Focus Questions

Background

The student book is organized around three to five investigations, each of which contain three to five problems and a Mathematical Reflection that students explore during class.

In the Teacher Guide the Goals for each unit include two to four big concepts with an elaboration of the essential understandings for each.

In the Teacher Guide, a Focus Question is provided for each problem in an investigation. The Focus Question collapses the mathematical understandings and strategies embedded in the problem into one overarching question. The teacher can use the Focus Question to guide his/her instructional decisions throughout his/her planning, teaching, and reflections on student understanding.

Description

The Goals of the unit describe the mathematics content developed in the unit. The Focus Questions provide a story line for the mathematical development of an investigation. The set of Mathematical Reflections in the student book provide a story line for the mathematical development of the unit. The following contain all of the Goals, Focus Questions and Mathematical Reflections for each unit in CMP3.

Purpose

These stories can serve as an overview of the unit and as a guide for planning, teaching and assessing.

The Goals, Mathematical Reflections, and Focus Questions can be laminated and used a bookmark for the Teacher.

8-8: Function Junction

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Functions : Understand equivalence of algebraic expressions and functions

- Describe domain and range of functions
- Use $f(x)$ notation to describe and operate with functions
- Construct and interpret inverses of functions
- Analyze function rates of change using graphs
- Identify contexts and graphs of step and piecewise defined functions
- Analyze polynomial functions and their graphs
- Identify, analyze, and solve problems related to arithmetic and geometric sequences
- Compare arithmetic and geometric sequences to linear and exponential functions
- Recognize and solve problems using special kinds of functions

Equivalence: Understand equivalence of algebraic expressions and functions

- Connect expressions for functions whose graphs are related by translation and/or stretching
- Develop and use vertex form to graph quadratic functions and solve quadratic equations
- Connect polynomial expressions and graphs of the polynomial functions they define, in order to identify max/min points, intercepts, and solutions of equations
- Use completing the square to write quadratics in equivalent vertex form
- Develop the quadratic formula for solving equations
- Develop complex numbers and operations
- Develop algorithms for adding, subtracting, and multiplying polynomials

Focus Questions and Mathematical Reflections

Investigation 1: The Families of Functions	Investigation 2: Arithmetic and	Investigation 3: Transforming Graphs,	Investigation 4: Solving Quadratic Equations	Investigation 5: Polynomial

	Geometric Sequences	Expressions, and Functions	Algebraically: Completing the Square and Using the Quadratic Formula	Expressions, Functions, and Equations
Problem 1.1: Filling Functions **Focus Question: How does the shape of a function graph tell the rate of change in the dependent variable as the independent variable changes?**	**Problem 2.1: Arithmetic Sequences** **Focus Question: What are the defining properties of an arithmetic sequence?**	**Problem 3.1: Sliding Up and Down: Vertical Translation of Functions** **Focus Question: If graphs of functions are related by sliding up and down, how are the expressions related?**	**Problem 4.1: Solving Quadratic Equations Algebraically** **Focus Question: What strategies allow you to solve quadratic equations algebraically, and how are the algebraic and graphical solutions related to each other?**	**Problem 5.1: Properties of Polynomial Expressions and Functions** **Focus Question: What are the patterns of change associated with polynomial expressions and functions of degree 2, 3, and 4, and how are those patterns shown in graphs?**
Problem 1.2: Domain, Range, and Function Notation **Focus Question: What do the terms domain and range tell about a function, and how is f(x) notation used to represent a function?**	**Problem 2.2: Geometric Sequences** **Focus Question: What are the defining properties of a geometric sequence?**	**Problem 3.2: Stretching and Flipping Up and Down: Multiplicative Transformations of Functions** **Focus Question: If graphs of functions are related by stretching away from or towards the x-axis and/or reflecting across that axis, how are the expressions related?**	**Problem 4.2: Completing the Square** **Focus Question: How can a quadratic expression be written in equivalent vertex form? How does this help solve any quadratic equation? Why is the process of re-writing in vertex form called completing the square?**	**Problem 5.2: Combining Profits: Operating with Polynomials I** **Focus Question: How are the sum and difference of two polynomials calculated?**

Problem 1.3: Taxi Fares, Time Payments, and Step Functions **Focus Question: What patterns of change can be modeled by functions called step-functions?**		**Problem 3.3: Sliding Left and Right: Horizontal Translations of Functions** **Focus Question: If graphs of functions are related by sliding left or right, how are the expressions related?**	**Problem 4.3: The Quadratic Formula** **Focus Question: What is the Quadratic Formula, and how do you use it to solve any equation in the form $q(x) = ax^2 + bx + c = 0$**	**Problem 5.3: Product Time: Operating with Polynomials II** **Focus Question: How is the product of two polynomials calculated?**
Problem 1.4: Piecewise defined functions **Focus Question: What patterns of change can be modeled by functions called piecewise defined?**		**Problem 3.4: Horizontal Translations of Functions** **Focus Question: How can you use the algebraic expression for any quadratic function $f(x) = a(x +/- b)^2 +/- c$ to predict the shape and location of the graph?**	**Problem 4.4: Complex Numbers** **Focus Question: How can the real number system be extended to a larger system that includes solutions for all quadratic equations?**	**Problem 5.4: The Factor Game Revisited** **Focus Question: How has your understanding of factors (and products) changed since you last played the factor (and product) game? What ideas about whole number factors are similar to ideas about polynomial factors?**
Problem 1.5: Inverse Functions **Focus Question: What makes one function g(x) the inverse of another function f(x)?**				

How can you find the inverse of a function f(x)?				
Mathematical Reflection: **This investigation was about functions and the ways mathematicians think and write about them.** **a. What is a function?** **b. What are the domain and range of a function?** **c. What does a statement such as f(6) = 23 say about the function f(x)?** **2. a. What is a step function?** **b. Describe what graphs of step functions look like.** **3. a. What is a piecewise defined function?**	**Mathematical Reflection:** **1. a. Describe the defining properties of an arithmetic sequence?** **b. What examples would you give to illustrate the idea for someone?** **2. a. Describe the defining properties of a geometric sequence.** **b. What examples would you give to illustrate the idea for someone?** **3. How are arithmetic and geometric sequences related to linear and exponential functions?**	**Mathematical Reflection:** **1. How will the rule for a function f(x) change if the graph is:** **a. Translated up or down by k?** **b. Stretched away from or toward the x-axis by a factor of k?** **c. Translated left or right by k?** **2. How does the vertex form of a quadratic equation like $f(x) = (x - h)^2 + k$ (where h and k are positive numbers) help to sketch the graph of a function?**	**Mathematical Reflection:** **1. What are the key steps in writing a quadratic expression like $x^2 + 6x + 11$ in vertex form?** **2. How does the Quadratic Formula help to solve equations in the form $ax^2 + bx + c = 0$?** **3. What methods do you have for solving quadratic equations other than the Quadratic Formula?** **4. What are the complex numbers? How are they added, subtracted, and multiplied?**	**Mathematical Reflection:** **1. What are polynomial expressions and functions?** **2. How can one analyze the graph of a polynomial function p(x) to discover** **a. solutions to the equations p(x) = 0** **b. intervals on which values of the function are increasing or decreasing?** **c. points that show relative maximum or minimum values of the function?** **3. What strategies give standard polynomial expressions for** **a. the sum or difference of two polynomials?**

b. Give an example to illustrate this idea. **4. a. When are two functions inverses of each other?** **b. What example would you give to illustrate this idea?**				**b. the product of two polynomials?**