Focus Questions

Background

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

Description

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

Purpose

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

6-6: Variables and Patterns

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Variables and Patterns (Relationships) Develop understanding of variables and how they are related
- Explore problem situations that involve variables and relationships
- Identify the dependent and independent variables and describe how they are related in a situation
- Interpret the "stories" told by patterns in tables and coordinate graphs of numeric (x, y) data
- Represent the pattern of change that relates two variables in words, data tables, graphs, and equations
- Investigate situations that change over time
- Examine increasing and decreasing patterns of change
- Compare linear and nonlinear patterns of change by using tables or graphs
- Use tables, graphs, and equations to find the value of a variable given the value of the associated variable
- Explore relationships that require graphing in all four quadrants
- Describe advantages and disadvantages of using words, tables, graphs, and equations to represent patterns of change relating two variables and make connections across those representations
- Write an equation to express the relationship between two variables in one and two operations: $y=mx$, $y=b+x$, and $y=b+mx$
- Calculate average speed and show how it is reflected in a table or graph and vice versa
- Recognize and express direct proportionality relationships with a unit rate ($y=mx$) and represent these relationships in rate tables and graphs
- Solve problems that involve variables

Expressions and Equations Develop understanding of expressions and equations
- Use properties of operations, including the Distributive Property and the Order of Operations, to write equivalent expressions for the dependent variable in terms of the independent variable
- Use tables, graphs, or properties of numbers such as the Distributive Property to show that two expressions are equivalent
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity
- Interpret and evaluate expressions in which letters stand for numbers and apply the Order of Operations as needed
- Recognize that equations are statements of equivalence between two expressions
- Solve linear equations of the forms $y=ax$, $y=b+x$, and $y=b+ax$ using numeric guess and check, tables of (x, y) values, and graphs or fact families

- Write an inequality and associate it with an equation to find solutions and graph the solutions on a number line

Focus Questions and Mathematical Reflections

Investigation 1 Variables, Tables and Graphs	Investigation 2 Analyzing Relationships among Variables	Investigation 3 Relating Variables with Equations	Investigation 4 Expressions, Equations, and Inequalities
Problem 1.1 Getting Ready to Ride: Data Tables and Graphs **Focus Question:** How can you construct a graph from a table of data that depicts change over time? How is the pattern of change represented in the graph?	**Problem 2.1** Renting Bicycles: Independent and Dependent Variables **Focus Question:** How do you analyze and compare the relationship between variables given in different representations?	**Problem 3.1** Visit to Wild World: Equations with One Operation **Focus Question:** In what kinds of situations will the equation between dependent and independent variables be in the form $y = x + k$? $y = x - k$? $y = kx$? $y = x/k$?	**Problem 4.1** Taking the Plunge: Equivalent Expressions I **Focus Question:** Is it possible to have two different, but equivalent, expressions for a given situation? Explain.
Problem 1.2 From Atlantic City to Lewes: Time, Rate, and Distance **Focus Question:** What are the advantages and disadvantages of tables and graphs in representing and describing the patterns of change in a variable over time?	**Problem 2.2** Finding Customers: Linear and Non-Linear Patterns **Focus Question:** How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1? How are the differences shown in tables and graphs of data?	**Problem 3.2** Moving, Texting, and Measuring: Using Rates and Rate Tables **Focus Question:** What can you tell about the relationship between dependent and independent variables in an equation of the form $y = mx$? How is that relationship shown in a table and a graph of sample (x, y) values? Why is the point (1, m) on every graph?	**Problem 4.2** More Than One way to Say it: Equivalent Expressions II **Focus Question:** What does it mean to say that two algebraic expressions are equivalent?
Problem 1.3 From Lewes to Chincoteague Island: Stories, Tables, and	**Problem 2.3** Predicting Profit: Four Quadrant Graphing	**Problem 3.3** Group Discounts and a Bonus Card: Equations with Two Operations	**Problem 4.3** Putting it All Together: Equivalent Expressions III

Graphs **Focus Question:** Which representation of data – table, graph, or written notes- seems to better show patterns of change in distance over time, and why?	**Focus Question:** How are the variables, *tour income* and *tour profit,* related to each other? How do you plot data points with one or both coordinates negative?	**Focus Question:** How do you calculate values of y from an equation like y = 3x + 5 when values of x are given? How about y = 5 + 3x? When do you need such equations that involve two operations?	**Focus Question:** How can expressions such as 3*x* + 7*x* or 3(*x* +2) be written in equivalent form?
Problem 1.4 From Chincoteague to Colonial Williamsburg: Average Speed **Focus Question:** How do you calculate average speed for a trip? How do a table and graph of (time, distance) data show speed?	**Problem 2.4** What's the Story? Interpreting Graphs **Focus Question:** When the relationship between dependent and independent variables is displayed in a graph, what can you learn about the relationship from a rising graph, a level graph, and a falling graph?	**Problem 3.4** Getting the Calculation Right: Expressions and Order of Operations **Focus Question:** When an equation relating two variables involves two or more operations, how do you use the equation to find values of the dependent variable from given values of the independent variable?	**Problem 4.4** Finding the Unknown Value: Solving Equations **Focus Question:** What strategies can you use to solve equations in the forms x + a = b, x – a = b, ax = b, and x ÷ a = b (a ≠ 0)?
			Problem 4.5 It's Not Always Equal: Solving Inequalities **Focus Question:** How can you represent and find solutions for inequalities?
Mathematical Reflections: 1. You can show patterns of change over time with tables, graphs, and written reports. a. What are the advantages and disadvantages of showing patterns with tables? b. What are the advantages and disadvantages of showing patterns with graphs?	**Mathematical Reflections:** 1. The word variable is used often to describe conditions in science and business. a. Explain what the word variable means when it is used in situations like those you studied in this investigation. b. When are the words independent and dependent	**Mathematical Reflections:** 1. What strategies help in finding equations to express relationships? 2 For relationships given by equations in the form y = mx: a. How does the value of y change as the value of x increases? b. How is the pattern of change	**Mathematical Reflections:** 1. What does it mean to say that two expressions are equivalent? How can you test the equivalence of two expressions? 2. What does it mean to *solve* an equation? What strategies are available for solving equations? 3. What does it mean to *solve* an inequality? What will graphs of

c. What are the advantages and disadvantages of showing patterns with written reports? 2. a. How do you see patterns in the speed of a moving object by studying (time, distance) data in tables? b. How do you see patterns in the speed of a moving object by studying (time, distance) data in coordinate graphs?	used to describe related variables? How are they used? 2. Suppose the values of a dependent variable increase as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following? a. a table of values for the two variables? b. a graph of values for the two variables? 3. Suppose the values of a dependent variable decrease as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following? a. a table of values for the two variables b. a graph of values for the two variables	shown in a table, graph, and equation of the function? 3. a. In this unit, you have represented relationships between variables with tables, graphs, and equations. List some advantages and disadvantages of each of these representations. b. If the value of one variable in a relationship is known, describe how you can use a table, graph, or equation to find a value of the other variable.	such solutions look like for inequalities in the form $ax > b$ and $a + x < b$ (Assume a and b are both positive numbers). 4. Describe how expressions, equations, inequalities, and representations are used in this Unit. How are they related?