



# Promoting Productive Disciplinary Engagement and Learning With Open Problems and “Just-in-Time” Supports in Middle School Mathematics

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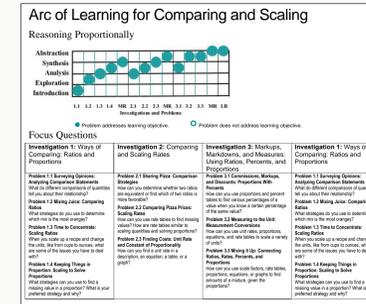
## Arc of Learning for *Connected Mathematics3*

Introducing <sup>Ⓜ</sup> Setting the Scene <sup>Ⓜ</sup>	Exploring <sup>Ⓜ</sup> Mucking About <sup>Ⓜ</sup>	Analyzing <sup>Ⓜ</sup> Going Deeper <sup>Ⓜ</sup>	Synthesizing <sup>Ⓜ</sup> Looking Across <sup>Ⓜ</sup>	Abstracting <sup>Ⓜ</sup> Going Beyond <sup>Ⓜ</sup>
<ul style="list-style-type: none"> <li>reveal the mathematical theme for the unit</li> <li>informally highlight the key mathematical concepts in the unit</li> <li>assess prior understandings related to the goals of the unit</li> </ul>	<ul style="list-style-type: none"> <li>establish a platform for developing key aspects of the understanding of the concepts and strategies</li> <li>explore (consider) a context that students can use to build, connect, and retrieve mathematical understandings</li> </ul>	<ul style="list-style-type: none"> <li>make connections between concepts and representations</li> <li>examine nuances in key aspects of the core mathematical ideas often with a variety of contextual situations</li> </ul>	<ul style="list-style-type: none"> <li>recognize core ideas across multiple contextual or problem situations</li> <li>begin to consolidate and refine emerging mathematical understanding(s) into a coherent structure</li> </ul>	<ul style="list-style-type: none"> <li>make judgments about which representations, operations, rules, or relationships are useful across various contexts</li> <li>look back on prior learning to generalize, extend, and abstract the underlying mathematical structure</li> <li>assess understandings at a more general level</li> </ul>

Connecting  
Reflecting  
Assessing  
Mathematical Practices

Arc of Learning for Comparing and Scaling

Investigation	1.1	1.2	1.3	1.4	1.5
1.1 Scaling Down: Analyzing Contexts	1.1	1.1			
1.2 Mixing Juice: Comparing Rates		1.2	1.2		
1.3 Time to Compare: Scaling Rates			1.3		
1.4 Mixing Juice: Comparing Rates				1.4	1.4
1.5 Scaling Rates: Comparing Strategies					1.5
1.6 Comparing Pizzas: Scaling Rates					1.6
1.7 Comparing Unit Rates and Concept of Proportionality					1.7
1.8 Making, Marking, and Measuring: Using Ratios, Percents, and Proportions					1.8
1.9 Comparing, Marking, and Measuring: Using Ratios, Percents, and Proportions					1.9
1.10 Measuring by the Unit: Measurement					1.10
1.11 Comparing Unit Rates, Rates, Percents, and Proportions					1.11
1.12 Looking Back					1.12



## Print Version of the Orange Juice Problem

1.2 Mixing Juice  
Comparing Rates

Every year, the Grade 7 students at Langston Hughes School go on an outdoor education camping trip. During the week-long trip, the students study nature and participate in recreational activities. Everyone pitches in to help with the cooking and cleanup.

This year, Andre and Maxine were in charge of making orange juice for the campers. They planned to make the juice by mixing equal amounts of orange juice concentrate. To test the mix that would last best, they decided to use water.

Which mix will make the juice that is most “orangey?” Explain.

1.1 Which mix will make the juice that is the most “orangey?” Explain your reasoning.  
1.2 Which mix will make the juice that is the most “orangey?” Explain your reasoning.  
1.3 Isabelle and Doug used fractions to express their reasoning. Do you agree with either of them? Explain.

## Redesigned Version of the Problem

Initial Challenge	What if...?	Now what do you know?
Which recipe will make juice that is most “orangey?” Least “orangey?” Explain.	<ol style="list-style-type: none"> <li>Max thinks that recipes A and C are both the most “orangey” since the difference between the number of cups of water and the number of cups of concentrate is 1. Is Max’s thinking correct? Explain.</li> <li>Isabelle and Doug used fractions to express their reasoning. Do you agree with either of them? Explain.</li> </ol>	<p>Reflect on the strategies that you and your classmates used to determine which recipe was the most “orangey.” Least “orangey.”</p> <p>Describe the strategies used by your classmates. How are they the same? Different?</p>

Isabelle:  $\frac{5}{9}$  of Mix B is concentrate. Doug:  $\frac{5}{14}$  of Mix B is concentrate.

## Digital Prototype

Collaborative Space  
Comparing and Scaling Unit 1.2

Initial Challenge

What if...?

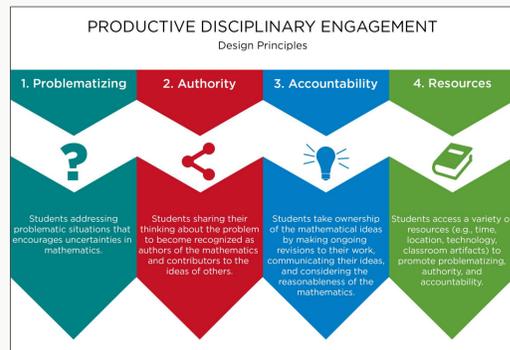
Now What Do You Know?

Which mix will make the juice that is the most “orangey?” Explain.

How could you represent the different recipes?

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## Productive Disciplinary Engagement



Embodiments of PDE	New Possibilities for PDE
<ul style="list-style-type: none"> <li>Students generate, discuss, and interpret complex problem situations (problematizing and authority)</li> <li>Students consider different ways to think about, assess, and refine problem-solving strategies (accountability)</li> <li>Students reflect on their learning – what they have learned, how it connects to prior knowledge, and possible new directions moving forward (accountability)</li> <li>Students consider the work of others as alternate thinking and approaches and form conceptual connections to the different ways of thinking (resources and authority)</li> <li>Students form connections between and among multiple representations (resources)</li> <li>Students access supports to extend perseverance (problematizing and resources)</li> </ul>	<ul style="list-style-type: none"> <li>Students can access high cognitive demand tasks that focus on conceptual understanding (problematizing)</li> <li>Students can use tool supports that do not reduce cognitive demand or limit focus on conceptual understanding (problematizing and resources)</li> <li>Students can make their own decisions on the problem solving approach without teacher intervention (problematizing and authority)</li> <li>Students can select their solution pathway and maintain authorship of ideas (authority)</li> <li>Students can model and press each other for answer completeness (authority and accountability)</li> <li>Students can probe deeper into mathematical justifications (accountability)</li> </ul>

## Research Questions

- How can productive disciplinary engagement be fostered in digital learning environments with open problems and “just-in-time” supports? How can student learning of mathematics be enhanced?
- What is the nature of productive disciplinary engagement and student learning of mathematics at key development points in a connected sequence of problems and lesson goals?
- What information do teachers draw upon when they use open problems and “just-in-time” supports? How do teachers adapt the supports for specific problems?

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