

## 8-1: Thinking with Mathematical Models

Unit Goals, Focus Questions, and Mathematical Reflections

### Unit Goals

**Linear and Nonlinear Relationships** Recognize and model linear and nonlinear relationships in bivariate data

- Represent data patterns using graphs, tables, word descriptions and algebraic expressions
- Use mathematical models to answer questions about linear relationships
- Investigate the nature of linear variation in contexts
- Write linear functions from verbal, numerical, or graphical information
- Analyze, approximate, and solve linear equations
- Model situations with inequalities expressed as “at most” and “at least” situations
- Investigate the nature of inverse variation in contexts
- Use mathematical models to answer questions about inverse variation relationships
- Compare inverse variation relationships with linear relationships

**Data Analysis** Measure variation in data and strength of association in bivariate data

- Use data patterns to make predictions
- Fit a line to data that show a linear trend and measure goodness of fit
- Analyze scatter plots of bivariate data to determine the strength of the linear relationship between the two variables.
- Use correlation coefficients informally to describe the strength of the linear relationship illustrated by scatter plots.
- Distinguish between categorical and numerical variables.
- Use 2-way tables and analysis of cell frequencies and relative frequencies to help in deciding whether two categorical variables are related.
- Use standard deviation to measure variability in data distributions

## 8-1 Thinking with Mathematical Models: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exploring Data Patterns	Investigation 2 Linear Models and Equations	Investigation 3 Inverse Variation	Investigation 4 Variability and Associations in Numerical Data	Investigation 5 Variability and Associations in Categorical Data
<p><b>Problem 1.1</b> <b>Bridge Thickness and Strength</b> FQ: How would you describe the relationship between bridge strength and bridge thickness revealed by your experiment?</p> <p><b>Problem 1.2</b> <b>Bridge Length and Strength</b> FQ: How would you describe the pattern relating bridge strength to bridge length shown in your experimental data?</p> <p><b>Problem 1.3</b> <b>Custom Construction Parts: Finding Patterns</b> FQ: How can you predict if a pattern between variables will be linear or nonlinear?</p>	<p><b>Problem 2.1</b> <b>Modeling Linear Data Patterns</b> FQ: How can you find a linear function that is a good model for a set of data and then measure the accuracy of that model with residuals?</p> <p><b>Problem 2.2</b> <b>Up and Down the Staircase: Exploring Slope</b> FQ: How do you write an equation for a linear function if you are given a graph, a table, or two points?</p> <p><b>Problem 2.3</b> <b>Tree Top Fun: Equations for Linear Functions</b> FQ: What strategies do you use in writing equations for linear functions?</p> <p><b>Problem 2.4</b> <b>Boat Rental Business: Solving Linear Equations</b> FQ: What strategies do you find useful to find solutions for linear equations?</p> <p><b>Problem 2.5</b> <b>Amusement Park or Movies: Intersecting Linear Models</b> FQ: When the graphs of two linear functions intersect, what do the coordinates of that intersection point tell you?</p>	<p><b>Problem 3.1</b> <b>Rectangles with Fixed Area</b> FQ: When the product of two variables is some fixed number, what is the pattern of change and how is that pattern of change reflected in tables and graphs of the relationship?</p> <p><b>Problem 3.2</b> <b>Distance, Speed and Time</b> FQ: What examples using distance, rate, and time show one variable inversely related to another?</p> <p><b>Problem 3.3</b> <b>Planning a Field Trip: Finding Individual Cost</b> FQ: How does the cost per person change if a fixed total cost is split among an increasing number of individual payers?</p> <p><b>Problem 3.4</b> <b>Modeling Data Patterns</b> FQ: What pattern in a table or graph of data suggests an inverse variation model and what strategies can you use to find an equation model for that kind of function?</p>	<p><b>Problem 4.1</b> <b>Vitruvian Man: Relating Body Measurements</b> FQ: If you have data relating two variables, how can you check to see whether a linear model is a good fit?</p> <p><b>Problem 4.2</b> <b>Older and Faster: Negative Correlations</b> FQ: From the scatter plot, how do you know if a linear model fits the data? How do you know if there are outliers? How do you know if the relationship is negative or positive?</p> <p><b>Problem 4.3</b> <b>Correlation Coefficients and Outliers</b> FQ: What does a correlation coefficient of 1, 0, or -1 suggest to you about the relationship between two variables?</p> <p><b>Problem 4.4</b> <b>Measuring Variability: Standard Deviation</b> FQ: How do you calculate the standard deviation for a data distribution and what does that statistic tell about the distribution?</p>	<p><b>Problem 5.1</b> <b>Wood or Steel? That's the Question</b> FQ: What does a two-way table show you about preferences among groups?</p> <p><b>Problem 5.2</b> <b>Politics of Girls and Boys: Analyzing Data in Two-Way Tables</b> FQ: Suppose you have recorded the counts of different preferences by group in a two-way table. How can you use those counts, or percents from the counts, to decide if two groups have the same preferences or not?</p> <p><b>Problem 5.3</b> <b>After-School Jobs and Homework: Working Backward: Setting up a Two-Way Table</b> FQ: Suppose you have data about the same trait in two groups. How can you organize the data to compare and decide if the groups are the same or not relative to the trait?</p>
<p><b>Mathematical Reflections</b></p> <p>1. You can represent a relationship between variables with a table, a graph, a description in words, or an equation. 1a. How can you decide whether a relationship is linear by studying the pattern in a data table? 1b. How can you decide whether a relationship is linear by studying the pattern in a graph? 1c. How can you decide whether a relationship is linear by studying the words used to describe the variables? 1d. How can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form?</p> <p>2. What are the advantages and disadvantages in finding patterns and making predictions?</p>	<p><b>Mathematical Reflections</b></p> <p>1. Why is it helpful to use a linear model for a set of data?</p> <p>2. When does it make sense to choose a linear function to model a set of data?</p> <p>3. How would you find the equation for a linear function in the following situations?</p> <p>3a. You are given a description of the variables in words. 3b. You are given a table of values for the variables 3c. You are given a graph of sample data points</p> <p>4. What strategies can you use to solve a linear equation such as <math>500 = 245 + 5x</math>?</p> <p>5. What kind of mathematical sentences express "at least" and "at most" questions about linear functions?</p>	<p><b>Mathematical Reflections</b></p> <p>1. Suppose the relationship between variables <math>x</math> and <math>y</math> is an inverse variation. 1a. How do the values of <math>y</math> change as the values of <math>x</math> increase? 1b. Describe the trend in a graph of <math>(x, y)</math> values. 1c. Describe the equation that relates the values of <math>x</math> and <math>y</math>.</p> <p>2. How is an inverse variation similar to a linear relationship? How is it different?</p>	<p><b>Mathematical Reflections</b></p> <p>1. Think about the pattern of points you see in a scatter plot. 1a. What pattern would you expect when two variables are related by a linear model with positive slope? 1b. What pattern would you expect when two variables are related by a linear model with negative slope? 1c. What would you expect to see in a scatter plot when two variables are unrelated?</p> <p>2. You assessed the accuracy of linear models. 2a. What do outliers on a scatter plot indicate? 2b. What can you learn from the errors of prediction or residuals? 2c. What do you know about a linear model from the correlation coefficient?</p> <p>3. What does the standard deviation tell you about a set of data?</p>	<p><b>Mathematical Reflections</b></p> <p>1. What are categorical variables and what do they measure?</p> <p>2. Suppose a survey asked teenagers and adults whether or not they use text messaging. 2a. How could you arrange the data to compare the groups? 2b. How would you decide that the two groups - teenagers and adults - were different in their use of text messaging? 2c. Suppose that one analysis compared only the numbers in each group - teenage text messenger, teenage non-text messenger, adult text messenger, and adult non-text messenger. How might the analysis result in misleading conclusions?</p>