Connected Mathematics Project

The Arc of Learning Framework: Transparency in Middle School Mathematics Curriculum

Introduction
(Setting the Scene)

- Reveal the mathematical theme (unit/investigation)
- Informally highlight the key mathematical concepts in the Unit
- Assess what students bring to the lesson in terms of the goals of the unit

Exploration
(Mucking About)

- Establish a platform for developing key aspects of the understanding of the concepts and strategies
- Explore (consider) a context that students can use to build, connect, and retrieve mathematical understandings

Analysis
(Going Deeper)

- Make connections between concepts and representations
- Examine nuances in key aspects of the core mathematical ideas often with a variety of contextual situations

Synthesis
(Looking Across)

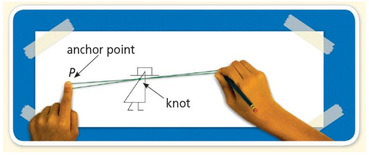
- Recognize core ideas across multiple contextual or problem situations
- Begin to consolidate and refine emerging mathematical understanding(s) into a coherent structure

Abstraction
(Going Beyond)

- Make judgments about which representations, operations, rules, or relationships are useful across various contexts
- Look back on prior learning to generalize, extend, and abstract the underlying mathematical structure
- Assess understandings at a more general level

Examples

1.1 Solving a Mystery
An Introduction to Similarity



Describe how the original figure and the image are alike and how they are different. Compare these features:

- the general shapes of the two figures
- the lengths of the line segments in the hats and bodies
- the areas and perimeters of the hats and bodies
- the angles in the hats and bodies
- the distance of corresponding points on each figure from *P*

Explain each comparison you make. For example, you may find that two lengths are different. Be sure to tell which lengths you are comparing and explain how they are different.

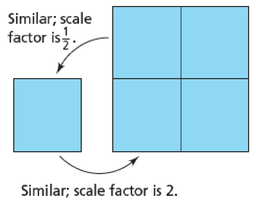
2.1 Drawing Wumps
Making Similar Figures

Coordinates of Game Characters

	Mug Wump	Zug	Lug	Bug	Glug
Rule	(x, y)	(2x, 2y)	(3x, y)	(3x, 3y)	(x, 3y)
Point	Part 1				
A	(0, 1)	(0, 2)			
B	(2, 1)	(4, 2)			
C	(2, 0)				
	Part 2 (Start Over)				
N	(2, 2)				
O	(6, 2)				
	Part 3 (Start Over)				
S	(3, 4)				
T	(4, 5)				
U	(5, 4)				
V	(3, 4)				
	Part 4 (Start Over)				
W	(2, 5) (make a dot)				
X	(6, 5) (make a dot)				

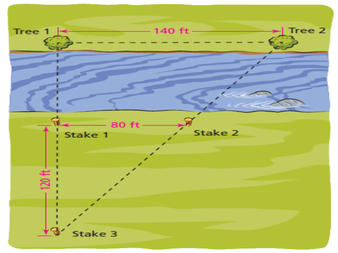
1. Compare the characters to Mug. Which are the impostors (*not* members of the Wump family)?
2. What things are the same about Mug and the others?
3. What things are different about the five characters?

3.1 Rep-Tile Quadrilaterals
Forming Rep-Tiles With Similar Quadrilaterals



1. Extend the rep-tile patterns you drew for Question A. Do this by sketching additional copies of the original figure to make even larger figures that are similar to the original. Show how the copies fit together.
2. Find the scale factor from each original figure to each new figure. Explain your reasoning.
3. What do the scale factors tell you about the corresponding side lengths, perimeters, angles, and areas?

3.4 Out of Reach
Finding Lengths With Similar Triangles



The triangles in the diagram on the previous page are similar. What is the distance across the river from Stake 1 to Tree 1? Explain your reasoning.

Describe the relationship between the perimeter of the smaller triangle and the perimeter of the large triangle.

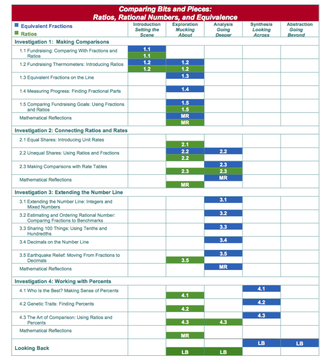
Mathematical Reflections 4

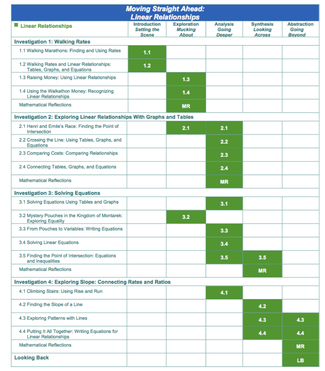
In this Investigation, you used ratios to describe and compare the size and shape of rectangles, triangles, and other figures. The following questions will help you summarize what you have learned.

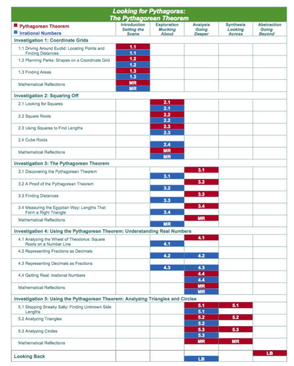
Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. If two triangles, rectangles, or parallelograms are similar,
   a. **How** does the ratio of two side lengths within one figure compare to the ratio of the corresponding side lengths in the other figure?
   b. **What** does the scale factor from one figure to the other tell you about the figures?
2. a. **Describe** at least two ways to find a missing side length in a pair of similar figures.
   b. **How** can you find the height of an object that cannot be measured directly?
3. **What** does it mean to say that two shapes are similar? After exploring with ratios, build on your statements from Mathematical Reflections 1, 2, and 3:

"Two geometric shapes are similar when. . ."







RESEARCH OVERVIEW

RATIONALE:

- Curriculum should be coherent and focused on important mathematics (NCTM, 2000)
- Coherence should attend to developing important mathematical ideas over time (NGA-CCSSO, 2010).
- A need to create greater transparency to teachers about curriculum. (Remillard, 2000)

RESEARCH PURPOSE

- To investigate what is meant by coherence when developing a single big idea.
- Examine how each problem contributes to a connected and coherent development of mathematical concepts.

RESEARCH QUESTION:

- How does the development of intended understanding of key mathematical concepts and methods in middle school curriculum materials evolve within sequences of problems?

IMPLICATIONS FOR PRACTICE

The Arc of Learning helps teachers to

- Recognize the complexities of understanding of a concept
- Understand how student thinking and learning might unfold within a unit and across units
- Understand how student thinking and learning is targeted within a problem in a sequence
- Plan, enact, and reflect on instruction with a guiding framework for discussing with colleagues the depth of student conceptual understanding

IMPLICATIONS FOR RESEARCH

The Arc of Learning informs the study of

- Curriculum design research
- Teacher planning and enactment in the classroom
- Enhancing teachers' understanding of the development of long-term mathematical goals as supported by the daily lessons
- The mathematical, pedagogical, and assessment decisions teachers make when planning or enacting lessons that respond to students' mathematical conceptions

FURTHER QUESTIONS

- To what extent do differences occur within and across mathematical strands?
- What differences occur between the intended and observed AoL as students solve problems?
- What are ways that teachers use (support, influence) the AoL in planning, teaching, assessing and reflecting?
- Particularly is there a difference in how experienced teachers and novice teachers use the AoL?
- How does classroom discourse change at various points in the AoL?
- How can the AoL support professional development settings?

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