major work of proportional reasoning. (7.RP.A)

Coherence: Indicator II

The amount of content designated for one grade level is viable for one school year in order to foster coherence between grades.

Claim

The instructional materials reviewed for Grade 7 partially meet the expectations for being able to be taught in one school year.

- The Grade 7 materials could be completed within the timeline of 170-190 days. This includes all lessons, mathematical reflections, *Looking Back* and *Looking Ahead* and all assessments.
- While overall it is viable for a school year, the amount of time on the major work for Grade 7 is less than 65% of the year (as reflected in the rating for 1b), which means that teachers will need to find supplemental materials to cover the standards

Response

As explained in the clarification provided for the Focus, Indicator I and II and Coherence, Indicator I, the work in Grade 7 represents the "major work" and hence is substantially greater than 65%.

Coherence: Indicator III

Materials are consistent with the progressions in the Standards

i. Materials develop according to the grade-by-grade progressions in the Standards. If there is content from prior or future grades, that content is clearly identified and related to grade-level work
ii. Materials give all students extensive work with grade-level problems
iii. Materials relate grade level concepts explicitly to prior knowledge from earlier grades.

Claim

The materials reviewed for the Grade 7 materials partially meet the expectations for being consistent with the progressions in the standards. The connections between standards to build understanding are strong. There are some off grade level topics that could be identified to help teachers and students know that these are topics that are beyond the CCSSM necessary for that grade.

All three grade levels have major work on equations, EE.A and EE.B:

- Grade 6: Reason about and solve one-variable equations and inequalities is found in several units (e.g., Let's Be Rational, Variables and Patterns) using informal methods of solving.
- Grade 7: Solve real-life and mathematical problems using numerical and algebraic expressions and equations is primarily in Moving Straight Ahead where they start using symbolic equations and properties of equality.
- Grade 8: Analyze and solve linear equations and pairs of simultaneous linear equations is found in It's in the System where various methods of solving systems are explored.

All three grade levels have major work on ratio and proportional reasoning, 6.RP to 7.RP:

- Grade 6: Comparing Bits and Pieces begins work with ratios/rates and proportions then continues the major work of Grade 6 ratio and proportion into Variables and Patterns.
- Grade 7: Stretching and Shrinking works with ratios using scale factors and Comparing and Scaling continues the work by solving proportions using many strategies learned from Grade 6 and Grade 7.
- Grade 8: Butterflies, Pinwheels and Wallpaper use the concepts of proportional reasoning in transformational geometry work.

All three grades have major work on the number system, 6.NS.A, 6.NS.B, 6.NS.C to 7.NS.A to 8.NS.A:

- In Prime Time, work with 6.NS.B.4 begins with finding the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- This leads to finding the LCM in order to find common denominators for fractions in Comparing Bits and Pieces, Let's be Rational and Decimal Ops in Grade 6 and extends

to ratios in Comparing and Scaling in Grade 7. This continues into Accentuate the Negative in Grade 7 with performing arithmetic operations with integers and rational numbers, which is 7.NS.A.

- Comparing Bits and Pieces begins developing the ideas of positive and negative numbers on a number line and absolute value which is 6.NS.C. This leads to 7.NS.A in Accentuate the Negative with operations on rational numbers. This also leads into 8.NS.A on approximating rational numbers (although not the major work of Grade 8).

- Let's Be Rational begins 6.NS.A with students dividing fractions. This continues in Grade 7 with 7.NS.A in Accentuate the Negative.

There is limited support for differentiation of instruction.

- There is guidance for the teacher in the book titled A Guide to Connected Mathematics 3 that discusses differentiation. This gives best practices from research to be used while working on the problem with all students.

- Differentiation is embedded within the instructional model for Connected Mathematics 3 that all students launch the problem and summarize the same way and that the differentiation comes during the explore phase of the problem.

- There were specific strategies and guidance for English language learners.

- To help make differentiation more explicit, strategies need to be discussed in the teacher's unit planning pages and it needs to be tied into the specific problems so the teachers have guidance.

- The guide has general best practices but what to use with specific parts of a unit would make it more accessible for teachers and students

There are many places where the materials relate grade level concepts to explicitly to prior knowledge from earlier grades. These can be found in the student editions in the problems and in the teacher editions in charts and in a narrative called Mathematics Background.

- Let's Be Rational in Grade 6: Page 3,"These situations require addition, subtraction, multiplication division of fractions, including mixed numbers. You will decided which operation makes sense in each situation.;" "You may already know shortcuts for working with fractions..."

- Comparing and Scaling in Grade 7: Problem 2.3 refers to work in unit rates in prior Grade 6 unit Comparing Bits and Pieces.

- Accentuate the Negative in Grade 7: Problem 4.2, refers to work with the distributive property in Grade 6.

- Accentuate the Negative in Grade 7: Page 3, "Most of the numbers you have worked with in math class have been greater than or equal to zero. However ... ;" "You will also learn more about the properties of operations on numbers." Page 4,

"You will extend your knowledge of negative numbers." Page 8,"You have worked with whole numbers, fractions, decimals in earlier units." Page 58, "You have already examined patterns in ..."

- Thinking With Mathematical Models in Grade 8: Page 3, "In earlier Connected Mathematics units, you explored relationships between two variables. You learned how to find linear relationships from tables and graphs and then write their equations. Using the equations, you solved problems.

Response

While some of the statements are fairly positive, the reviewers still fail to recognize the focus of the "major work" in all of the units. We provide clarification that all of the units in grade 7 focus on the major work in 7.EE.A, 7.EE.A, and 7.RP.A.

All three grade levels have major work on equations, EE.A and EE.B:
We agree with this statement, but we also contend that *Shapes and Designs*, *Accentuate the Negative*, *Filling and Wrapping* also strengthen the work of 7.EE.A and 7. EE.B. See the discussion in Indicator II for Focus.

All three grade levels have major work on ratio and proportional reasoning, 6.RP to 7.RP:
We agree with this statement, but it is interesting to note that the reviewers now state that the *Stretching and Shrinking* unit provides major work for proportional reasoning, while in earlier claims, this unit was deemed not appropriate for 7th grade as similarity is addressed in 8th grade.
We also contend that the major focus of the *Filling and Wrapping*, *Moving Straight Ahead*, and *What do you Expect*? units strongly support the work of 6.RB. See the discussion in Indicator II for Focus.

All three grades have major work on the number systems, 6.NS.A, 6.NS.B, 6.NS.C to 7.Ns.A to 8.NS.A:
There are many places where the materials relate grade level concepts to explicitly to prior knowledge from earlier grades. These can be found in the student editions in the problems and in the teacher editions in charts and in a narrative called Mathematics Background
We agree with this statement, but we also contend that *Samples and Populations* and *Filling and Wrapping* also provide critical understanding to the work of numbers.

There is limited support for differentiation:

We provide clarification that differentiation is a strong component of Connected Mathematics and that its curriculum and pedagogical model is designed to enhance the learning experience of *all* students.

For over 25 years of field-testing, revision, and evaluation, differentiation is and has been an important concern of the authors of Connected Mathematics. We take seriously our overarching goal:

> *The overarching goal of CMP is to help students and teachers develop mathematical knowledge, understanding, and skill along with an awareness of and appreciation for the rich connections among mathematical strands and between mathematics and other disciplines. The CMP curriculum development has been guided by our single mathematical standard:*
>
> All students should be able to reason and communicate proficiently in mathematics. They should have knowledge of and skill in the use of the vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics, including the ability to define and solve problems with reason, insight, inventiveness, and technical proficiency.

Connected Mathematics is a problem-centered curriculum. The problems were carefully selected to provide multiple access points and time for students to acquire the understanding of important mathematical understandings embedded in the Problems. The development of a concept moves gradually from informal to formal mathematics through a sequence of problems within Investigations in a Unit and continues when students revisit the concept to build understandings of related ideas in other units throughout Grades 6, 7, an 8.

The reviewers note that the Explore is a significant time to attend to differentiation, which we agree with. But we also contend that differentiation starts with the teacher's planning and is implemented throughout the lesson, ending with the teacher's reflection. The Launch, Explore, Summarize phases of the lesson were developed to provide guidance on differentiation at each stage. For example, the first part of a Launch connects the challenge of the problem to prior knowledge. This is an important step in scaffolding and hence differentiation. Throughout the Launch the teacher is asking questions and taking note of strengths and weaknesses, which are used to guide the rest of the lesson during the Explore and Summarize. Each phase, Launch, Explore and Summarize, contain numerous questions, possible student responses and suggestions for follow-up to each response. *Going Further* and *Check for Understanding* are also important features that occur in the Explore or Summarize, which the teacher can use to attend to the individual needs of students. The homework, Applications, Connections, and Extensions are also designed to provide for individual needs.

Further, the authors have provided numerous examples of possible student responses, stumbling blocks, and misconceptions with suggestions that the teacher can use in these situations throughout the Teacher Guide, the Math Background, Unit, Investigation and Problem Overviews, Mathematical Reflections, Looking Back, Labsheets, Assessments, Self Assessments and in the Guide to CMP3: Understanding, Implementing, and Teaching.

Comment

> *It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. CCSSM*, P. 4

Finally, we are curious as to why the comments on Differentiation occur with Gateway I. According to the EdReports.org *Quality Instructional Materials Tool: Grades K–8 Mathematics* differentiation, is not mentioned until Gateway III. p. 19

Rating Sheet 4: Differentiated Instruction

- For 'Differentiated Instruction' to attain a score of 'Meets Expectations,' material must earn at least 10 points.

CRITERION	INDICATORS	RATING	EVIDENCE
Differentiated instruction: Materials support teachers in differentiating instruction for diverse learners within and across grades. Earned: _____ of 12 points ☐ **Meets expectations (10-12 points)** ☐ **Partially meets expectations (8-9 points)** ☐ **Does not meet expectations (<8 points)**	**3r.** Materials provide strategies to help teachers sequence or scaffold lessons so that the content is accessible to all learners.	0 1 2	
	3s. Materials provide teachers with strategies for meeting the needs of a range of learners.	0 1 2	
	3t. Materials embed tasks with multiple entry-points that can be solved using a variety of solution strategies or representations.	0 1 2	
	3u. Materials suggest support, accommodations, and modifications for English Language Learners and other special populations that will support their regular and active participation in learning mathematics (e.g., modifying vocabulary words within word problems).	0 1 2	
	3v. Materials provide opportunities for advanced students to investigate mathematics content at greater depth.	0 1 2	
	3w. Materials provide a balanced portrayal of various demographic and personal characteristics.	0 1 2	
	3x. Materials provide opportunities for teachers to use a variety of grouping strategies.		
	3y. Materials encourage teachers to draw upon home language and culture to facilitate learning.		

Coherence: Indicator IV

Materials foster coherence through connections at a single grade, where appropriate and required by the Standards

i. Materials include learning objectives that are visibly shaped by CCSSM cluster headings.

ii. Materials include problems and activities that serve to connect two or more clusters

in a domain, or two or more domains in a grade, in cases where these connections are natural and important.

Claim

The materials reviewed for Grade 7 meet the expectations for coherence. Each investigation within each unit lists the CCSSM that are taught. The mathematical highlights for each unit stress the clusters from CCSSM. All investigations in the student books contain the standards included in that lesson. Every investigation includes activities that connect two or more clusters in a domain, or two or more domains. An example of this is in Accentuate the Negative where two of the highlights are to understand the relationship between a number and its opposite and to develop algorithms for adding, subtracting, multiplying and dividing positive and negative numbers.

There are many links between major clusters in this curriculum.

- In *Comparing and Scaling* investigation 1, students are asked to recognize and represent proportional relationships between quantities (7.RP.A) and use variables to represent quantities in a real work or a mathematical problem and construct simple equations (7.EE.B).
- In *Accentuate the Negative* investigation 2, students are asked to understand subtraction of rational numbers as adding the additive inverse (7.NS.A) and solve multi-step real-world and mathematical problems posed with positive and negative rational numbers (7.EE.B)
- In *Moving Straight Ahead* investigation 3, students are asked to explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation (7.RP.A) and understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related (7.EE.A).
- There is no unit or investigation that only focuses on one aspect of the CCSSM. Connections are evident in all grade levels and in all units. This is a very strong aspect of Connected Mathematics 3.

Response

We agree with all of the statements in this Indicator. However, this Indicator *contradicts* many of the statements made earlier in this report on Grade 7 in Connected Mathematics. In particular, the first paragraph would suggest that Connected Mathematics is fully aligned with both the Focus and Coherence Indicators for 7th grade.

Conclusion

We have provided clarification that Connected Mathematics does meet the criteria for the Indicators for both Focus and Coherence