

## **Focus Questions**

### **Background**

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

### **Description**

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

### **Purpose**

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

# 8-5: Butterflies, Pinwheels, and Wallpaper

Unit Goals, Focus Questions, and Mathematical Reflections

## Unit Goals

**Transformations** Describe types of transformations that relate points by the motions of reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures

Recognize properties of reflection, rotation, and translation transformations

Explore techniques for using rigid motion transformations to create symmetric designs

Use coordinate rules for basic rigid motion transformations

**Congruence and Similarity** Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes

Recognize that two figures are congruent if one is derived from the other by a sequence of reflection, rotation, and/or translation transformations

Recognize that two figures are similar if one can be obtained from the other by a sequence of reflections, rotations, translations, and/or dilations

Use transformations to describe a sequence that exhibits the congruence between figures

Use transformations to explore minimum measurement conditions for establishing congruence of triangles

Use transformations to explore minimum measurement conditions for establishing similarity of triangles

Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations

Use properties of congruent and similar triangles to solve problems about shapes and measurements

## Focus Questions and Mathematical Reflections

<b>Investigation 1</b> <b>Symmetry and Transformations</b>	<b>Investigation 2</b> <b>Transformations and Congruence</b>	<b>Investigation 3</b> <b>Transforming Coordinates</b>	<b>Investigation 4</b> <b>Dilations and Similar Figures</b>
<b>Problem 1.1 Butterfly</b>	<b>Problem 2.1 Connecting</b>	<b>Problem 3.1 Flipping on a</b>	<b>Problem 4.1 Focus on</b>

<p><b>Symmetry: Line Reflections</b></p> <p><b>Focus Question</b> What does it mean to say that a figure has reflection or flip symmetry? How is each point related to its image under transformation by reflection in a line?</p>	<p><b>Congruent Polygons</b></p> <p><b>Focus Question</b> What does it mean to say two geometric shapes are congruent to each other and how could you demonstrate congruence with movable copies of the figures?</p>	<p><b>Grid: Coordinate Rules for Reflections</b></p> <p><b>Focus Question</b> How can you describe the ‘motions’ of points under reflections with coordinate rules in the form <math>(x,y) \rightarrow (\square, \square)</math> tells how to ‘move’ any point to its image under a translation?</p>	<p><b>Dilations</b></p> <p><b>Focus Question</b> What coordinate rules model dilations and how do dilations change or preserve characteristics of the original figure?</p>
<p><b>Problem 1.2 In a Spin: Rotations</b></p> <p><b>Focus Question</b> What does it mean to say that a figure has rotation or turn symmetry? How is each point related to its image under transformation by rotation?</p>	<p><b>Problem 2.2 Supporting the World: Congruent Triangles I</b></p> <p><b>Focus Question</b> How much information do you need to decide that two triangles are probably congruent or not congruent? How do you go about planning transformations that ‘move’ one triangle onto another?</p>	<p><b>Problem 3.2 Sliding on a Grid: Coordinate Rules for Translations</b></p> <p><b>Focus Question</b> What kind of coordinate rule <math>(x,y) \rightarrow (\square, \square)</math> tells how to ‘move any point to its image under a translation?</p>	<p><b>Problem 4.2 Return of Super Sleuth: Similarity Transformations</b></p> <p><b>Focus Questions</b> How can you use transformations to check whether two figures are similar or not?</p>
<p><b>Problem 1.3 Sliding Around: Translations</b></p> <p><b>Focus Question</b> What does it mean to say that a figure has translation or slide symmetry? How is each point related to its image under transformation by translation?</p>	<p><b>Problem 2.3 Minimum Measurement: Congruent Triangles II</b></p> <p><b>Focus Question</b> What is the smallest number of side and angle measurements that will allow you to conclude that two triangles are congruent?</p>	<p><b>Problem 3.3 Spinning on a Grid: Coordinate Rules for Rotations</b></p> <p><b>Focus Question</b> What are the coordinate rules that describe ‘motion’ of points on a grid under turns of <math>90^\circ</math> and <math>180^\circ</math>?</p>	<p><b>Problem 4.3 Checking Similarity Without Transformations</b></p> <p><b>Focus Question</b> What information about the sides and angles of two triangles will guarantee that they are similar?</p>
<p><b>Problem 1.4 Properties of Transformations</b></p> <p><b>Focus Question</b> How, if at all, will the shape, size, and</p>		<p><b>Problem 3.4 A Special Property of Translations and Half-Turns</b></p> <p><b>Focus Question</b> How are lines</p>	<p><b>Problem 4.4 Using Similar Triangles</b></p> <p><b>Focus Question</b> What facts about similar triangles allow</p>

<p>position of a geometric figure change after each of the transformations studied in this investigation – flip, turn, or slide?</p>		<p>and their images under translations and half-turns related to each other?</p>	<p>you to find lengths in very large figures even when they can't be reached to measure?</p>
		<p><b>Problem 3.5 Parallel Lines, Transversals, and Angle Sums</b></p> <p><b>Focus Question</b> When two parallel lines are cut by a transversal, what can be said about the angles formed? What is always true about the angle measures in a triangle? How do you know that your answers are correct?</p>	
<p><b>Mathematical Reflection</b></p> <ol style="list-style-type: none"> <li>How would you explain to someone how to make a design with:             <ol style="list-style-type: none"> <li>reflectional symmetry?</li> <li>rotational symmetry?</li> <li>translational symmetry?</li> </ol> </li> <li>How are points and their images related by each of these geometric transformations?             <ol style="list-style-type: none"> <li>reflection in line <math>m</math></li> <li>rotation of <math>d^\circ</math> about point <math>P</math></li> <li>translation with distance and direction set by</li> </ol> </li> </ol>	<p><b>Mathematical Reflection</b></p> <ol style="list-style-type: none"> <li>How can you find a sequence of flips, turns, and slides to “move” one figure exactly onto another to show that they are congruent?</li> <li>What information about the sides and angles of two triangles will guarantee you can “move” one triangle onto the other?</li> <li>How could you convince someone that two given triangles are not congruent?</li> </ol>	<p><b>Mathematical Reflection</b></p> <ol style="list-style-type: none"> <li>What are the general forms of the coordinate rules for these transformations?             <ol style="list-style-type: none"> <li>reflection in the <math>y</math>-axis</li> <li>reflection in the <math>x</math>-axis</li> <li>counterclockwise rotation of <math>90^\circ</math> about the origin</li> <li>counterclockwise rotation of <math>180^\circ</math> about the origin</li> <li>translation that “moves” points <math>a</math> units horizontally and <math>b</math> units vertically</li> </ol> </li> <li>What is the effect of translation and half-turns on</li> </ol>	<p><b>Mathematical Reflection</b></p> <ol style="list-style-type: none"> <li>How would you explain what it means for two geometric shapes to be similar using             <ol style="list-style-type: none"> <li>everyday words that most people could understand?</li> <li>technical terms of mathematics?</li> </ol> </li> <li> <ol style="list-style-type: none"> <li>Suppose you dilate a polygon to form a figure of a different size. How will the side lengths, angle measures, perimeters, areas, and</li> </ol> </li> </ol>

<p>the segment from point X to point X'.</p> <p>3. How do reflections, rotations, and translations change the size and shape of line segments, angles, and/or polygons, if at all?</p>		<p>lines?</p> <p>3. How has your knowledge of transformations changed or extended what you already knew about the angles formed by two parallel lines and a transversal?</p> <p>4. How has your knowledge of transformations changed or extended what you already knew about the sum of the angle measures of a triangle?</p>	<p>slopes of the sides of the two figures be alike? How will they be different?</p> <p>b. How has your knowledge of dilations changed or extended what you already knew about similarity.</p> <p>3. What is the least amount of information you need in order to be sure that two triangles are similar?</p> <p>4. How do you use similarity to find the side lengths of similar figures?</p>
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