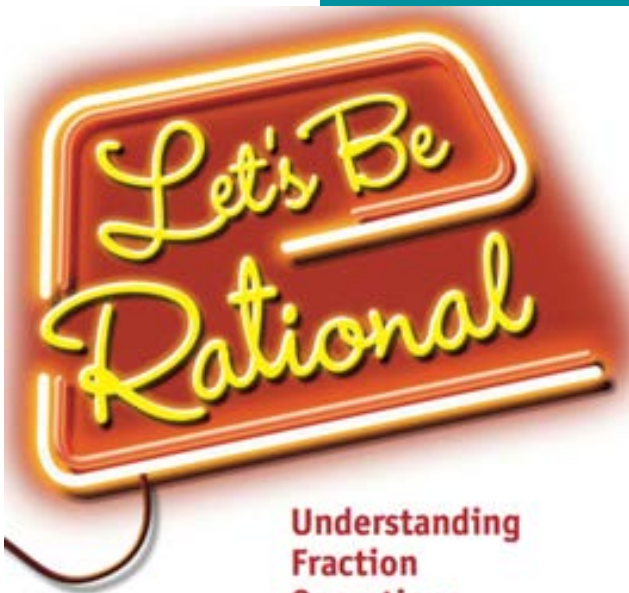




Grade 6 Student Work

Let's Be Rational Problem 3.3



Understanding
Fraction
Operations

3.3 Sharing a Prize

Dividing a Fraction by a Whole Number

At a recent school carnival, teams of students competed in contests. The members of each winning team shared prizes donated by store owners. Sharing the prizes leads to a new kind of division.

What does it mean to divide a fraction by a whole number?

From the Teacher's Guide

Focus Question: What does it mean to divide a fraction by a whole number? What strategies help you divide a fraction by a whole number?

Problem 3.3

- A** Ms. Li gave peanuts as a prize for a relay race. The members of the winning team share the peanuts equally among themselves. What fraction of a pound of peanuts does each team member get in each situation? Use diagrams and number sentences to explain your reasoning.
- Four students share $\frac{1}{2}$ pound of peanuts.
 - Three students share $\frac{1}{4}$ pound of peanuts.
 - Two students share $\frac{3}{4}$ pound of peanuts.
 - Four students share $1\frac{1}{2}$ pounds of peanuts.
- B** Find each quotient and explain how you thought about it.
- $\frac{2}{3} \div 5$
 - $\frac{3}{2} \div 2$
 - $\frac{2}{5} \div 3$
 - $\frac{4}{5} \div 4$
- C** Write a story problem that can be represented by $\frac{8}{3} \div 4$. Explain why the division makes sense.
- D** Describe a strategy for dividing a fraction by a whole number.

The examples of student strategies are from Problem 3.3 Part A #3. In this problem, students are given the following information:

Two students share $\frac{3}{4}$ pound of peanuts. Students are asked to use diagrams and number sentences to determine what fraction of a pound of peanuts each member gets.



Strategy 1

STRATEGY # 1:

Multiply the whole number by the denominator of the fraction. This product becomes the denominator of your answer. Take the numerator of the fraction and make that numerator the numerator of your answer.

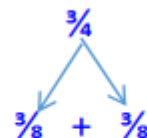
Example: $\frac{3}{4} \div 2 = ?$

$4 \times 2 = 8$ ← This is the denominator of your answer.

Keep the numerator of the fraction as the numerator of your answer (in this case it is 3).

Answer = $\frac{3}{8}$

This works because without changing the numerator, we can just change the size of the pieces to share them.



Now we have two equal size pieces to share.

One share is $\frac{3}{8}$.

If we were sharing with 3 people, as in $\frac{3}{4} \div 3$, we would multiply the denominator by 3, to split the fraction into 3 equal sized pieces to share.

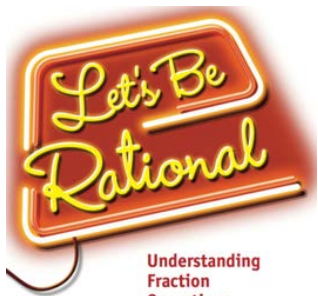
$$\frac{3}{4} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$$

Now we have 3 equal pieces to share. We are changing the size of the piece to accommodate the sharing.

“One share” or the answer would be $\frac{3}{12}$ or $\frac{1}{4}$.

Note that students often develop this strategy by seeing a pattern and are not always able to explain it. This strategy seems to be an intuitive understanding of dividing by a whole number is the same as multiplying by the reciprocal. For example, if a fraction is divided by 2, my share is $\frac{1}{2}$ the amount. If a fraction is divided by 3, my share is $\frac{1}{3}$ the amount.

This strategy works with any fraction divided by a whole number.

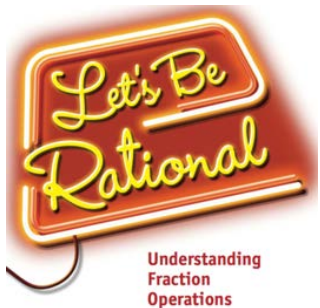


Understanding
Fraction
Operations



Strategy 2

<p>STRATEGY # 2: Take the whole number and make it the denominator of a unit fraction. Multiply this new unit fraction by the fraction in the original problem.</p> <p>Example: $\frac{3}{4} \div 2 = ?$</p> $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$	<p>This works because dividing by 2 is the same as taking $\frac{1}{2}$ of a quantity. In other words, dividing by 2 is the same as multiplying by $\frac{1}{2}$.</p> <p>This strategy works with any fraction divided by a whole number.</p>
---	---



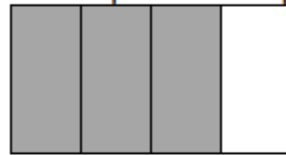


Strategy 3

STRATEGY # 3: Draw a picture...it will be a Brownie Pan Model.

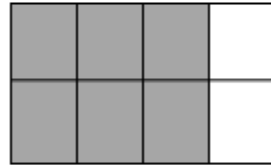
$$\frac{3}{4} \div 2$$

Draw a picture to represent the fraction.



This represents $\frac{3}{4}$

Split each shaded part above, into the number of equal size pieces that the whole number represents.



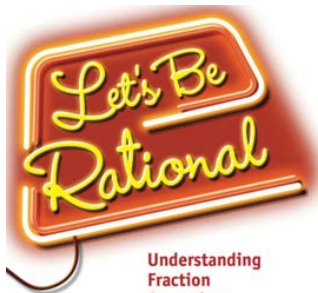
Shade in one share and determine what fraction the shaded part is of the whole.



This represents $\frac{3}{8}$ of the whole.

This looks like the Brownie Pan Model (from Problem 2.1 in the last Investigation) because dividing by 2 is the same as taking $\frac{1}{2}$ of a quantity. In other words, dividing by 2 is the same as multiplying by $\frac{1}{2}$.

This strategy works with any fraction divided by a whole number.



Understanding
 Fraction
 Operations



Strategy 4

STRATEGY #4: Find a fraction that is equivalent to the fraction in the problem whose numerator is divisible by the whole number. Divide the numerator by the whole number and keep the denominator the same.

Example: $\frac{3}{4} \div 2$

$\frac{3}{4} = \frac{6}{8}$ ← Re-write the fraction with a numerator that is divisible by the whole number.

$\frac{6}{8} \div 2 = \frac{3}{8}$

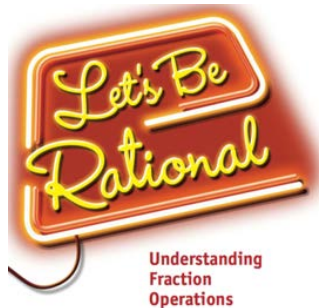
This strategy can be thought of as sharing the pieces equally.

Note that this also works when the numerator does not divide equally. However, students get a fraction in the numerator. So, they have to write an equivalent fraction at the end. Students don't usually like having the fraction in the numerator.

$\frac{3}{4} \div 2 = ?$

$\frac{1\frac{1}{2}}{4} = \frac{3}{8}$

This strategy works with any fraction divided by a whole number.





Strategy 5

STRATEGY # 5: Write the whole number as a fraction with the same denominator as the fraction. Divide the numerators of the fractions to find the answer.

$$\frac{3}{4} \div 2 = ?$$

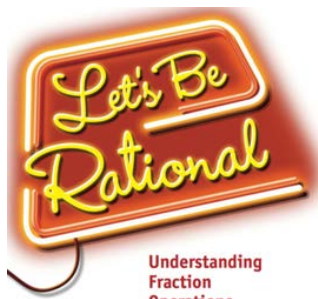
$$\frac{3}{4} \div \frac{8}{4} = ?$$

$$3 \div 8 = \frac{3}{8}$$

This strategy rewrites the whole number as a fraction. When the fractions have the same size pieces (determined by the same denominators), you can divide the number of pieces (the numerators).

In this case you have 3 pieces divided between 8, so the result is $\frac{3}{8}$. Also, this can be thought of as having 3 pieces and it takes 8 to make a group, so we have $\frac{3}{8}$ of a group.

This strategy will work for any fraction division situation.





General Note

Some of the strategies the students generate work for a specific type of fraction situation. Here it is a fraction divided by a whole number. Other strategies will work in any fraction division situation, like strategy #5. It is important to eventually sort-out with students when a strategy is specific and when it is generalizable.

