An Analytic Framework for Examining Curriculum-Generated Student Work

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Introduction

The past several decades in mathematics education policy documents have increased attention on supporting students' engagement in cognitively demanding tasks (National Council of Teachers of Mathematics, 1989, 2000, 2014). During the same time, instruction and curriculum materials have increased attention on supporting students to communicate their mathematical ideas with their peers. As a result of this increased attention, mathematics educators have described the learning benefits of students interacting around mathematical ideas by emphasizing problem solving (Hiebert et al., 1996), promoting the quality of classroom discourse (e.g. Ball, 1993; Chapin, O'Connor & Anderson, 2009; Lampert, 2001), and establishing productive norms for interaction in mathematics classrooms (e.g. Yackel & Cobb, 1996). Taken together, these ideas help contribute to a goal of ensuring that mathematics is meaningful and accessible to all students.

For instance, a set of instructional practices that exemplifies the goal of ensuring mathematics that is meaningful and accessible is the *Five Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011). Teachers observe students as they explore an open task, and then thoughtfully select and sequence the strategies that would be useful to share during a whole-class discussion to advance the mathematical goals of the lesson. The driving force of this instructional approach is to leverage student thinking while using student-generated ideas to construct a shared understanding of mathematics within the classroom community. A key aspect of facilitating discussions requires the teacher to have a sense of what strategies are likely to surface. In some cases, strategies may be more likely to occur, but less useful in advancing the mathematical goal. In other cases, unique strategies may be less likely to occur, but quite productive because they explore a nuance or a misconception that can be fruitful for students to develop understanding.

The potential mismatch between the likelihood of a strategy occurring and its relative importance to advancing the mathematical goal of the lesson may result in an instructional obstacle. In this case, the teacher may choose to "seed" student strategies from other classes as a germinating point for discussion. Here, teachers often impose student work that emerged in previous classes or when anticipating student responses. We refer to this source of student work as *teacher-generated student work* (TGSW) as it differs from the student work that is produced by students in the classroom – referred to as *student-generated student work* (SGSW). Yet, another source of student work occurs in mathematics classrooms, namely, *curriculum-generated student work* (CGSW). The purpose of the research is to examine the latter, student work that is embedded in curriculum materials.

Background Literature

Extensive research literature (e.g., Bell, 1993; Lannin, Townsend, & Barker, 2006; Herbel-Eisenmann & Phillips, 2005; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Silver & Suh, 2014) focuses on the pivotal role of using student work to (a) develop teachers' knowledge of mathematics, pedagogy, and assessment, (b) strengthen teachers' instructional practice, and (c) build teacher community around practice-based professional learning. Less attention, however, has been placed on students attending to the process of examining student work and how this practice impacts student learning.

In our work, we differentiate CGSW from TGSW and SGSW in that it appears directly in written curriculum materials. While similarities may exist for providing opportunities for student to engage and discuss mathematics in all three sources of student work, we hypothesize that

CGSW offers qualitatively different opportunities for students. Unlike work that is generated by the teacher or students in the classroom, the author of the work is uniquely positioned as external to the classroom interaction. The purpose of this study is to report on an analytic framework for investigating the student work opportunities that exist in mathematics curriculum materials.

Analytic Framework for Curriculum Generated Student Work

In this section, we describe the analytic framework designed to support researchers in the coding and analysis processes of student work found in mathematics curriculum materials. The analytic framework is composed of three different dimensions relevant for examining student work in mathematics curriculum materials. They include:

1. Location – Exposition and Homework Practice
2. Mathematical Task – Conjectures and Strategies
3. Intended Mathematical Learning Purpose

The dimensions of the framework were drawn from the relevant literature in mathematics education related to SGSW, including work related to error-analysis (e.g. Lannin et al., 2006), distinctions between conceptual and procedural understanding (Hiebert & Lefevre, 1986), and classifications of student work (e.g., methods to solve problems, methods to categorize problems, correct methods used, incorrect methods used, and concepts in methods used) (Rittle-Johnson & Star, 2011). Further elaborations of the different dimensions are discussed later.

We selected three different textbooks that were aligned to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) to represent a range of different types of curricula in how they were developed and their underlying philosophy. The materials are also commonly found in middle school mathematics classrooms. The selected materials include *Big Ideas* (Larson &

Boswell, 2014), *College Preparatory Mathematics* (Dietiker et al., 2013), and *Connected Mathematics3* (Lappan et al., 2014).

Seven researchers coded sections of the curriculum materials (i.e., unit, chapter) that focused on the topic of scaling. We selected the topic of scale drawings/similarity because it was treated in Grade 7 curriculum materials and it provided many instances of curriculum generated student work in the curriculum materials. The group reached consensus on the criteria for which tasks should be coded as student work embedded in curriculum materials. Additional chapters were inspected across all grades to determine how well the developed framework aligned with non-similarity topics. Minor changes to the framework were made as a result of further inspection of additional topics. While the sample size was limited for the topic of scaling/similarity, the additional inspection of topics across the middle grades seemed sufficient for providing an analytic framework for curriculum generated student work in middle school mathematics texts.

In the following sections, we report on the definition for identifying instances of CGSW. An example of a mathematics task and the related codes for all the dimensions are shown in Table 1. Tables 2 and 3 report on the inclusion/exclusion of what counts as CGSW. This is followed by a report on the descriptive and interpretive aspects of the CGSW. Tables 4-6 provide examples for each dimension of the analytic framework.

What Counts as Curriculum Generated Student Work?

An important aspect in the development of the analytic framework was identifying instances of CGSW. We developed three criteria to identify whether a curriculum tasks counts as including student work. Existence of the three criteria indicates a CGSW task that closely reflects what might be generated in the classroom as student work during the course of a discussion. In our criteria, we refer to the "character" as the speaker/author referenced in the curriculum materials and the "reader" as the student in the classroom reading the text. Table 2 shows examples of problems that meet all three criteria. Table 3 shows non-examples that fail at least one of the three criterion. The three criteria of CGSW are:

1. The mathematical task must mention at least one person (the *character*) to which the work is attributed.
2. The task must include a character's thinking or actions or prompt the reader to determine the character's thinking or actions. Thinking might include a written mathematical claim, a conjecture, a strategy, some form of reasoning, an observation or measurement, an algorithm, or a reflection on a mathematical idea.
3. There must be an expected activity for the reader of the text. These activities might include analyzing, critiquing, or reflecting on the mathematical thinking/actions of the character in the written materials.

Dimension 1: Location – Exposition and Homework Practice

The first dimension of the analytic framework focuses on the location of the student work in the curriculum materials. Each lesson of the text contains two well-defined sections: the exposition portion and homework practice. The exposition section refers to the location where students and teachers directly interact with the development of the mathematics expressed in the text. This includes the written activity, a description of the mathematical ideas, any directions to the students, and any surrounding text for the activity. In contrast, the homework practice section is typically located at the end of a lesson, where students interact with problems independently or outside of class time. Most materials demark these sections or provide teachers with support for assigning tasks to be completed at home (i.e., homework practice). We did not examine supplementary materials or assessments in the curriculum programs. Examples of the first dimension are shown in Table 4.

The inclusion of the location dimension underscores the premise that students will have different experiences based where the mathematical task occurs in the text. There is a stronger likelihood that students will engage in the tasks found in the exposition section and that the experience involves collaboration with others. This contrasts with their experience with tasks in the homework practice, where students typically complete mathematical tasks individually. In our work, noted differences in the relative frequency of CGSW in the exposition across the three curricula were found. That is, student work in *Big Ideas* was located in the homework practice, student work in *College Preparatory Mathematics* was primarily located in the exposition, and student work in *Connected Mathematics 3* was located in the exposition and homework practice sections.

Dimension 2: Mathematical Task – Strategies and Conjectures

The second dimension of the analytic framework focuses on the mathematical task (see Table 5 for coding examples). That is, this dimension captures either a stand-alone conjecture or a strategy with support and/or reasoning. In this dimension, *conjectures* refer to mathematical claims – attributed by a character in the problem – that do not include any support or reasoning. S*trategies* refer to a character's written process for solving a problem or /given claim. It is noteworthy that both conjectures and strategies may include claims, however, if a claim is supported then it is considered as a strategy. If it is not supported, then it is considered a conjecture. While the definition for conjectures is somewhat restrictive, it provided a mechanism to discuss the expected activity of the reader. For example, conjectures typically provided the opportunity for the reader to provide support for the claim, whereas strategies provided the opportunity for the reader to analyze the support or reasoning of the character in the task.

The second dimension includes four components for both conjectures and strategies that further describe the reader's experience with the written task. They include:

- The number of conjectures or strategies (one or multiple).
- The validity of the conjecture or strategy in the task is known (valid, not valid) or unknown.
- The conjecture or strategy is explicitly given or at least partially hidden (implicit).
- The type of representations (e.g., table, graph) in the conjecture or strategy.

Examples of the second dimension are shown in Table 5.

The first component for the location dimension is the number of embedded conjectures or strategies in the mathematical task. We included this component because readers are provided with different opportunities when analyzing more than one piece of student work in a mathematical task. For instance, students may be asked to compare and contrast a number of strategies. This differs from tasks where the reader finds or resolves an error involving a single strategy.

The second component is the assumption that a conjecture or strategy is viable, not viable, or unknown to the reader. This component determines if the embedded conjecture or strategy in the mathematical task was explicitly written as valid (or true, correct, makes sense), not valid (or false, incorrect, not make sense), or unknown. The three options for this component provide a different experience for readers when they explore and solve problems.

The third component for this dimension is whether the strategy or conjecture is *given* (explicit for the reader) or at least partially *hidden* (implicit). Most instances of student work in tasks provide a strategy or conjecture that is explicit for the reader to analyze. In contrast, some tasks involve readers having to determine or think about possible conjectures or strategies that were suggested by the character's work (hidden or implicit). It is noteworthy that the inclusion of this component could be viewed as overly-broadening the definition of student work to include all story problems. The crucial difference between standard story problems (where the character is simply acting) and the *hidden* component, is that the prompt in the written materials for problems that are identified as hidden explicitly asks the reader to consider what a character did to solve or reason about a problem.

The fourth component for this dimension is the type of representations included in the student work. The embedded student work may include representations including a graph, a table/numeric representation, symbols, diagram/picture, or a verbal/written representation. Inclusion or exclusion of each representation provides various supports for readers as they engage in the mathematical activity.

Dimension 3: Intended Mathematical Learning Purpose

The third dimension of the analytic framework examines the emphasis on mathematical understanding. Kilpatrick, Swafford, and Findell (2001) suggested that students need conceptual understanding, procedural understanding, strategic competence, adaptive reasoning, and productive disposition to be able to use mathematics to solve new problems. Further, Hiebert and colleagues (1996) suggested developing mathematical understanding involves exposing students to tasks that have problematic scenarios of others, such as mathematics tasks involving student work. Further, they suggested that insight into the structure of mathematics comes out in analyzing procedures and concepts within a mathematical context. In analyzing these problems, a variety of strategies for solving problems emerge, including applying particular procedures and, more deeply, the thought required in using those particular procedures. The deeper thought allows for students to construct strategies and adjust strategies to solve different problems later on. They suggest that “students who treat the development of procedures as problematic must rely on their conceptual understanding to drive their procedural advances. The two necessarily are linked” (Hiebert et al., 1996, p. 17). Therefore, the conceptual and procedural – or the “why” and “how” – of doing and learning mathematics are viewed more as complementary aspects.

Building on this foundation of mathematics learning, we identified three components for describing the intended mathematical learning purpose of the task:

- *Providing new relationships or insight to a mathematical concept or structure*. According to Hiebert and colleagues (1996), “Insights into the structure of the subject matter are left behind when problems involve analyzing patterns and relationships within the subject… In fact, the evidence suggests that young students who are presented with just these kinds of problems and engage in just these kinds of discussions do develop deeper structural understandings” (p. 17).
- *Adapting and constructing strategies to solve problems*. According to Hiebert and colleagues (1996), “By working through problematic situations, students learn how to construct strategies and how to adjust strategies to solve new kinds of problems… students who have been encouraged to treat situations problematically and develop their own strategies can adapt them later, or invent new ones, to solve new problems” (p. 17).
- *Refining or practicing a particular strategy or procedure*. According to Hiebert and colleagues (1996), “The procedures that get left behind depend on the kinds of problems that are solved. These procedures make up the kinds of skills that ordinarily are taught in school mathematics. The evidence suggests that students who are allowed to

problematize arithmetic procedures perform just as well on routine tasks as their more traditionally taught peers” (p. 17).

It is noteworthy that a task identified as curriculum generated student work may fit more than one of the three components. Examples for all three components for the third dimension are shown in Table 6.

Discussion and Conclusion

In this paper, we reported on an analytic framework for investigating the student work that is embedded in curriculum materials and their tasks. The analytic framework was developed to systematically investigate a variety of curricula in middle school mathematics. The analytic framework was composed of three dimensions that examine the location of student work in the texts, the mathematical task (conjectures and strategies), and the intended mathematical learning purpose of the task. Each dimension was discussed and examples were provided.

CGSW may provide opportunities for students to practice analyzing another student’s work or critiquing other students’ reasoning because the nature of the character generating the work being external to the classroom may support students attending to the idea, instead of the person stating the idea. While the character in CGSW is external to the classroom, his/her work may still be viewed as that of a student, as opposed to a traditional authority like a teacher or textbook. This may promote the idea that students can be the generators and creators of mathematics in the classroom. CGSW may offer opportunities for modeling norms for both students and the teacher in the mathematics classroom including conventions for communicating mathematics, appropriate and productive ways to respond to student mistakes and misconceptions, and the importance of valuing multiple approaches towards solving mathematical problems. Likewise, teachers might benefit from CGSW because it may provide representations of ways that students could communicate in their classroom such as looking across strategies to determine the most reasonable approach. Additionally, CGSW may help teachers in planning for anticipated student strategies and possible mistakes or misconceptions by providing examples of hypothetical student thinking in the work of the characters in the problems. Thus, CGSW could prove to be a productive vehicle for improving social and mathematical classroom norms, student-student interactions, student-teacher interactions, and students' and teachers' understanding of mathematics content.

Although a limitation of the research is that the main focus is on the framework for analyzing middle school mathematics texts, there is potential for this analytic framework to be adapted and used for studying student work in curriculum materials in other grades and subjects. This work can be useful for the field of mathematics education to better understand the benefits of student work for teaching and learning mathematics. Future work will involve classroom observations to study whether students engage in student work that is embedded in curriculum materials differently than other curricular tasks. Also, future research is also needed to study whether students learn productive norms for the evaluation and critique of student work that is embedded in curriculum materials and apply these norms to the evaluation and critique of the student work generated by their peers. In addition, future work is also needed to examine how curriculum generated student work is set up by teachers, enacted in the classroom, and is used in assessments.

Table 1

Examples of Tasks and the Analytic Framework Dimensions

<table>
<tr><th>Analytic Framework Dimension</th><th>Mathematical Task</th></tr>
<tr><td>

1. Location – Exposition and Homework Practice

The task is coded as Homework Practice.

2. Mathematical Task – Conjectures and Strategies

The task is coded as Strategies with:

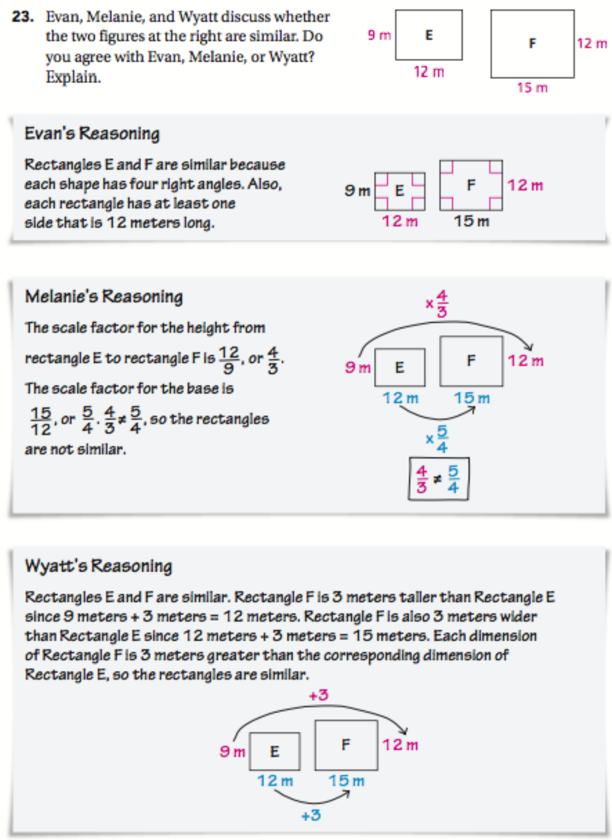
- There are three strategies in the task.
- The validity of the strategies are unknown to the reader.
- The strategies are given (or explicit) to the reader.
- The representations in the strategies include diagram/picture, symbolic and verbal/written.

3. Intended Mathematical Learning Purpose

The mathematical task is coded as refining or practicing a particular strategy or procedure.

- Wyatt's reasoning shows a common error is adding a constant rather than multiplying by a constant to maintain similar shapes. Melanie provides a correct explanation for why they are not similar figures. Evan attends to corresponding angles, but misses the scale factor criteria for similar shapes.

</td><td>



23. Evan, Melanie, and Wyatt discuss whether the two figures at the right are similar. Do you agree with Evan, Melanie, or Wyatt? Explain.

Evan's Reasoning

Rectangles E and F are similar because each shape has four right angles. Also, each rectangle has at least one side that is 12 meters long.

Melanie's Reasoning

The scale factor for the height from rectangle E to rectangle F is $\frac{12}{9}$, or $\frac{4}{3}$. The scale factor for the base is $\frac{15}{12}$, or $\frac{5}{4}$. $\frac{4}{3} \neq \frac{5}{4}$, so the rectangles are not similar.

Wyatt's Reasoning

Rectangles E and F are similar. Rectangle F is 3 meters taller than Rectangle E since 9 meters + 3 meters = 12 meters. Rectangle F is also 3 meters wider than Rectangle E since 12 meters + 3 meters = 15 meters. Each dimension of Rectangle F is 3 meters greater than the corresponding dimension of Rectangle E, so the rectangles are similar.

(Connected Mathematics 3, Grade 7, p. 281)

</td></tr>
</table>

Table 2

Examples of Curriculum Generated Student Work

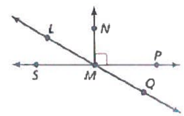
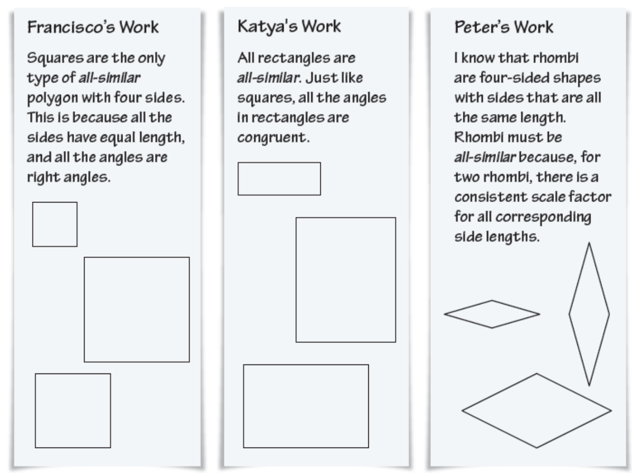
Criteria and Task Evidence	Mathematical Task
The mathematical task meets all three criteria: • The problem involves a person, namely, “your friend.” • The person’s claim is that the angles are complementary. • The expected activity for the reader is to explain if she is correct.	 **24. LOGIC** Your friend says that $\angle LMN$ and $\angle PMQ$ are complementary angles. Is she correct? Explain. (Big Ideas, Grade 7, p. 281)
The mathematical task meets all three criteria: • The problem involves three people. • Each person provides an argument for why the shapes are all-similar or not. • The reader is expected to determine which student’s reasoning is correct.	**50.** Francisco, Katya, and Peter notice that all squares are similar. They wonder if other shapes that have four sides are *all-similar*. Who is correct?  (Connected Mathematics 3, Grade 7, p. 103)

Table 3

Non-Examples of Curriculum-Generated Student Work

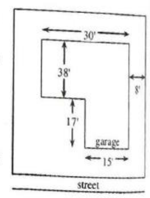
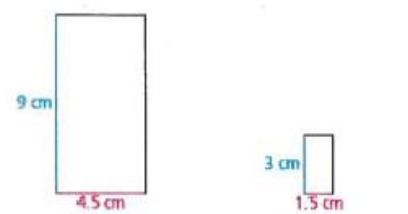
Criteria and Lack of Task Evidence	Mathematical Task
The mathematical task fails the first criteria: • There is no mention of a person (Cr.1). • This problem does mention a strategy (Cr. 2). • The problem and has an expected activity for the reader (Cr. 3).	17. **ERROR ANALYSIS** A scale is 1 cm : 20 m. Describe and correct the error in finding the actual distance that corresponds to 5 centimeters. ✗ $\frac{1 \text{ cm}}{20 \text{ m}} = \frac{x \text{ m}}{5 \text{ cm}}$ $x = 0.25$ m (Big Ideas, Grade 7, p. 303)
The mathematical task fails the second criteria: • The problem does mention a person (Cr. 1). • The thinking is not Guillermo's (Cr. 2). • The problem has an expected activity for the (Cr. 3).	4-12. Guillermo needs a scale drawing of his house placed on its suburban lot. The lot is 56' wide and 100' deep. The garage is 20' back from the street. He has a sketch of his house – *not* drawn to scale – with the measurements shown at right.  a. Use graph paper and a ruler to create a scale drawing of Guillermo's house on its lot. Be sure to state your scale. b. Guillermo wants to put a rectangular swimming pool in his backyard. What is the largest pool you would advise him to have installed? (College Preparatory Math, Grade 7, p. 192)
The mathematical task fails the third criteria: • The expository text mentions a person (Cr. 1). • The text mentions a student's claim (Cr. 2). • The text does not expect the reader to engage in any activity (Cr. 3).	Curtis says that rectangles are similar if the ratios of corresponding adjacent sides within each shape are proportional.  ratio of height to width 9 to 4.5 ratio of height to width 3 to 1.5 $\frac{9}{4.5} = \frac{3}{1.5}$ You need to compare more than just side lengths of polygons to understand their shapes. In this Problem, you will use angle measures and side-length ratios to find similar triangles. (Connected Mathematics 3, Grade 7, p. 84)

Table 4

Location – Exposition and Homework Practice

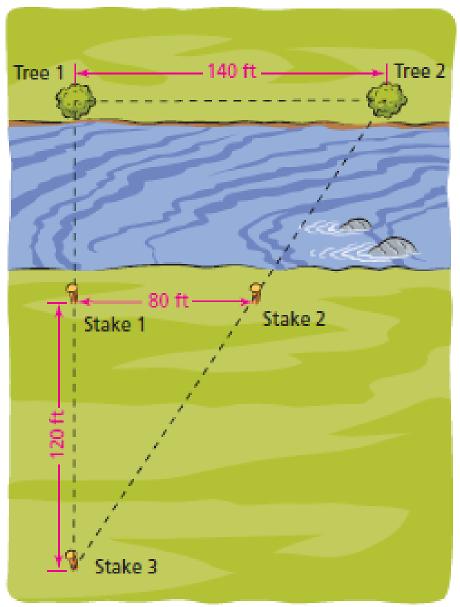
<table>
<tr><th>Dimension 1:
Location – Exposition and Homework Practice</th><th>Mathematical Task</th></tr>
<tr><td>The mathematical task is coded as Exposition.</td><td>

3.4 Out of Reach

Finding Lengths With Similar Triangles

Durell, Angie, and Tonya are designing a triangular boardwalk that crosses a river for a class project. They make several measurements and sketch the diagram below.

The students use similar triangles to find distances that are difficult to measure. The diagram shows a specific type of similar triangles, **nested triangles,** which are triangles that share a common angle.

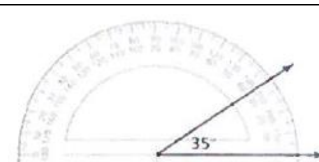


- The angles that look like right angles are right angles. What are the angle measures in each triangle?
- Durell claims that he can use rep-tiles to show that the smaller right triangle is similar to the larger right triangle. Is he correct?

(Connected Mathematics 3, Grade 7, p. 58 – box added for emphasis)

</td></tr>
<tr><td>The mathematical task is coded as Homework Practice.</td><td>

20. LOGIC You are constructing a triangle. You draw the first angle, as shown. Your friend says that you must be constructing an acute triangle. Is your friend correct? Explain your reasoning.



(Big Ideas, Grade 7, p. 287)

</td></tr>
</table>

Table 5

Mathematical Task – Conjectures and Strategies

<table>
<tr><th>Dimension 2:
Mathematical Task –
Conjectures and Strategies</th><th>Mathematical Task</th></tr>
<tr><td>The mathematical task is coded as a stand-alone conjecture because there is no rationale or strategy provided to support the claims.
• There are two conjectures.
• The validity of the conjecture is unknown.
• The conjectures are explicitly given to the reader.
• The representation embedded in the conjecture is verbal/written.</td><td>51. Ernie and Vernon are having a discussion about all-similar shapes. Ernie says that regular polygons and circles are the only types of all-similar shapes. Vernon claims isosceles right triangles are all-similar, but they are not regular polygons. Who is correct? Explain.

(Connected Mathematics 3, Grade 7, p. 103)</td></tr>
<tr><td>The mathematical task (part b) is coded as a strategy, because part B asks the reader to determine a strategy.
• There is one strategy in the task.
• The validity of the strategy is unknown.
• The strategy is hidden (or implicit).
• The representation is tabular/numeric.</td><td>4-48. Lexie claims that she can send 14 text messages in 22 minutes. Her teammates Kenny and Esther are trying to predict how many text messages Lexie can send in a 55-minute lunch period if she keeps going at the same rate.

a. Is the relationship between the number of text messages and time in minutes proportional? Why or why not?

b. Kenny represented the situation using the table shown below. Explain Kenny's strategy for using the table.

Kenny's work<table><tr><th>Messages</th><th>Minutes</th></tr><tr><td>14</td><td>22</td></tr><tr><td>7</td><td>11</td></tr><tr><td></td><td>55</td></tr><tr><td></td><td></td></tr></table>
c. Esther wants to solve the problem using an equation. Help her write an equation to determine how many text messages Lexie could send in any number of minutes.

d. Find the missing value in Kenny's table.

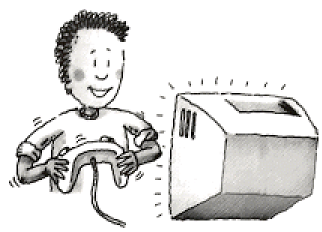
e. Solve Esther's equation. Will she get the same answer as Kenny?

f. What is Lexie's unit rate? That is, how many text messages can she send in 1 minute?

(College Preparatory Mathematics, Course 2, p. 209)</td></tr>
</table>

Table 6

Intended Mathematical Learning Purpose

Dimension 3: Intended Mathematical Learning Purpose	Mathematical Task
The mathematical task is coded as providing new relationships or insight to a mathematical concept or structure. • The reader of the task is asked to reflect on additive reasoning as a way to answer a ratio problem. First the reader is given a solution using subtraction. Then the reader is asked to consider why the subtraction may not work. The reader is being asked to consider the structure of a proportional relationship.	**B** Selena wonders whether a person at a small table or a person at a large table gets more pizza. She uses two ratios, 8 : 3 and 10 : 4, and says *The difference of 10 and 4 is 6. The difference of 8 and 3 is 5. The large table has more people, so the people at the small table will get more pizza.* **1.** Do you agree with Selena's reasoning? Explain. **2.** Tony disagrees with Selena. He says *If you place five pizzas on the large table and three pizzas on the small table, Selena's method would show that the campers at the large table and the campers at the small table get the same amount of pizza.* *If ten people share five pizzas, however, each person gets $\frac{1}{2}$ pizza. That's more pizza than each of the eight people who share three pizzas will get.* Do you agree with Tony's reasoning? Explain. (Connected Mathematics 3, Grade 7, p. 42)
The mathematical task is coded as adapting and constructing strategies to solve problems. • The reader of the task is being challenged to consider how a percentage off might be thought of as a scale factor. The reader must integrate a previous strategy of using a multiplier to scale down with reducing the percentage off of a price. The reader must decide if the percentage discount works as the multiplier or if using the percentage as the scale factor is not the way to find the sale price.	7-20. While shopping for a computer game, Isaiah found one that was on sale for 35% off. He was wondering if he could use $\frac{35}{100}$ as a multiplier to scale down the price to find out how much he would have to pay for the game.  a. If Isaiah uses $\frac{35}{100}$ as a scale factor (multiplier), will he find the price that he will pay for the game? Why or why not? b. There is scale factor (multiplier) other than 35% that can be used to find the sale price. What is it? Draw a diagram to show how this scale factor is related to 35%. Label the parts of your diagram "discount" and "sale price" along with the relevant percents. c. How much will Isaiah have to pay for the game if the original price is $40? Show your strategy. (College Preparatory Mathematics, Course 2, p. 386)

The mathematical task (part b and c) is coded as refining or practicing a particular strategy or procedure.

- The reader is asked to complete a strategy to check an answer. This is an example of the reader being asked to apply a particular procedure for checking the accuracy of a solution to an equation.

6-62. CHECKING YOUR SOLUTION

When you solve an equation that has one solution, you get a value for the variable. But how do you know that you have done the steps correctly and that your answer "works"?

a. Look at your answer for problem 6-61. How could you verify that your solution is correct and convince someone else? Discuss your ideas with your team.

b. When Kelly and Madison compared their solutions for the equation $2x-7=-2x+1$, Kelly got a solution of $x=2$ and Madison got a solution of $x=-1$. To decide whether the solutions were correct, the girls decided to check their answers to see if they made the expressions equal.

Finish their work below to determine whether either girl has the correct solution.

Kelly's Work	Madison's Work
$2x-7\stackrel{?}{=}-2x+1$	$2x-7\stackrel{?}{=}-2x+1$
$2(2)-7\stackrel{?}{=}-2(2)+1$	$2(-1)-7\stackrel{?}{=}-2(-1)+1$

c. When checking, Kelly ended up with $-3=-3$. Does this mean that her answer is correct or incorrect? If it is correct, does this mean the solution is $x=-3$ or $x=2$? Explain.

d. Go back to problem 6-61 and show how to check your solution for that problem.

(College Preparatory Mathematics, Course 2, p. 343)

The mathematical task is coded as providing new relationships or insight to a mathematical concept or structure and as adapting and constructing strategies to solve problems.

- The reader must extend the relationships of finding similar figures to a situation where the unit or measurementchanges from one figure to another. The work of two students is presented to the reader. One student applies a scale factor without attention to the units. The other student other student claims the figures cannot be similar because the units are different. The reader must consider new relationships in similarity while adapting strategies.

18. Triangle A has sides that measure 4 inches, 5 inches, and 6 inches. Triangle B has sides that measure 8 feet, 10 feet, and 12 feet. Taylor and Landon are discussing whether the two triangles are similar. Do you agree with Taylor or with Landon? Explain.

Taylor's Explanation

The triangles are similar. If you double each of the side lengths of Triangle A, you get the side lengths for Triangle B.

OR

Landon's Explanation

The triangles are not similar. Taylor's method works when the two measures have the same units. However, the sides of Triangle A are measured in inches, and the sides of Triangle B are measured in feet. So, they cannot be similar.

(Connected Mathematics 3, Grade 7, p. 94)

The mathematical task is coded as providing new relationships or insight to a mathematical concept or structure and as refining or practicing a particular strategy or procedure.

- The reader must determine if the three students in the task have the correct solution. The first student, Corey, is using the structure of the terms in the equation. This fits the first criteria of providing new relationships or insight to a mathematical concept or structure. The other two students, Hadden and Jackie, use the distributive property and properties of equality to justify their solutions to the equation. These are examples of the third criteria where students are practicing a procedures learned in previous tasks.

B Below are examples of students' solutions the equations from Question A, part (3) above. Is each solution correct? If not, explain what the error is.

$3(x + 1) = 21$

Corry's Solution

3 times something in the parentheses must be 21.
So $3(\quad) = 21$.
The something is 7.
So $x + 1 = 7$, and
$x = 6$.

$2 + 3(x + 1) = 6x$

Hadden's Solution

$2 + 3(x + 1)$ is equivalent to $5(x + 1)$.
So I can rewrite the original equation as $5(x + 1) = 6x$.
Using the Distributive Property, this is the same as $5x + 5 = 6x$.
Subtracting $5x$ from each side, I get $5 = 1x$.
So $x = 5$.

$-2(2x - 3) = -2$

Jackie's Solution

By using the Distributive Property on the left-hand side of the equality, I get $-4x - 6 = -2$.
By adding 6 to each side, I get $-4x = 4$.
By dividing both sides by -4, I get $x = -1$.

(Connected Mathematics 3, Grade 7, p. 65)

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