8-8: Function Junction
Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

**Functions** Understand equivalence of algebraic expressions and functions

- Describe domain and range of functions
- Use \( f(x) \) notation to describe and operate with functions
- Construct and interpret inverses of functions
- Analyze function rates of change using graphs
- Identify contexts and graphs of step and piecewise defined functions
- Analyze polynomial functions and their graphs
- Identify, analyze, and solve problems related to arithmetic and geometric sequences
- Compare arithmetic and geometric sequences to linear and exponential functions
- Recognize and solve problems using special kinds of functions

**Equivalence** Understand equivalence of algebraic expressions and functions

- Connect expressions for functions whose graphs are related by translation and/or stretching
- Develop and use vertex form to graph quadratic functions and solve quadratic equations
- Connect polynomial expressions and graphs of the polynomial functions they define, in order to identify max/min points, intercepts, and solutions of equations
- Use completing the square to write quadratics in equivalent vertex form
- Develop the quadratic formula for solving equations
- Develop complex numbers and operations
- Develop algorithms for adding, subtracting, and multiplying polynomials
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<td>FQ: How does the shape of a function graph tell the rate of change in the dependent variable as the independent variable changes?</td>
<td>FQ: What are the defining properties of an arithmetic sequence?</td>
<td>FQ: If graphs of functions are related by sliding up and down, how are the expressions related?</td>
<td>FQ: What strategies allow you to solve quadratic equations algebraically, and how are the algebraic and graphical solutions related to each other?</td>
<td>FQ: What are the patterns of change associated with polynomial expressions and functions of degree 2, 3, and 4, and how are those patterns shown in graphs?</td>
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<td>FQ: What do the terms domain and range tell about a function, and how is f(x) notation used to represent a function?</td>
<td>FQ: What are the defining properties of a geometric sequence?</td>
<td>FQ: If graphs of functions are related by stretching away from or towards the x-axis and/or reflecting across that axis, how are the expressions related?</td>
<td>FQ: How can a quadratic expression be written in equivalent vertex form? How does this help solve any quadratic equation? Why is the process of rewriting in vertex form called completing the square?</td>
<td>FQ: How are the sum and difference of two polynomials calculated?</td>
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<td>FQ: What patterns of change can be modeled by functions called step-functions?</td>
<td>FQ: If graphs of functions are related by sliding left or right, how are the expressions related?</td>
<td>FQ: How can you use the algebraic expression for any quadratic function ( f(x) = ax^2 + bx + c ) to predict the shape and location of the graph?</td>
<td>FQ: What is the Quadratic Formula and how do you use it to solve any quadratic equation in the form ( a(x^2 + bx + c) = 0 )?</td>
<td>FQ: How is the product of two polynomials calculated?</td>
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<td>FQ: What patterns of change can be modeled by functions called piecewise defined?</td>
<td>FQ: What makes one function ( g(x) ) the inverse of another function ( f(x) )? How can you find the inverse of a function ( f(x) )?</td>
<td>FQ: How can you use the Quadratic Formula and how do you use it to solve any quadratic equation?</td>
<td>FQ: How has your understanding of factors (and products) changed since you last played the factor (and product) game? What ideas about whole number factors are similar to ideas about polynomial factors?</td>
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### Mathematical Reflections

1. This investigation was about functions and the ways mathematicians think and write about them.
   1a. What is a function?
   1b. What are the domain and range of a function?
   1c. What does a statement such as \( f(8) = 23 \) say about the function \( f(x) \)?
   1d. What is a step function?
   1e. Describe what graphs of step functions look like.
   1f. What is a piecewise defined function?
   1g. Give an example to illustrate this idea.

2. When are two functions inverses of each other?

3. What example would you give to illustrate the idea for someone?