8-8: Function Junction

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Functions Understand equivalence of algebraic expressions and functions

- Describe domain and range of functions
- Use $f(x)$ notation to describe and operate with functions
- Construct and interpret inverses of functions
- Analyze function rates of change using graphs
- Identify contexts and graphs of step and piecewise defined functions
- Analyze polynomial functions and their graphs
- Identify, analyze, and solve problems related to arithmetic and geometric sequences
- Compare arithmetic and geometric sequences to linear and exponential functions
- Recognize and solve problems using special kinds of functions

Equivalence Understand equivalence of algebraic expressions and functions

- Connect expressions for functions whose graphs are related by translation and/or stretching
- Develop and use vertex form to graph quadratic functions and solve quadratic equations
- Connect polynomial expressions and graphs of the polynomial functions they define, in order to identify max/min points, intercepts, and solutions of equations
- Use completing the square to write quadratics in equivalent vertex form
- Develop the quadratic formula for solving equations
- Develop complex numbers and operations
- Develop algorithms for adding, subtracting, and multiplying polynomials

8-8 Function Junction: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 The Families of Functions	**Investigation 2** Arithmetic and Geometric Sequences	**Investigation 3** Transforming Graphs, Expressions, and Functions	**Investigation 4** Solving Quadratic Equations Algebraically: Completing the Square and Using the Quadratic Formula	**Investigation 5** Polynomial Expressions, Functions, and Equations
Problem 1.1 **Filling Functions** FQ: How does the shape of a function graph tell the rate of change in the dependent variable as the independent variable changes? **Problem 1.2** **Domain, Range, and Function Notation** FQ: What do the terms domain and range tell about a function, and how is f(x) notation used to represent a function? **Problem 1.3** **Taxi Fares, Time Payments, and Step Functions** FQ: What patterns of change can be modeled by functions called step-functions? **Problem 1.4** **Piecewise defined functions** FQ: What patterns of change can be modeled by functions called piecewise defined? **Problem 1.5** **Inverse Functions** FQ: What makes one function g(x) the inverse of another function f(x)? How can you find the inverse of a function f(x)?	**Problem 2.1** **Arithmetic Sequences** FQ: What are the defining properties of an arithmetic sequence? **Problem 2.2** **Geometric Sequences** FQ: What are the defining properties of a geometric sequence?	**Problem 3.1** **Sliding Up and Down: Vertical Translation of Functions** FQ: If graphs of functions are related by sliding up and down, how are the expressions related? **Problem 3.2** **Stretching and Flipping Up and Down: Multiplicative Transformations of Functions** FQ: If graphs of functions are related by stretching away from or towards the x-axis and/or reflecting across that axis, how are the expressions related? **Problem 3.3** **Sliding Left and Right: Horizontal Translations of Functions** FQ: If graphs of functions are related by sliding left or right, how are the expressions related? **Problem 3.4** **Horizontal Translations of Functions** FQ: How can you use the algebraic expression for any quadratic function f(x) = $a(x \pm b)^2 \pm c$ to predict the shape and location of the graph?	**Problem 4.1** **Solving Quadratic Equations Algebraically** FQ: What strategies allow you to solve quadratic equations algebraically, and how are the algebraic and graphical solutions related to each other? **Problem 4.2** **Completing the Square** FQ: How can a quadratic expression be written in equivalent vertex form? How does this help solve any quadratic equation? Why is the process of re-writing in vertex form called completing the square? **Problem 4.3** **The Quadratic Formula** FQ: What is the Quadratic Formula and how do you use it to solve any equation in the form $q(x) = ax^2 + bx + c = 0$? **Problem 4.4** **Complex Numbers** FQ: How can the real number system be extended to a larger system that includes solutions for all quadratic equations?	**Problem 5.1** **Properties of Polynomial Expressions and Functions** FQ: What are the patterns of change associated with polynomial expressions and functions of degree 2, 3, and 4, and how are those patterns shown in graphs? **Problem 5.2** **Combining Profits: Operating with Polynomials I** FQ: How are the sum and difference of two polynomials calculated? **Problem 5.3** **Product Time: Operating with Polynomials II** FQ: How is the product of two polynomials calculated? **Problem 5.4** **The Factor Game Revisited** FQ: How has your understanding of factors (and products) changed since you last played the factor (and product) game? What ideas about whole number factors are similar to ideas about polynomial factors?
Mathematical Reflections 1. This investigation was about functions and the ways mathematicians think and write about them. 1a. What is a function? 1b. What are the domain and range of a function? 1c. What does a statement such as f(6) = 23 say about the function f(x)? 2a. What is a step function? 2b. Describe what graphs of step functions look like. 3a. What is a piecewise defined function? 3b. Give an example to illustrate this idea. 4a. When are two functions inverses of each other? 4b. What example would you give to illustrate this idea?	**Mathematical Reflections** 1a. Describe the defining properties of an arithmetic sequence? 1b. What examples would you give to illustrate the idea for someone? 2a. Describe the defining properties of a geometric sequence. 2b. What examples would you give to illustrate the idea for someone? 3. How are arithmetic and geometric sequences related to linear and exponential functions?	**Mathematical Reflections** 1. How will the rule for a function f(x) change if the graph is: 1a. Translated up or down by k? 1b. Stretched away from or toward the x-axis by a factor of k? 1c. Translated left or right by k? 2. How does the vertex form of a quadratic equation like $f(x) = (x - h)^2 + k$ (where h and k are positive numbers) help to sketch the graph of a function?	**Mathematical Reflections** 1. What are the key steps in writing a quadratic expression like $x^2 + 6x + 11$ in vertex form? 2. How does the Quadratic Formula help to solve equations in the form $ax^2 + bx + c = 0$? 3. What methods do you have for solving quadratic equations other than the Quadratic Formula? 4. What are the complex numbers? How are they added, subtracted, and multiplied?	**Mathematical Reflections** 1. What are polynomial expressions and functions? 2. How can one analyze the graph of a polynomial function p(x) to discover 2a. solutions to the equations p(x) = 0 2b. intervals on which values of the function are increasing or decreasing? 2c. points that show relative maximum or minimum values of the function? 3. What strategies give standard polynomial expressions for 3a. the sum or difference of two polynomials? 3b. the product of two polynomials?