8-5: Butterflies, Pinwheels, and Wallpaper

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Transformations Describe types of transformations that relate points by the motions of Reflections, rotations, and translations, and describe methods for identifying and creating symmetric plane figures

- Recognize properties of Reflections, rotation, and translation transformations
- Explore techniques for using rigid motion transformations to create symmetric designs
- Use coordinate rules for basic rigid motion transformations

Congruence and Similarity Understand congruence and similarity and explore necessary and sufficient conditions for establishing congruent and similar shapes

- Recognize that two figures are congruent if one is derived from the other by a sequence of Reflections, rotation, and/or translation transformations
- Recognize that two figures are similar if one can be obtained from the other by a sequence of Reflections, rotations, translations, and/or dilations
- Use transformations to describe a sequence that exhibits the congruence between figures
- Use transformations to explore minimum measurement conditions for establishing congruence of triangles
- Use transformations to explore minimum measurement conditions for establishing similarity of triangles
- Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations
- Use properties of congruent and similar triangles to solve problems about shapes and measurements

8-5 Butterflies, Pinwheels and Wallpaper: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Symmetry and Transformations	**Investigation 2** Transformations and Congruence	**Investigation 3** Transforming Coordinates	**Investigation 4** Dilations and Similar Figures
Problem 1.1 **Butterfly Symmetry: Line Reflections** FQ: What does it mean to say that a figure has flip or reflectional symmetry? How is each point related to its image under transformation by reflection in a line? **Problem 1.2** **In a Spin: Rotations** FQ: What does it mean to say that a figure has turn or rotation symmetry? How is each point related to its image under transformation by rotation? **Problem 1.3** **Sliding Around: Translations** FQ: What does it mean to say that a figure has slide or translational symmetry? How is each point related to its image under transformation by translation? **Problem 1.4** **Properties of Transformations** FQ: How, if at all, will the shape, size, and position of a figure change after each of the transformations – reflection, rotation, or translation?	**Problem 2.1** **Connecting Congruent Polygons** FQ: What does it mean to say two geometric shapes are congruent to each other and how could you demonstrate congruence with movable copies of the figures? **Problem 2.2** **Supporting the World: Congruent Triangles I** FQ: How much information do you need to decide that two triangles are probably congruent or not congruent? How do you go about planning transformations that 'move' one triangle onto another? **Problem 2.3** **Minimum Measurement: Congruent Triangles II** FQ: What is the smallest number of side and/or angle measurements needed to conclude two triangles are congruent?	**Problem 3.1** **Flipping on a Grid: Coordinate Rules for Reflections** FQ: How can you describe how points "move" under a reflection with coordinate rules in the form (x,y)→(□,□) when the reflection line is (1) the x-axis? (2) the y-axis? (3) the line y=x? **Problem 3.2** **Sliding on a Grid: Coordinate Rules for Translations** FQ: What kind of coordinate rule (x,y)→(□,□) tells how to "move" any point to its image under a translation? **Problem 3.3** **Spinning on a Grid: Coordinate Rules for Rotations** FQ: What are the coordinate rules that describe "motion" of points on a grid under turns of 90° and 180°? **Problem 3.4** **A Special Property of Translations and Half-Turns** FQ: How are lines and their images under translations and half-turns related to each other? **Problem 3.5** **Parallel Lines, Transversals, and Angle Sums** FQ: When two parallel lines are cut by a transversal, what can be said about the angles formed? What is always true about the angle measures in a triangle? How do you know that your answers are correct?	**Problem 4.1** **Focus on Dilations** FQ: What coordinate rules model dilations and how do dilations change or preserve characteristics of the original figure? **Problem 4.2** **Return of Super Sleuth: Similarity Transformations** FQ: How can you use transformations to check whether two figures are similar or not? **Problem 4.3** **Checking Similarity Without Transformations** FQ: What information about the sides and angles of two triangles will guarantee that they are similar? **Problem 4.4** **Using Similar Triangles** FQ: What facts about similar triangles allow you to find lengths in very large figures that you are unable to reach?
Mathematical Reflections 1. How would you explain to someone how to make a design with: 1a. reflectional symmetry? 1b. rotational symmetry? 1c. translational symmetry? 2. How are points and their images related by each of these geometric transformations? 2a. reflections in line *m* 2b. rotation of d° about point P 2c. translation with distance and direction set by the segment from point *X* to point *X'*. 3. How do reflections, rotations, and translations change the size and shape of line segments, angles, and/or polygons, if at all?	**Mathematical Reflections** 1. How can you find a sequence of flips, turns, and slides to "move" one figure exactly onto another to show that they are congruent? 2. What information about the sides and angles of two triangles will guarantee you can "move" one triangle onto the other? 3. How could you convince someone that two given triangles are not congruent?	**Mathematical Reflections** 1. What are the general forms of the coordinate rules for these transformations? 1a. reflection in the y-axis 1b. reflection in the x-axis 1c. counterclockwise rotation of 90° about the origin 1d. counterclockwise rotation of 180° about the origin 1e. translation that "moves" points *a* units horizontally and *b* units vertically 2. What is the effect of translation and half-turns on lines? 3. How has your knowledge of transformations changed or extended what you already knew about the angles formed by two parallel lines and a transversal? 4. How has your knowledge of transformations changed or extended what you already knew about the sum of the angle measures of a triangle?	**Mathematical Reflections** 1. How would you explain what it means for two geometric shapes to be similar using 1a. everyday words that most people could understand? 1b. technical terms of mathematics? 2a. Suppose you dilate a polygon to form a figure of a different size. How will the side lengths, angle measures, perimeters, areas, and slopes of the sides of the two figures be alike? How will they be different? 2b. How has your knowledge of dilations changed or extended what you already knew about similarity. 3. What is the least amount of information you need in order to be sure that two triangles are similar? 4. How do you use similarity to find the side lengths of similar figures?