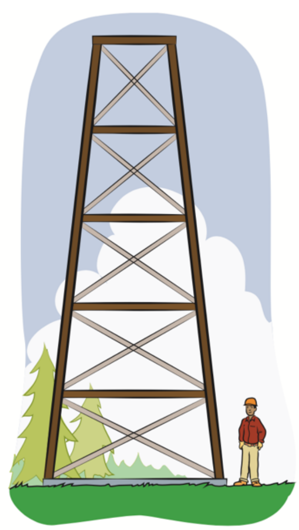
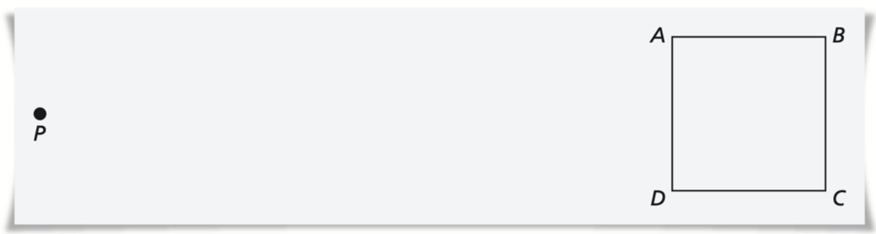
STRETCHING AND SHRINKING
Applications-Connections-Extensions
With Answers & Problem Correlations

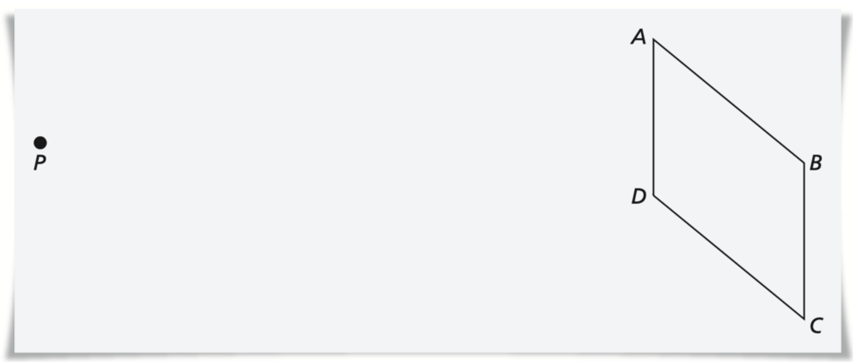
Investigation1

	Applications	Connections	Extensions	Total
1.1	2	4	1	7
1.2	2	2	2	6
1.3	3	2	1	6
Total	7	8	4	19

Applications

Problem #	Answer	CMP4 Problem #	Note
For Exercises 1 and 2, use the drawing at the right, which shows a person standing next to a ranger's outlook tower. 			
1	Find the approximate height of the tower if the person is a. 6 feet tall 30 ft b. 5 feet 6 inches tall	1.1	

	27 ft 6 in.		
2	Find the approximate height of the person if the tower is a. 28 feet tall Approximately 5ft 7 in. b. 36 feet tall Approximately 7ft $2\frac{1}{2}$ in.	1.1	
3	Copy square ABCD and anchor point P onto a sheet of paper. Use the rubber-band method to enlarge the figure. Then, answer parts (a)-(e) below.  a. How do the side lengths of the original figure compare to the side lengths of the image? The original lengths are half the new lengths. Or the lengths are 2 (scale factor) times the original lengths. b. How does the perimeter of the original figure compare to the perimeter of the image? The perimeter of the original figure is half the perimeter of the new figure. Or the perimeter of the new figure is 2 (scale factor) times the original perimeter. c. How do the angle measures of the original compare to the angle measures of the image? Angles remain the same, 90°. d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image?	1.2	

	The area of the original figure is ¼ the area of the new image. The area of the new figure is 4 times the original area. It takes 4 copies of the original figure to cover its stretched image. e. How does the distance between each point in the original figure and P compare to the corresponding distances in the image? The distance of each point in the original figure from P is half the distance of the corresponding point in the image. Or, each image point is twice the distance from P as its corresponding point on the original figure.		
4	Copy parallelogram ABCD and anchor point P onto a sheet of paper. Use the rubber-band method to enlarge the figure. Then, answer parts (a)-(e) from Exercise 3 for your diagram.  a. How do the side lengths of the original figure compare to the side lengths of the image? The original lengths are half the new lengths. Or the lengths are 2 (scale factor) times the original lengths. b. How does the perimeter of the original figure compare to the perimeter of the image? The perimeter of the original figure is half the perimeter of the new figure. Or the perimeter of the new figure is 2 (scale factor) times the original perimeter. c. How do the angle measures of the original compare to the angle measures of the image? Angles remain the same.	1.2	

<table>
<tr>
<td></td>
<td>d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image?

The area of the original figure is ¼ the area of the new image. The area of the new figure is 4 times the original area. It takes 4 copies of the original figure to cover its stretched image.

e. How does the distance between each point in the original figure and P compare to the corresponding distances in the image?

The distance of each point in the original figure from P is half the distance of the corresponding point in the image. Or, each image point is twice the distance from P as its corresponding point on the original figure.</td>
<td></td>
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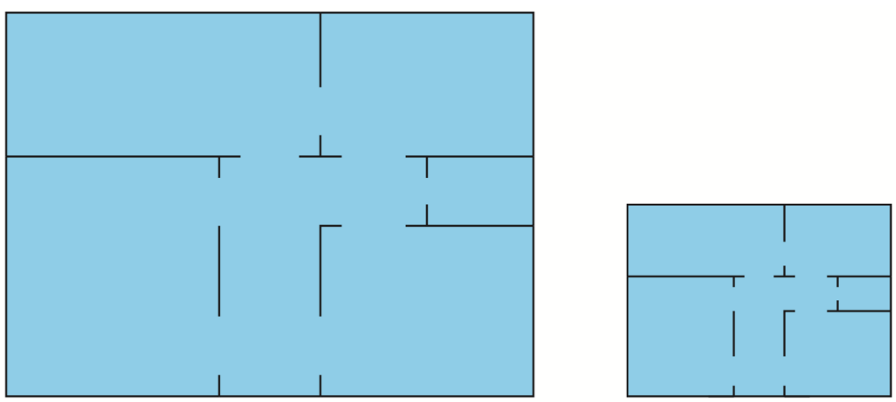
<tr>
<td>5</td>
<td>The diagram on the left is the floor plan for a model house. The diagram on the right is a scale drawing of the floor plan. The scale drawing was made by reducing the original on a copy machine.


Original   Reduced Image

a. Estimate the copier size factor used. Give your answer as a percent.

50%; Students can use a side of a piece of paper to compare the side lengths of the floor plan.

b. How do the segment lengths in the original plan compare to the corresponding segment lengths in the reduced image?

The line segment in the original plan are twice the lengths of the corresponding sides in the reduced image. Or, the line</td>
<td>1.3</td>
<td></td>
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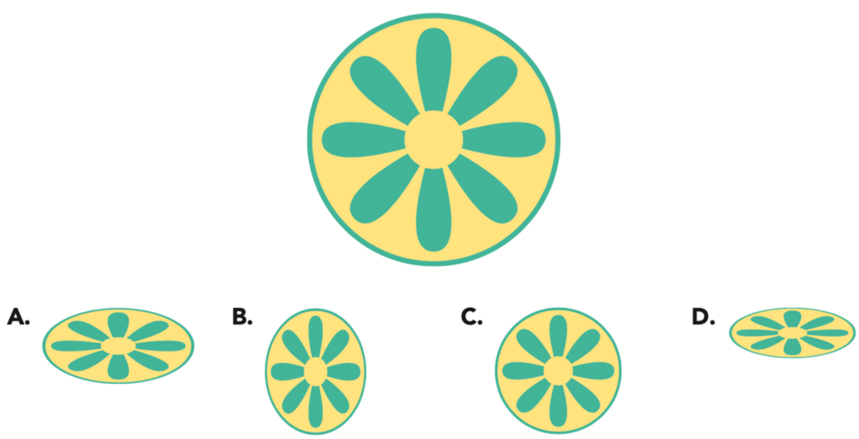
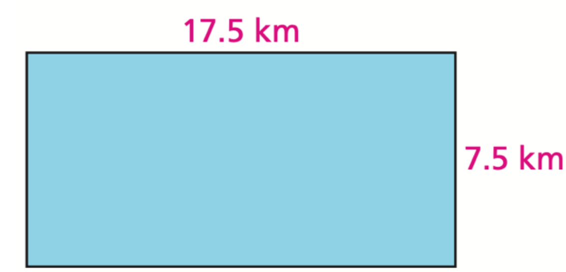
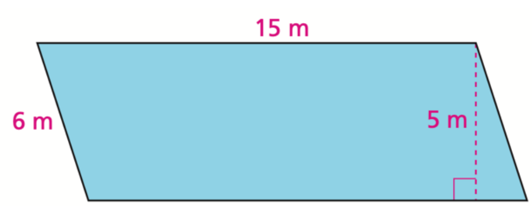
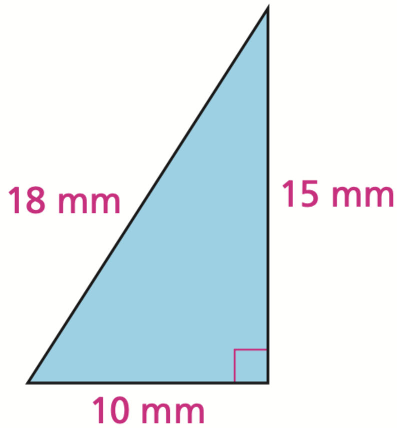
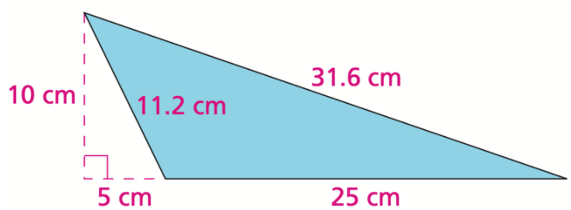
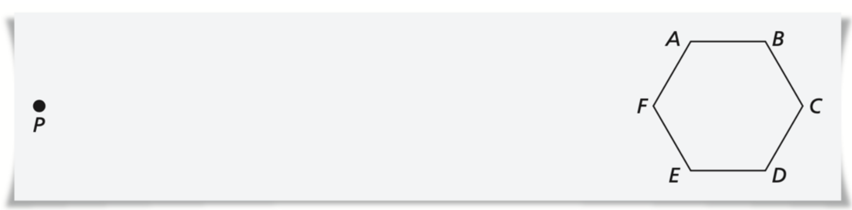
</table>

	segments in the reduced image are half as long as the corresponding line segments in the original plan. c. Compare the area of the entire original floor plan to the area of the entire reduced image. Then, do the same with one room in the plan. Is the relationship between the areas of the rooms the same as the relationship between the areas of the whole plans? Explain. The area of the whole house in the original plan is about 4 times the area of the reduced image. The relationship between a room in the original plan and in the reduced image is the same as the relationship between the whole plans. d. The scale on the original plan in 1 inch = 1 foot. This means that 1 inch on the floor plan represents 1 foot on the model house. What is the scale on the reduced plan? 1 inch represents 2 ft		
6	**Multiple Choice** Suppose you reduce the design below with a copy machine. Which of the following can be the image?  C; Its diameters have a ratio of 2 to 1 from the original to the image.	1.3	
7	Suppose you copy a drawing of a polygon using the given size factor. How will the side lengths, angle measures, and perimeter of the image compare to those of the original? a. 200% Angle measures do not change. The side lengths and the	1.3	

	perimeter are 2 times as long. b. 150% Angle measures do not change. The side lengths and the perimeter are 1.5 times as long. c. 50% Angle measures do not change. The side lengths and the perimeter are 1/2 times as long. d. 75% Angle measures do not change. The side lengths and the perimeter are 3/4 times as long.		

Connections

Problem #	Answer	CMP4 Problem #	Note
For Exercises 8-11, find the perimeter and the area of each figure. In Exercises 10 and 11, the measurements are rounded.			
8	 The perimeter = 50 km, and the area = 131.25 km²	1.1	
9	 The perimeter = 42 m, and the area = 75 m²	1.1	

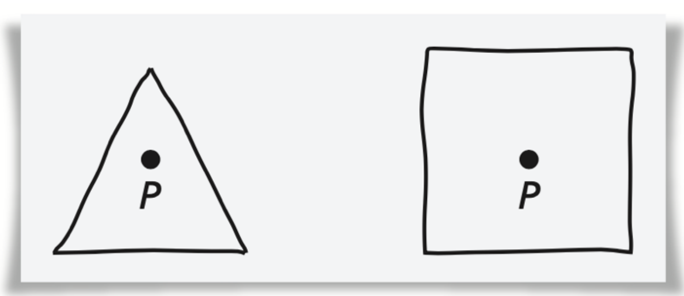
10	 The perimeter = 43 mm, and the area = 75 mm^2	1.1	
11	 The perimeter = 67.8 cm, and the area = 125 cm^2	1.1	
12	Copy hexagon ABCDEF and anchor point P onto a sheet of paper. Make an enlargement of the hexagon using your two-band stretcher.  a. How do the side lengths of the two hexagons compare? The side lengths of the image hexagon are 2 times as long as the side of the original hexagon. b. How do the angles of the hexagons compare? The angles of the two hexagons are the same. c. How do the areas of the hexagons compare?	1.2	

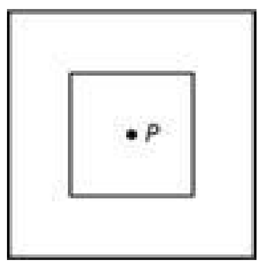
	The area of the image hexagon is 4 times as big as the area of the original hexagon. d. How do the perimeters of the hexagons compare? The perimeter of the image hexagon is 2 times as long as the perimeter of the original hexagon.		
13	Make a three-band stretcher by tying three rubber bands together. Use this stretcher to enlarge the "Super Sleuth" drawing from Problem 1.1. Note that there are two possible interpretations of this Exercise. Most students will use the knot closer to the anchor point to trace the original figure. Some students may use the knot closer to the pencil. Both answers are given. Answers for the knot closer to the anchor point are listed first. a. How does the shape of the image compare to the shape of the original figure? The shapes are similar to each other. b. How do the lengths of the segments in the two figures compare? The lengths in the image figure are 3 times as long as the lengths in the original figure. (Or the lengths in the image figure are 1.5 times as long as the lengths in the original figure.) c. How do the areas of the two figures compare? The areas in the image figure are 9 times as big as the areas in the original figure. (Or the areas in the image figure are 2.25 times as long as the areas in the original figure. Students may estimate and say that the areas in the image are a little more than twice those in the original.) d. How do the distances from *P* compare? Each point on the image is 3 times as far from *P* as its corresponding point on the original figure. (Or each point on the image is 1.5 times as far from *P* as its corresponding point on the original figure.)	1.2	
14	**Multiple Choice** What is the 28% tax on a $600,000 cash prize?	1.3	

	A. $16,800 B. $21.429 C. $168,000 D. $214,290 C; 28/100 x 600,000 = 28 x 6,000 = 168,000.		
15	While shopping for sneakers, Ling finds two pairs she likes. One pair costs $55 and the other costs $165. She makes the following statements about the prices. “The expensive sneakers cost $110 more than the cheaper sneakers.” “The cost of the expensive sneakers is 300% of the cost of the cheaper sneakers.” “The cheaper sneakers are 1/3 the cost of the expensive sneakers.” a. Are all statements accurate? Explain. All the statements are accurate. Sample explanation: The first statement gives the difference between the costs of the two pairs of shoes. The second statement uses a percent comparison. The last statement uses a fraction to compare the two costs. Even though the statements are different, they are each accurate way to compare items. b. How are the comparison methods Ling uses like the methods you use to compare the sizes and shapes of similar figures? Answers will vary. One can use similar statement in comparing sizes of shapes (the length is 300% larger than the original; the length is 1/3 the length of the original, etc.) c. Which statements are appropriate for comparing the size	1.3	

	and shape of an image to the original figure? Explain. Answers will vary. Sample answer: The second and third statements are most appropriate because they compare the two quantities in terms of percent and scale factors.		

Extensions

Problem #	Answer	CMP4 Problem #	Note
16	A movie projector that is 6 feet away from a large screen shows a rectangular picture that is 3 feet wide and 2 feet high. a. Suppose the projector is moved to a point 12 feet from the screen. What size will the picture be (width, height, and area)? The width and height would be 2 times as large as the first picture. Width = 6ft, height = 4ft, and area = 24ft² b. Suppose the projector is moved to a point 9 feet from the screen. What size will the picture be (width, height, and area)? The width and height would be 1.5 times as large as the first picture. Width = 4.5ft, height = 3ft, and area = 13.5ft²	1.1	
17	Suppose you enlarge some triangles and squares with a two-band stretcher. You use an anchor point inside the original figure, as shown in the sketches below.  a. In each case, how do the shape and position of the image compare to the shape and position of the original?	1.2	

	The size of the image would still be the same as in the case when the anchor point is outside. However, in this case, the image figure would enclose the original figure. b. What relationships do you expect to find among the side lengths, angle measures, perimeters, and areas of the figures, and the distances from P? The lengths of sides and perimeters would be 2 times as long as the original figure. Angle measures would not change. Area would be 4 times as big as the area of the original figure. The distance each image point from P is twice the distance its corresponding point is from P. c. Test your ideas with larger copies of the given shapes. Make sure the shortest distance from the anchor point to any side of a shape is at least one band length. Answers will vary. One possibility is shown.  		
18	Suppose you make a stretcher with two different-sized rubber bands. Suppose the band attached to the anchor point is twice as long as the band attached to the pencil. a. If you used the stretcher to enlarge polygons, what relationships would you expect to find among the side lengths, angle measures, perimeters, and areas of the figures? The lengths are 1.5 times as long as the original figure. Angle measures do not change. The perimeter is 1.5 times as long as the original figure. The Area would be 1.5 x 1.5 = 2.25 times as large as the original figure. b. Test your ideas with copies of some basic geometric shapes.	1.2	

	Answers will vary.		
19	Amy's friend gave her a picture from Field Day. The picture is 3 in. by 2 in. Amy has a picture frame that is 6 in. by 4 in. She wants the photo to fit in the frame exactly. What percent enlargement does she need to make?  To use her picture frame for the picture, Amy will have the picture enlarged to 200%. It is because the scale factor is 2.	1.3	

Investigation 2

	Applications	Connections	Extensions	Total
2.1	3	3	1	7
2.2	4	3	3	10
2.3	6	4	3	13
Total	13	10	7	30

Applications

Problem #	Answer	CMP4 Problem #	Note
1	The table below gives key coordinates for drawing Mug Wump's mouth and nose. It also gives rules for finding the corresponding points for four other characters-some members of the Wump family and some imposters.	2.1	

Coordinates of Characters

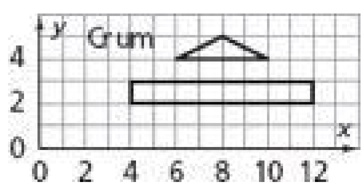
	Mug Wump	Glum	Sum	Tum	Crum
Rule	**(x, y)**	**(1.5x, 1.5y)**	**(3x, 2y)**	**(4x, 4y)**	**(2x, y)**
Point	**Mouth**				
M	(2, 2)	■	■	■	■
N	(6, 2)	■	■	■	■
O	(6, 3)	■	■	■	■
P	(2, 3)	■	■	■	■
Q	(2, 2) (connect *Q* to *M*)	■	■	■	■
	Nose (Start Over)				
R	(3, 4)	■	■	■	■
S	(4, 5)	■	■	■	■
T	(5, 4)	■	■	■	■
U	(3, 4) (connect *U* to *R*)	■	■	■	■

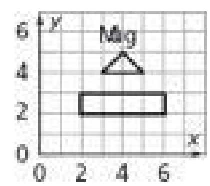
a. Before you find the coordinates or plot points, predict which characters are the imposters.

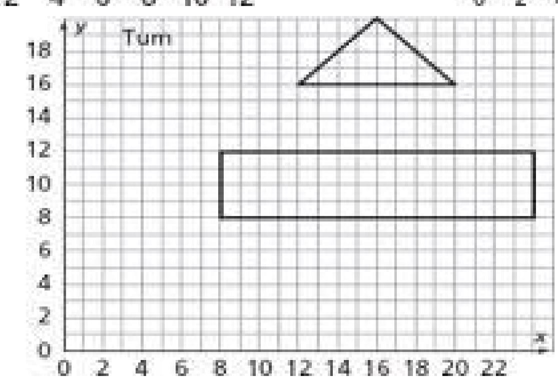
Sum and Crum are imposters. They have different coefficients for x and y.

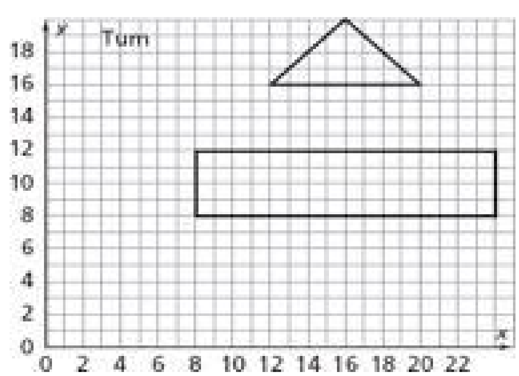
b. Copy and complete the table. Then, plot the figures on grid paper. Label each figure.

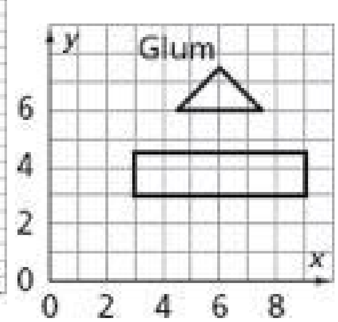
	Mug Wump	Glum	Sum	Tum	Crum
Rule	**(*x*, *y*)**	**(1.5*x*, 1.5*y*)**	**(3*x*, 2*y*)**	**(4*x*, 4*y*)**	**(2*x*, *y*)**
Point	**Mouth**				
M	(2, 2)	(3, 3)	(6, 4)	(8, 8)	(4, 2)
N	(6, 2)	(9, 3)	(18, 4)	(24, 8)	(12, 2)
O	(6, 3)	(9, 4.5)	(18, 6)	(24, 12)	(12, 3)
P	(2, 3)	(3, 4.5)	(6, 6)	(8, 12)	(4, 3)
Q	(2, 2)	(3, 3)	(6, 4)	(8, 8)	(4, 2)
	Nose				
R	(3, 4)	(4.5, 6)	(9, 8)	(12, 16)	(6, 4)
S	(4, 5)	(6, 7.5)	(12, 10)	(16, 20)	(8, 5)
T	(5, 4)	(7.5, 6)	(15, 8)	(20, 16)	(10, 4)
U	(3, 4)	(4.5, 6)	(9, 8)	(12, 16)	(6, 4)





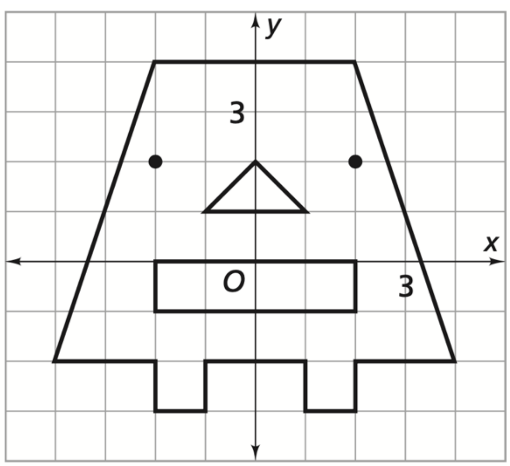






c. Which of the new characters (Glum, Sum, Tum, and Crum) are members of the Wump family? Which are imposters?

	Glum and Tum are members. Sum and Crum are imposters. d. Choose one of the new Wumps. How do the mouth and nose measurements (side lengths, perimeter, area, angle measures) compare with those of Mug Wump? For Glum: Mouth lengths, nose lengths, and perimeters are 1.5 times as long as the corresponding lengths of Mug. The angles are the same. The areas are 2.25 times as large. For Tum: Mouth lengths, nose lengths, and perimeters are 4 times as long as the corresponding lengths and perimeter of Mug. The angles are the same. The areas are 16 times as large. The dimensions of the mouth are 16 units by 4 units and the nose has a width of 8 units and a height of 4 units. e. Choose one of the imposters. What are the dimensions of this imposter's mouth and nose? How do the mouth and nose measurements compare with those of Mug Wump? For Sum: The height of the mouth and the height of the nose are 2 times as long while the width of the mouth and width of the nose are 3 times as long as the corresponding lengths of Mug. The mouth is 12 units wide and 2 units high and the nose is 6 units wide and 2 units high. For Crum: The heights of the mouth and the nose are the same as the corresponding heights of Mug. The widths of the mouth and the nose are 2 times as long as the corresponding widths of Mug. f. Do your findings in parts (b)-(e) support your prediction from part (a)? Explain. Yes, the findings support the prediction that the imposters will be Sum and Crum. Imposters are those who have different scale factors applied to both the x- and y- coordinates, while family members have the same scale factor applied.		
2	a. Design a Mug-like character of your own grid paper. Give your character eyes, a nose, and a mouth. Answers will vary.	2.1	

	b. Give coordinates so that someone else could draw your character. Answers will vary. c. Write a rule for finding coordinates of a member of your character's family. Check your rule by plotting the figure. The rule is that one should multiply both x and y-coordinates by the same number k: (kx, ky). d. Write a rule for finding the coordinates of an imposter. Check your rule by plotting the figure. Choose different numbers multiplying the x- and y- coordinates: (kx, ry), where k is not equal to r.		
3	The diagram below shows Mug Wump drawn on a coordinate grid. Use this diagram to answer the questions.  a. Use the diagram, complete the first column of a table like the one shown to record coordinates of key points needed to draw Mug. (You will need to determine the number of points needed for each body part.)	2.1	

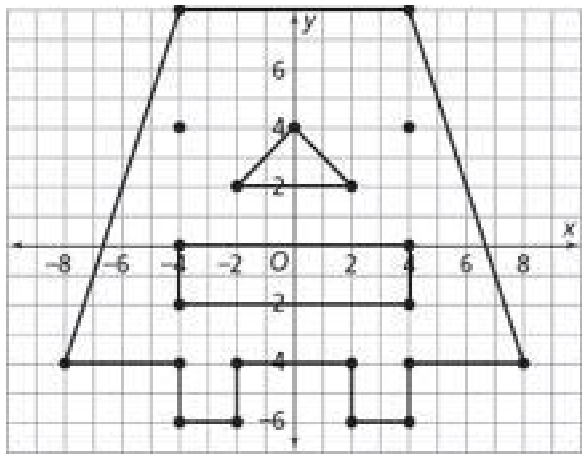
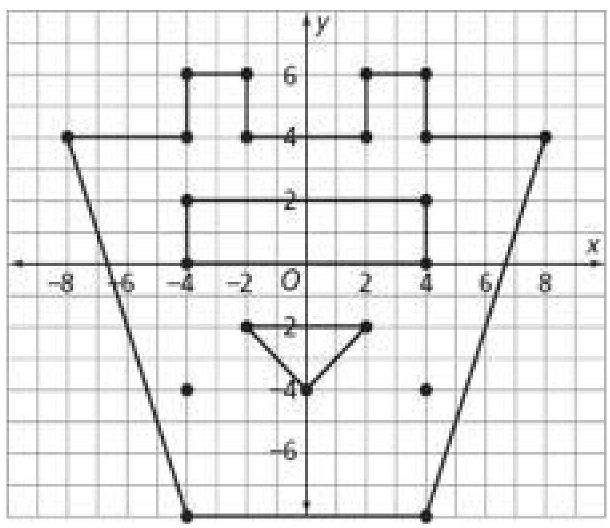
Coordinates for Mug and Variations

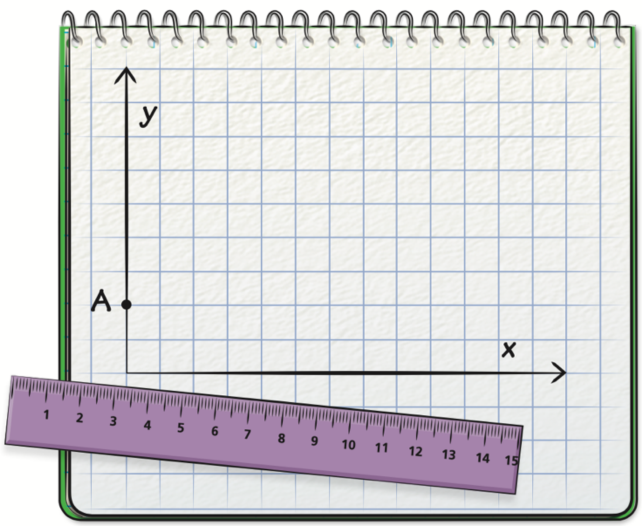
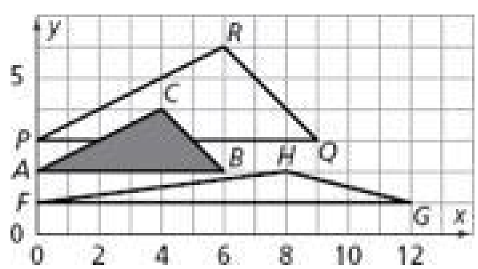
Rule	(x, y)	(2x, 2y)	(−2x, −2y)
Head Outline	(−4, −2)	■	■
	(−2, −2)	■	■
	(−2, 3)	■	■
	■	■	■
	■	■	■
Nose	(−1, 1)	■	■
	■	■	■
	■	■	■
Mouth	(−2, −1)	■	■
	■	■	■
	■	■	■
	■	■	■
Eyes	(−2, 2)	■	■
	■	■	■

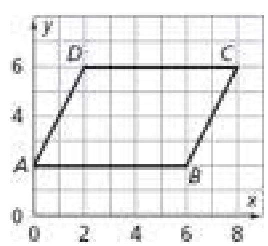
See Table below.

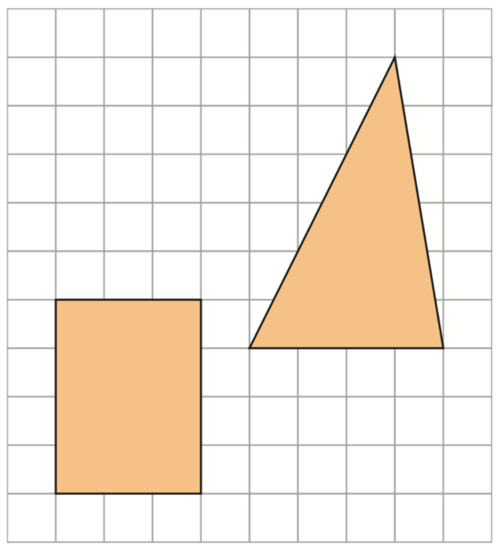
b. Suppose you make scale drawings with rules (2x, 2y) and (-2x, -2y)

Rule	(x, y)	(2x, 2y)	(–2x, –2y)
Head Outline	(–4, –2)	(–8, –4)	(8, 4)
	(–2, –2)	(–4, –4)	(4, 4)
	(–2, –3)	(–4, –6)	(4, 6)
	(–1, –3)	(–2, –6)	(2, 6)
	(–1, –2)	(–2, –4)	(2, 4)
	(1, –2)	(2, –4)	(–2, 4)
	(1, –3)	(2, –6)	(–2, 6)
	(2, –3)	(4, –6)	(–4, 6)
	(2, –2)	(4, –4)	(–4, 4)
	(4, –2)	(8, –4)	(–8, 4)
	(2, 4)	(4, 8)	(–4, –8)
	(–2, 4)	(–4, 8)	(4, –8)
	(–4, –2)	(–8, –4)	(8, 4)

	c. On graph paper, plot the images of Mug Wump produced by the new sets of coordinates in part (b).   d. Compare the length, width, and area of Mug's mouth to those of the figures drawn in part (c). Explain how you could have predicted those results by studying the coordinate rules for the drawings. In both diagrams, the length of Mug's new mouth is 8 units, twice as long as Mug's original mouth. The width of Mug's new mouth is 2 units, twice as wide as Mug's original mouth. The area of Mug's new mouth is 16 square units, 4 times greater than that of Mug's original mouth.			

	The coefficients of x and y correspond to the scale factor from the original diagram of Mug to both images. The square of the coefficients gives the factor by which the area is increased. All these comparisons are the same for both images because the second diagram is simply a reflection of the first. A reflection preserves measurements, such as length and area.		
4	a. On grid paper, draw triangle ABC with vertex coordinates A(0, 2), B(6, 2), and C(4, 4).  Answer for a, b, c drawings  b. Apply the rule (1.5x, 1.5y) to the vertices of triangle ABC to get triangle PQR. Compare the corresponding measurements (side lengths, perimeters, areas, angle measures) of the two triangles. The side lengths and perimeter of triangle PQR are 1.5 times the side lengths and perimeter of triangle ABC. The angle	2.2	

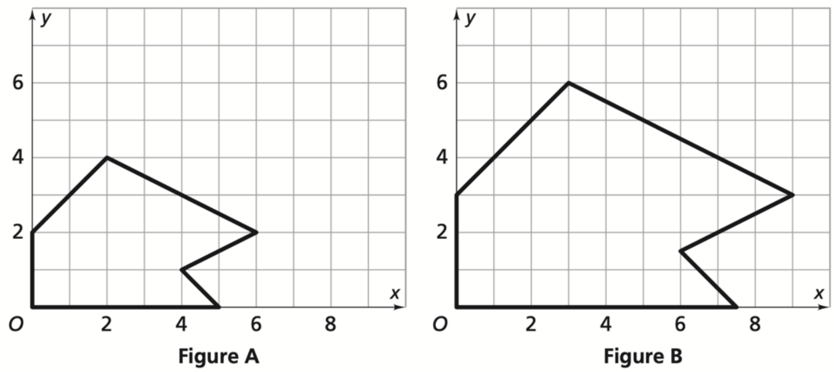
	measures of triangles ABC and PQR are the same. The area of triangle PQR is 2.25 times (the scare factor squared) the area of triangle ABC. **Note**: At this point, students may still use area formulas to calculate each area and then compare the results. The area of triangle ABC is 6 square units, and the area of triangle PQR is 13.5 square units. 13.5 is equal to 2.25 x 6. c. Apply the rule (2x, 0.5y) to the vertices of triangle ABC to get triangle FGH. Compare the corresponding measurements (side lengths, perimeters, areas, angle measures) of the two triangles. In comparing triangle ABC to triangle FGH, the side lengths of triangle FGH grew by different-size scale factors. Therefore, the perimeter of triangle FGH did not grow by the same scale factor as the side lengths, and the angle measures are not the same. Finally, the area of triangle FGH is the same as the area of triangle ABC. **Note:** Doubling the base and halving the height makes the areas equal. d. Which triangle, PQR or FGH, seems similar to triangle ABC? Why? Triangle PQR is similar to triangle ABC since the corresponding lengths are enlarged by the same factor.		
5	a. On grid paper, draw parallelogram ABCD with vertex coordinates A(0, 2), B(6, 2), C(8, 6), and D(2, 6). Answer:  b. Write a rule to find the vertex coordinates of a parallelogram PQRS that is smaller than, but similar to, ABCD. Test your rule to see if it works.	2.2	

	Choose any positive number *s* less than 1. The rule is (*s*x, *s*y). Students may test their rules in the same manner as in part (b) above. c. Write a rule to find the vertex coordinates of a parallelogram TUVW that is the same size as parallelogram ABCD but is in a different position on the grid? Using the rule (x, y) students would add or subtract values to the x and/or the y. Students may test their rules in the same manner as in part (b) above.		
6	Copy the figures below accurately onto your own grid paper.  a. Draw a figure similar, but not identical, to the rectangle. Answers will vary. b. Draw a figure similar, but not identical, to the triangle. Answers will vary. c. How do you know your scale drawings are similar to the given figures? Answers will vary. Possible answer: The comparison of small sides with each other and the larger sides with each other gives the same scale factor.	2.2	

7

2.2

The diagram below shows two similar polygons.



a. Write a rule for finding the coordinates of a point on Figure B from the corresponding point on Figure A.

(1.5x, 1.5y)

b. Write a rule for finding the coordinates of a point on Figure A from the corresponding point of Figure B.

$(\frac{1}{1.5}x,\ \frac{1}{1.5}y)$ or $(\frac{2}{3}x,\ \frac{2}{3}y)$

c. i. What is the scale factor from Figure A to Figure B?

1.5

ii. Use the scale factor to describe how the perimeter and area of Figure B are related to the perimeter and area of Figure A.

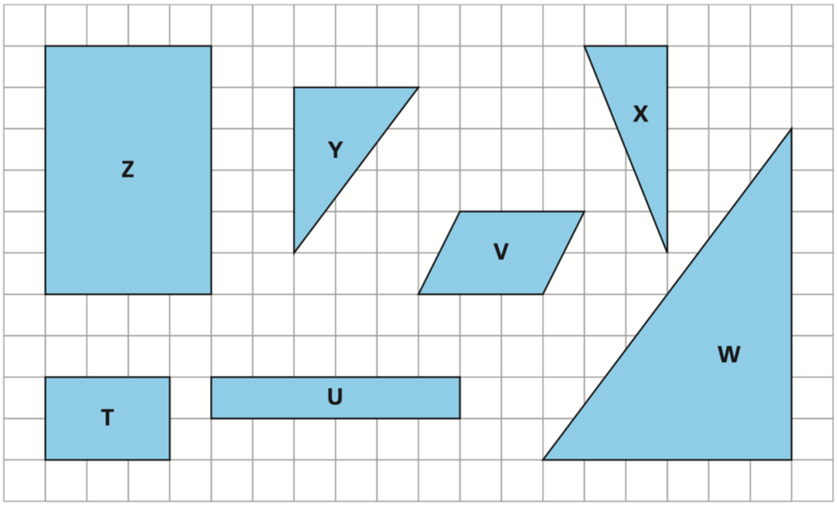
The perimeter of B is 1.5 times as large as the perimeter of A, and the area of B is 2.25 times as large as the area of A. The perimeter relationship is given by the same factor as the constant number multiplying the x- and y- coordinates (the scale factor). The area relationship is given by the square of this number.

d. i. What is the scale factor from Figure B to Figure A?

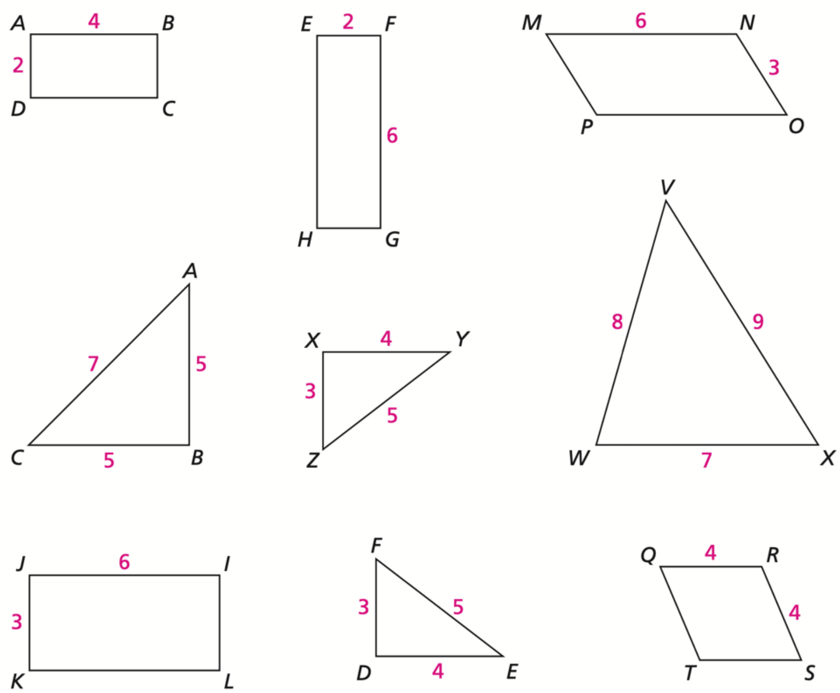
$\frac{1}{1.5}$ or $\frac{2}{3}$

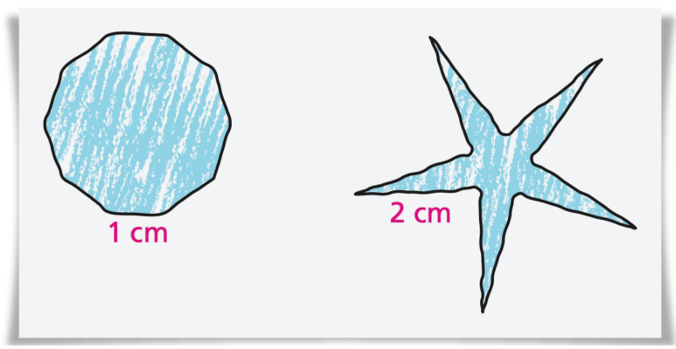
ii. Use the scale factor to describe how the perimeter and area of Figure A are related to the perimeter and area of Figure B.

The perimeter of A is 2/3 times as small as the perimeter of B while the

	area of A is 4/9 times as small as the area of B. The perimeter is given by the same factor as the constant number multiplying the x- and y-coordinates. The area relationship is given by the square of this number.		
For Exercises 8 and 9, study the size and shape of the polygons below. 			
8	**Multiple Choice** Choose the pair of similar figures. A. Z and Y B. V and T C. X and Y D. Y and W D	2.3	
9	Find another pair of similar figures Explain your reasoning. Z and T are similar. The comparison of small sides with each other and the larger sides with each other gives the scale factor, 2 or ½.	2.3	
10	What is the scale factor from an original figure to its image if the image is made using the given method? a. a two-rubber-band stretcher 2 b. a copy machine with size factor 150%	2.3	

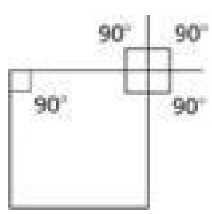
	1.5 c. a copy machine with size factor 250% 2.5 d. the coordinate rule (0.75x, 0.75y) 0.75		
11	a. Use the polygons below. Which pairs of polygons are similar figures? Rectangles ABCD and JILK seem to be similar. Triangles DFE and XYZ seem to be similar. You need to know angle measures to be sure they are similar. b. For each pair of similar figures, list corresponding sides and angles. For Rectangles ABCD and JILK: The corresponding angles are: A and J (or L), B and I (or K), C and L (or J), D and K (or I) The corresponding sides are: AB and JI (or LK), BC and IL (or KJ), CD and LK (or JI), DA and KJ (or IL) For Triangles DFE and XYZ: The corresponding angles are: F and Z, E and Y, D and X The corresponding sides are: FE and ZY, ED and YX, DF and XZ c. For each pair of similar figures, find the scale factor that relates side lengths of the larger figure to the corresponding side lengths of the smaller figure. The scale factor from the larger to the smaller figure for the rectangles is 2/3. The scale factor for the triangles is 1. Note that triangles FDE and ZXY have corresponding sides of equal length. These are congruent triangles.	2.3	

			
12	On grid paper, draw a rectangle with an area of 14 square centimeters. Label it ABCD. a. Write and use a coordinate rule that will make a rectangle similar to rectangle ABCD that is three times as long and three times as wide. Label it EFGH. (3x, 3y) b. How does the perimeter of rectangle EFGH compare to the perimeter of rectangle ABCD? The perimeter of rectangle EFGH varies because the perimeter of rectangle ABCD varies. It is three times as long as the perimeter of rectangle ABCD. c. How does the area of rectangle EFGH compare to the area of rectangle ABCD? Area of rectangle EFGH is nine times as large as the area of the rectangle ABCD. d. How do your answers to parts (b) and (c) relate to the scale factor from rectangle ABCD to rectangle EFGH?	2.3	

	The answer to part (b) is the same as the scale factor and the answer to part (c) is the same as the square of the scale factor.		
13	A student drew the figures below. The student says the two shapes are similar because there is a common scale factor for all of the sides. The sides of the figure on the right are twice as long as those of the figure on the left. What do you say to the student to explain why the figures are *not* similar?  Answers will vary. Student answers should mention the fact that the angles in the two figures are different from each other. In the figure on the left, the angles are all the same measure and obtuse. In the figure on the right, there are some obtuse angles and some acute angles.	2.3	

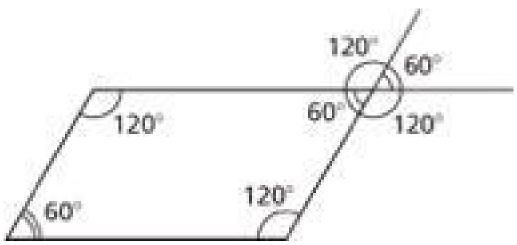
Connections

Problem #	Answer	CMP4 Problem #	Note
For Exercises 14 and 15, the rule $(x, \frac{3}{4}y)$ is applied to a polygon.			
14	Is the image similar to the original polygon? Explain. No, because 1 ≠ 3/4. The image will look shorter as it will shrink vertically.	2.1	
15	Each of the following points is on the original polygon. Find the	2.1	

	coordinates of each corresponding point on the image. a. (6, 8) (6, 6) b. (9, 8) (9, 6) c. (3/2, 4/3) (3/2. 1)		
16	One angle measure is given for each of the parallelograms below. ● Find the measure of the other three angles in the parallelogram. ● List all pairs of supplementary angles in the diagram. Then, classify each angle as *acute, right*, or *obtuse*. a.  90°, 90°, and 90°; The supplementary angle of 90° is 90°. All angles are right angles. 	2.1	

<table>
<tr><td></td><td>b.


60°, 120°, and 60° (clockwise); The supplementary angle of 120° is 60°, and the supplementary angle of 60° is 120°. 60° is an acute angle, and 120° is an obtuse angle.


</td><td></td><td></td></tr>
<tr><td colspan="4">Multiple Choice For Exercises 17 and 18, what is the percent reduction or enlargement that will result if the rule is applied to a figure or an coordinate gird?</td></tr>
<tr><td>17</td><td>(1.5x, 1.5y)

A. 150% B. 15% C. 1.5% D. None of these

A</td><td>2.2</td><td></td></tr>
<tr><td>18</td><td>(0.7x, 0.7y)

F. 700% G. 7% H. 0.7% J. None of these

J; It will be 70% reduction.</td><td>2.2</td><td></td></tr>
<tr><td>19</td><td>The rule $(x + \frac{2}{3}, y - \frac{3}{4})$ is applied to a polygon. For each vertex below of the polygon, find the coordinates of the corresponding vertex on the image.

a. (5, 3)

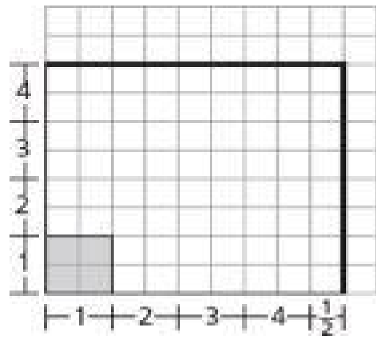
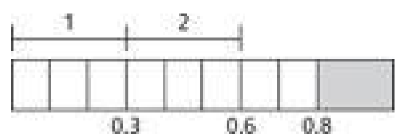
$(5 + \frac{2}{3}, 3 - \frac{3}{4}) = (\frac{17}{3}, \frac{9}{4})$

b. (1/6, 11/12)

$(\frac{1}{6} + \frac{2}{3}, \frac{11}{12} - \frac{3}{4}) = (\frac{1+4}{6}, \frac{11-9}{12}) = (\frac{5}{6}, \frac{2}{12}) = (\frac{5}{6}, \frac{1}{6})$</td><td>2.2</td><td></td></tr>
</table>

	c. (9/12, 4/5) $(\frac{9}{12}+\frac{2}{3}, \frac{4}{5}-\frac{3}{4}) = (\frac{9+8}{12}, \frac{16-15}{20}) = (\frac{17}{12}, \frac{1}{20})$		
20	An accurate map is a scale drawing of the place it represents. Below is a map of South Africa.  a. Use the scale to estimate the distance from Cape Town to Port Elizabeth. About 700 km, or 400 miles b. Use the scale to estimate the distance form Johannesburg to East London. About 800 km, or 440 miles c. What is the relationship between the scale for the map and a "scale factor"? The scale on the map gives the lengths of two corresponding sides – one from the map and one from the real world. The ratio of those lengths gives the scale factor between the map and the real world.	2.3	
Find each quotient.			

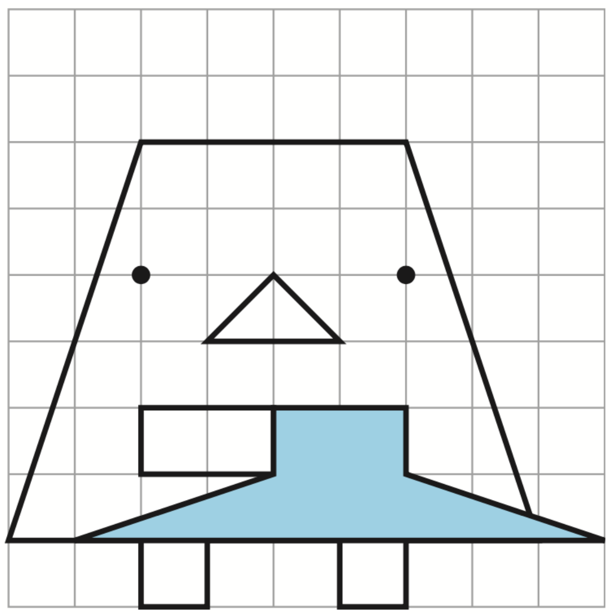
21	**Find each quotient** a. $\frac{1}{2}\ \ \frac{1}{4}$ 2 b. $\frac{1}{4}\ \ \frac{1}{2}$ ½ c. $\frac{3}{7}\ \ \frac{4}{7}$ **3/4** d. $1\frac{1}{2}\ \ \frac{3}{8}$ **4**	2.3	
22	At a bake sale, 0.72 of a pan of corn bread has not been sold. A serving is 0.04 of a pan. a. How many serving are left? 0.72 0.04 = 18 servings b. Use a hundredths grid to show your reasoning. Answers will vary. One possible answer: The larger rectangle represents 0.72, the left over bread, and the smaller rectangle represents 0.04, one serving. There are 18 servings in the larger rectangle. $4\ \ 4\frac{1}{2} = 18$	2.3	

			
23	Each pizza takes 0.3 of a large block of cheese. Charlie has 0.8 of a block of cheese left. a. How many pizzas can he make? 0.8 0.3 = 8/3; 2 pizzas and 2/3 (or 0.66) of another. Or, 2 pizzas and a remainder of 0.2 of a large block of cheese. b. Use a diagram to show your reasoning. 	2.3	

Extensions

Problem #	Answer	CMP4 Problem #	Note
24	Select a drawing of a comic strip character from a newspaper or magazine. Draw a grid over the figure or tape a transparent grid on top of the figure. Identify key points on the figure and then enlarge it using each of these rules. Which figures are similar? Explain. a. (2x, 2y) b. (x, 2y) c. (2x, y)	2.1	

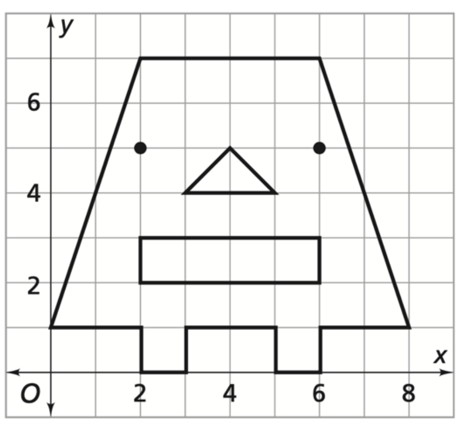
	Drawings will vary. In part (a), one gets a similar figure, which is two times as wide and two times as high. In part (b) and part (c), the image will not be similar. It will be two times as high as in part (b) while keeping the same width and two times as wide in part (c) while keeping the same height.		
25	Suppose you use the rule (3x+1, 3y-4) to transform Mug Wump into a new figure. a. How will the angle measures in the new figure compare to corresponding angle measures in Mug? Angle measures remain the same. b. How will the side lengths of the new figure compare to corresponding side lengths of Mug? Side lengths will be three times as long. c. How will the area and perimeter of this new figure compare to the area and perimeter of Mug? Area will be nine times as large. Perimeter will be three times as large.	2.2	
26	The vertices of three similar triangles are given. ● triangle ABC: A(1, 2), B(4, 3), C(2, 5) ● triangle DEF: D(3, 6), E(12, 9), F(6, 15) ● triangle GHI: G(5, 9), H(14, 12), I(8, 18) a. Find a rule that changes the vertices of triangle ABC to the vertices of triangle DEF. (3x, 3y) b. Find a rule that changes the vertices of triangle DEF to the vertices of triangle GHI. (x+2, y+3) c. Find a rule that changes the vertices of triangle ABC to the	2.2	

	vertices of triangle GHI. (3x+2, 3y+3)		
27	If you drew Mug and his hat on the same grid, his hat would be at his feet instead of on his head.  a. Write a rule that puts Mug's hat centered on his head. (x-1, y+6) b. Write a rule that changes Mug's hat to fit Zug and puts the hat on Zug's head. (2x-2, 2y+12) c. Write a rule that changes Mug's hat to fit Lug and puts the hat on Lug's head. (3x-3, y+6)	2.2	
28	Films are sometimes modified to fit a TV screen. Find out what that means. What exactly is modified? If Mug is in a movie, is he still a Wump when you see the video on TV?	2.3	

<table>
<tr><td></td><td>The rectangle of a movie screen is not similar to the rectangle of a TV screen, in general. The width of the movie screen is usually much longer than its height, while the width and height of a TV screen are close to each other, i.e., more like a square. The reduction may be performed in three different ways;

(1) It is performed so that the width of the theater picture fits exactly onto the width of the TV screen, and the same scale is used to reduce the height. In this case Mug will still be a Wump but there will be a blank area at the bottom or the top of the TV screen.

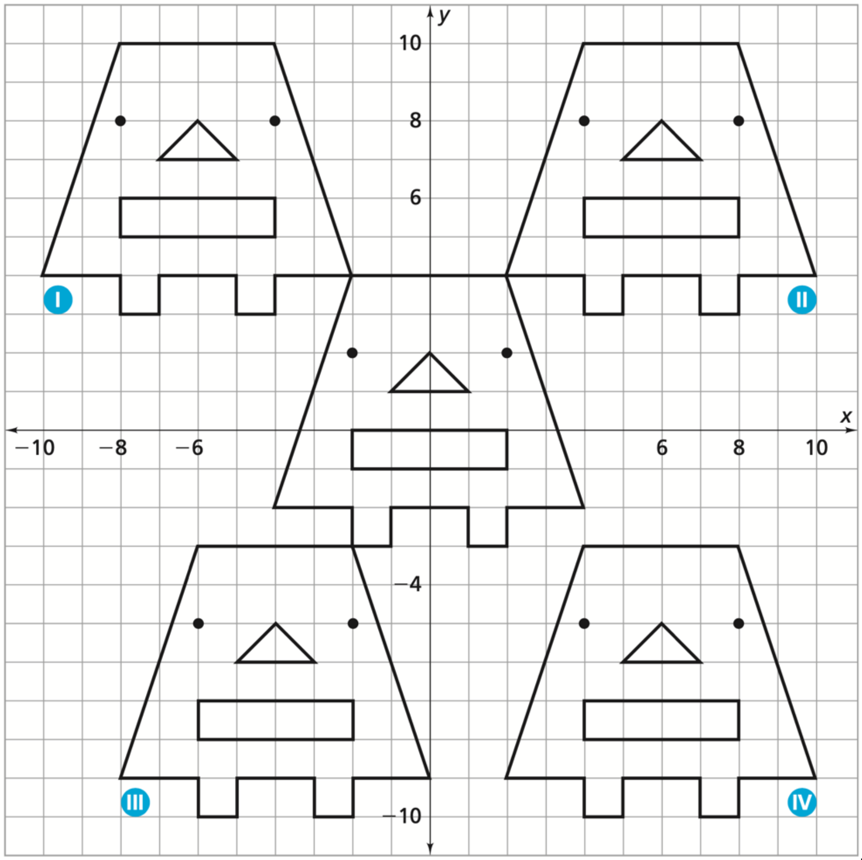
(2) The reduction is performed so that the height of the movie screen fits exactly onto the height of the TV screen, and the same scale is used to reduce the width. In this case Mug will still be a Wump but a part of the picture will be cut from the left and/or right side since it will be outside of the TV screen range.

(3) Different scales are used to reduce the width and the height so that the whole picture will fit onto the TV screen. However, in this case, the images will be distorted a little bit and Mug will not be a Wump anymore. Because of this, the reduction method is not usually applied in practice.</td><td></td><td></td></tr>
<tr><td>29</td><td>Explain how each rule changes the original shape, size, and location of Mug Wump.


a. (-x, y)

The y-axis acts as a mirror.</td><td>2.3</td><td></td></tr>
</table>

	b. (x, -y) The x-axis acts as a mirror. c. (-0.5x, -0.5y) Both the x- and the y- axes are used as mirrors, and the image is half the size of the original. Another way to look at this is that the figure gets rotated 180° around the point (0, 0) and shrunk by a factor of ½. d. (-0.5x, y) The y-axis acts as a mirror and, in addition, the width of the image is ½ the width of the original. The height stays the same. e. (-3x, -3y) Both the x- and y- axes are used as mirrors, and the image is 3 times the size of the original. f. (3x+5, -3x-4) Both the x- and y- axis are used as mirrors, the image is 3 times the size of the original, and the image is moved 5 units to the right and 4 units down.		
30	The diagram below shows Mug Wump drawn at the center of a coordinate grid and in four other positions.	2.3	


y
10
8
6
x
−10
−8
−6
6
8
10
−4
−10
I
II
III
IV

Coordinates for Mug and Variations

Rule	(x, y)	(2x, 2y)	(−2x, −2y)
Head Outline	(−4, −2)	■	■
	(−2, −2)	■	■
	(−2, 3)	■	■
	■	■	■
	■	■	■
Nose	(−1, 1)	■	■
	■	■	■
	■	■	■
Mouth	(−2, −1)	■	■
	■	■	■
	■	■	■
	■	■	■
Eyes	(−2, 2)	■	■
	■	■	■

a. Find a sequence of coordinates to draw Mug's body at the center of the grid. Make a table to keep track of the points. For parts (b) and (c) below, use this Mug as the original Mug.

Rule	(x, y)
Head Outline	(–4, –2)
	(–2, –2)
	(–2, –3)
	(–1, –3)
	(–1, –2)
	(1, –2)
	(1, –3)
	(2, –3)
	(2, –2)
	(4, –2)
	(2, 4)
	(–2, 4)
	(–4, –2)

Nose Outline	(–1, 1)
	(0, 2)
	(1, 1)
Mouth Outline	(–2, 0)
	(2, 0)
	(2, –1)
	(–2, –1)
	(–2, 0)
Eyes	(–2, 2)
	(2, 2)

b. You can write a coordinate rule to describe the movement of points from one location to another. For example, the coordinate rule (x-2, y+3) moves a point (x, y) to the left 2 units

	and up 3 units from its original location. Which of the other drawings is produced by the coordinate rule (x+6, y-7)? The rule (x+6, y-7) moves Mug to position IV. c. Find coordinate rules for moving the original Mug to the other positions on the grid. The rule (x-6, y+6) moves Mug to position I. The rule (x+6, y+6) moves Mug to position II. The rule (x-4, y-7) moves Mug to position III.		

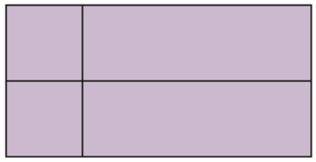
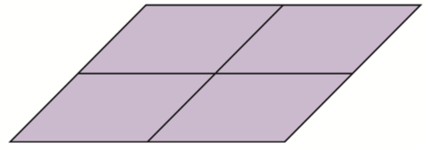
Investigation 3

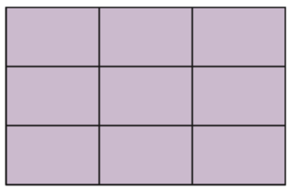
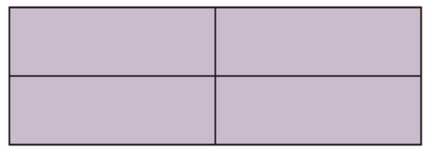
	Applications	Connections	Extensions	Total
3.1	6	4	2	12
3.2	6	6	3	15
3.3	4	4	1	9
Total	16	14	6	36

Applications

Problem #	Answer	CMP4 Problem #	Note
1	Look for rep-tile patterns in the designs below. For each design, • Decide whether the small quadrilaterals are similar to the large quadrilateral. Explain. • If the quadrilaterals are similar, give the scale factor from each small quadrilateral to the large quadrilateral.	3.1	

<table>
<tr><td></td><td>

a.  b. 

c.  d. 

a. No, they are not similar. One of the small figures is a square, so it does not have the same shape as the original rectangle, which is not a square.

b. Yes, they are similar because their corresponding interior angles are congruent. Also, each side of the smaller quadrilateral increases by the same scale factor to form the larger quadrilateral. The side lengths of the larger shape are double that of the smaller shape, so the scale factor is 2.

c. Yes, they are similar because their corresponding interior angles are congruent. Also, each side of the smaller quadrilateral increases by the same scale factor to form the larger quadrilateral. The side lengths of the larger shape are triple that of the smaller shape. The scale factor is 3.

d. Yes, they are similar because their corresponding interior angles are congruent. Also, each side of the smaller quadrilateral increases by the same scale factor to form the larger quadrilateral. The side lengths of the larger shape are double that of the smaller one, so the scale factor is 2.

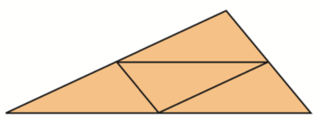
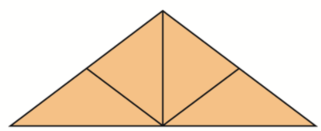
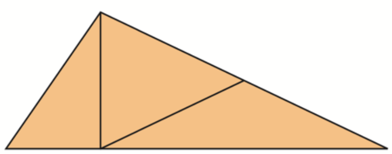
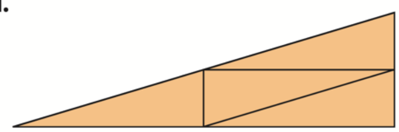
</td><td></td><td></td></tr>
<tr><td>2</td><td>

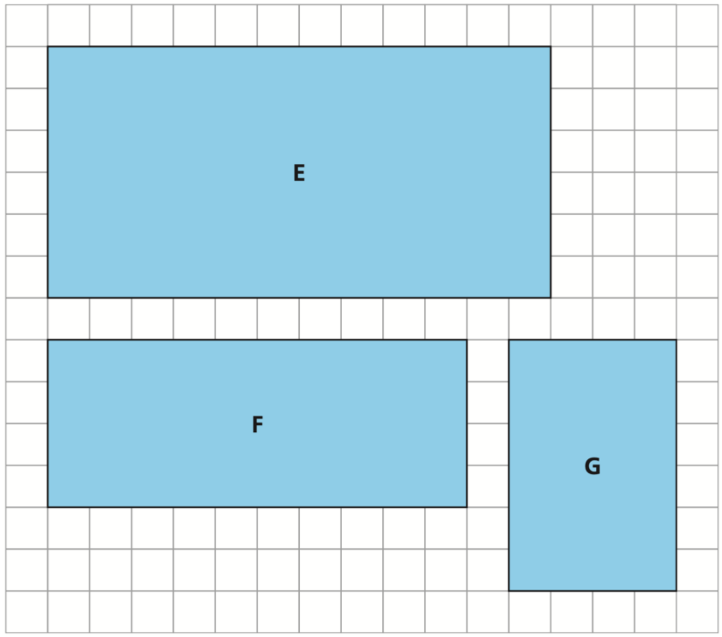
Suppose you divide a rectangle into 25 smaller rectangles such that each rectangle is similar to the original rectangle.

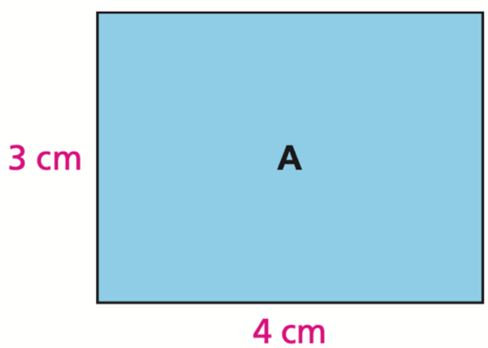
a. How is the area of each of the smaller rectangles related to the area of the original rectangle?

The area of each small rectangle is 1/25 the area of the large rectangle.

</td><td>3.1</td><td></td></tr>
</table>

<table>
<tr><td></td><td> Note: You might suggest that students provide a sketch to verify their answer.
b. What is the scale factor from the original rectangle to each of the smaller rectangles?
1/5 </td><td></td><td></td></tr>
<tr><td>3</td><td> Look for rep-tile patterns in the figures below.
Tell whether the small triangles are similar to the large triangle. Explain.
If the triangles are similar, give the scale factor from each small triangle to the large triangle.
a.

b.

c.

d.

a. The small triangles are similar to the large triangle because the corresponding angles of the triangles are congruent. The scale factor is 2.
b. The small triangles on the left and right corners are similar to the large triangle with scale factor 2, but the other two small triangles are not similar to the large triangle. Since the non-similar triangles are not formed by the connection of midpoints, we cannot assume that the triangles are similar.
c. None of the small triangles is similar to the large one.
d. The small triangles are similar to the large triangle since the angles in each small triangle are congruent to the angles in the large triangle. The scale factor is 2.
Note: You can compare this figure with the figure from part (a). </td><td>3.1</td><td></td></tr>
</table>

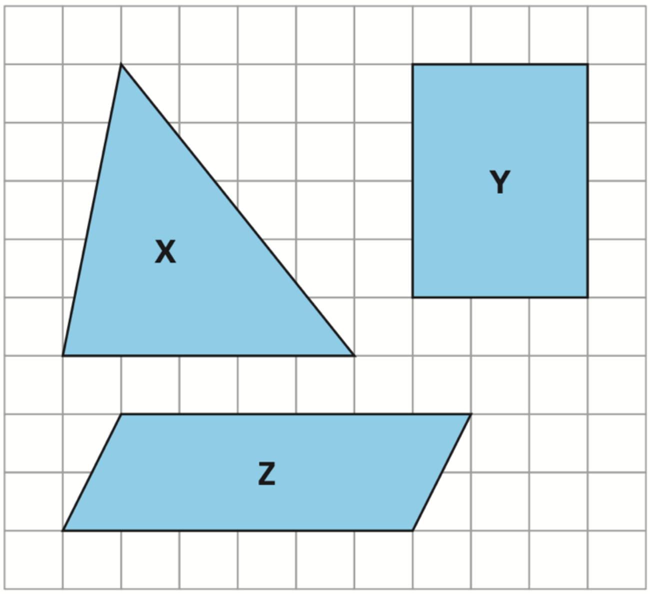
	They look different, but their constructions are essentially the same.		
4	a. For rectangles E-G, give the length and width of a different, similar rectangle. Explain how you know the new rectangles are similar.  b. Give the scale factor from each original rectangle in part (a) to the similar rectangles you described. Explain what the scale factor tells you about the corresponding lengths, perimeters, and areas. Answers will vary. **Rectangle E:** a. Any rectangle with dimensions 6k by 12k, where k is any positive number, is similar to Rectangle E. The ratio of the corresponding sides will be the same. b. The scale factor from Rectangle E to the new rectangle is k. The side lengths and perimeter of the new rectangle are k times the corresponding lengths and perimeter of Rectangle E. The area of the new rectangle is k^2 times the area of Rectangle E. **Rectangle F:** a. Any rectangle with dimensions 4k by 10k, where k is any positive number, is similar to Rectangle F. The ratio of the corresponding sides will be the same.	3.1	

	b. The scale factor from Rectangle F to the new rectangle is *k*. The side lengths and perimeter of the new rectangle are k times the corresponding lengths and perimeter of Rectangle F. The area of the new rectangle is k^2 times the area of Rectangle F. **Rectangle G:** a. Any rectangle with dimensions 6*k* by 4*k*, where *k* is any positive number, is similar to Rectangle G. The ratio of the corresponding sides will be the same. b. The scale factor from Rectangle G to the new rectangle is *k*. The side lengths and perimeter of the new rectangle are k times the corresponding lengths and perimeter of Rectangle G. The area of the new rectangle is k^2 times the area of Rectangle G.		
5	Suppose Rectangle B is similar to Rectangle A below. The scale factor from Rectangle A to Rectangle B is 4. What is the area of Rectangle B?  192 cm^2 The side lengths of Rectangle B are 12cm (3cm x scale factor 4) and 16 (4cm x scale factor 4). So, the area of Rectangle B is 12 x 16 = 192 cm^2 Or, when the scale factor from A to B is 4, the area of B will be 4^2 times larger than the area of A. Since the area of A is 3 cm x 4 cm = 12 cm^2, the area of B is 4^2 x 12 cm^2 = 192 cm^2	3.1	
6	Suppose Rectangle E has an area of 9 square centimeters and Rectangle F has an area of 900 square centimeters. The two rectangles are similar.	3.1	

<table>
<tr><td></td><td>What is the scale factor from Rectangle E to Rectangle F?

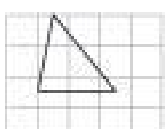
10; The area of Rectangle F is 100 times greater than the area of Rectangle E. Since 10x10=100, the scale factor from E to F will be 10.

For example, Rectangle E can be 3 by 3 square and Rectangle F can be 30 by 30 square. They are similar and areas are 9 and 900, respectively. The scale factor from E to F will be 10.</td><td></td><td></td></tr>
<tr><td>7</td><td>For parts (a)-(c), use grid paper.



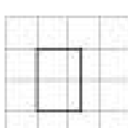
a. Sketch a triangle similar to Triangle X with an area that is ¼ the area of Triangle X.

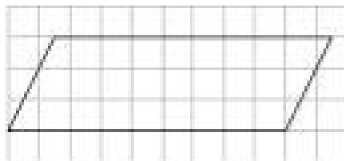
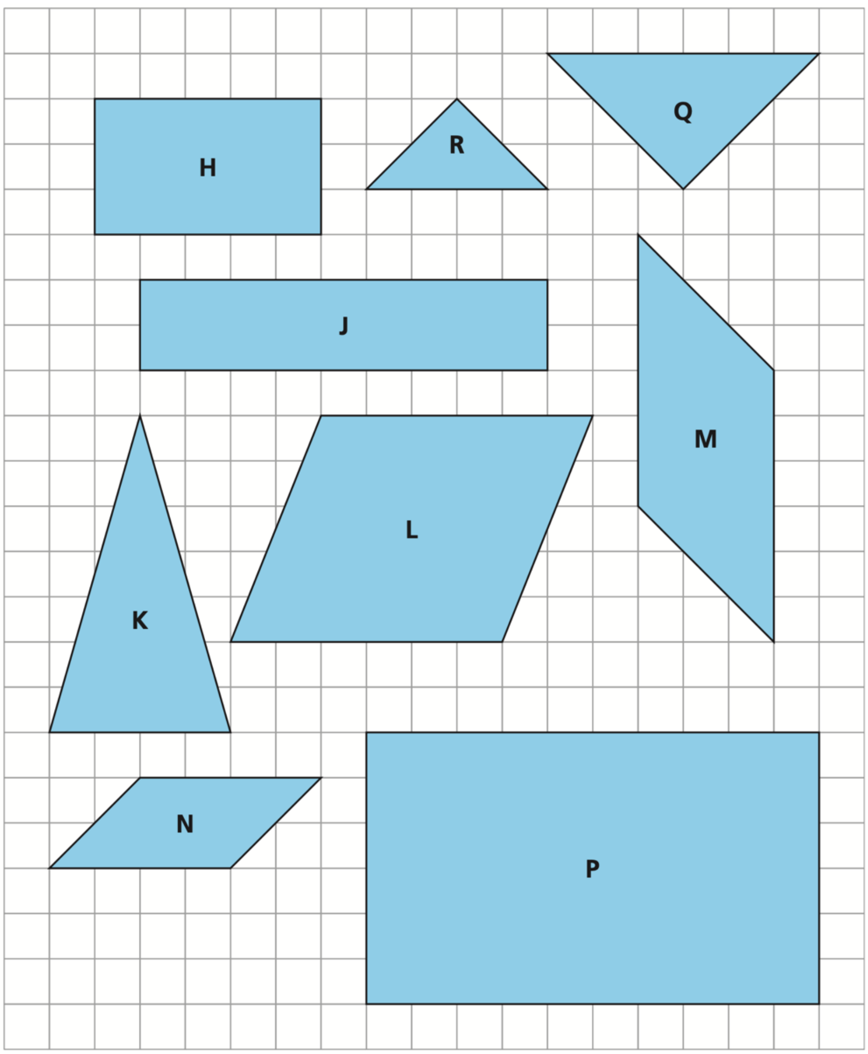
base: 2.5, height: 2.5

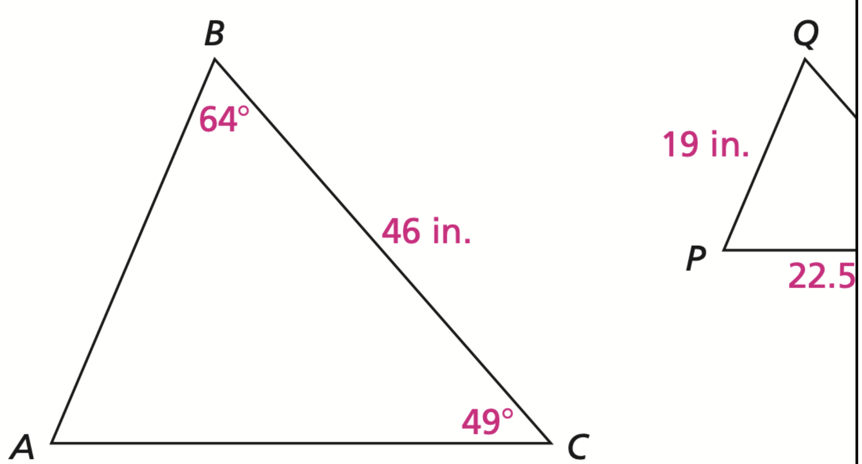


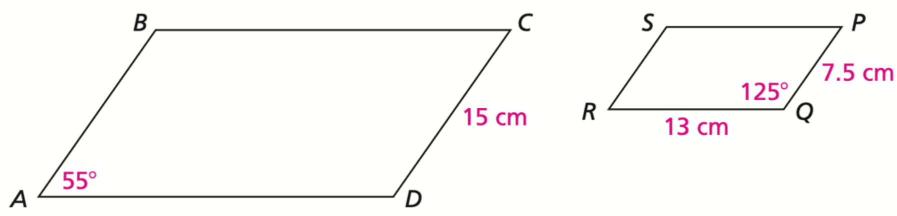
b. Sketch a rectangle similar to Rectangle Y with a perimeter that is 0.5 times the perimeter of Rectangle Y.

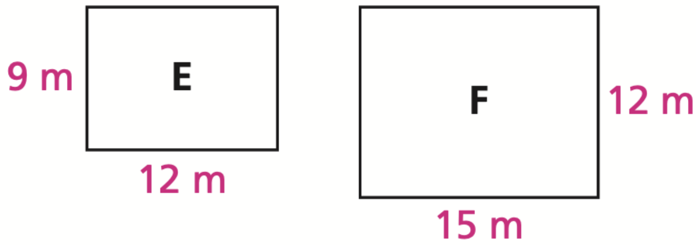
base: 1.5, height: 2

</td><td>3.2</td><td></td></tr>
</table>

	c. Sketch a parallelogram similar to Parallelogram Z with side lengths that are 1.5 times the side lengths of Parallelogram Z. base: 9, height: 3 		
8	Use the polygons below.  	3.2	

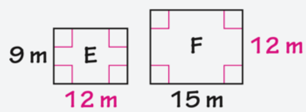
	a. List pairs of similar shapes. Rectangles H and P, Triangles R and Q, and Parallelograms M and N. b. For each pair of similar shapes, find the scale factor from the smaller shape to the larger shape. The scale factor from H to P is 2, from R to Q is 6/4 or 3/2, and from N to M is 6/4 or 3/2.		
9	Triangle ABC is similar to triangle PQR. Find the indicated angle measure or side length.  a. angle A 67°; 180 – 64 – 49 = 67. b. angle Q 64°; The measure of angle Q is same as the corresponding angle B since two triangles are similar. c. angle P 67°; The measure of angle P is same as the corresponding angle A. d. length of side AB 38 in.; The scale factor from Triangle PQR to Triangle ABC is 2. Side AB is two times longer than its corresponding side, PQ. e. length of side AC	3.2	

	45 in.; The length of side AC is twice the length of PR. f. perimeter of triangle ABC 129 in.; Perimeter of triangle ABC is the sum of side lengths, AB+BC+CA: 38 + 46 + 45 = 129.		
10	**Multiple Choice:** Use the similar parallelograms below.  a. What is the measure of angle D? A. 55° B. 97.5° C. 125° D. 135° C; The angle D corresponds to the angle Q. b. What is the measure of angle R? F. 55° G. 97.5° H. 125° J. 135° F; The angle R corresponds to the angle A. c. What is the measure of angle S? A. 55° B. 97.5° C. 125° D. 135° C; The measure of angle S is same as the measure of angle Q, because it is a parallelogram. d. What is length of side AB? F. 3.75cm G. 13 cm H. 15 cm J. 26 cm H; The length of side AB is equal to the length of side CD, because it is a parallelogram	3.2	
11	Evan, Melanie, and Wyatt discuss whether the two figures E and F are similar. Do you agree with Evan, Melanie, and Wyatt? Explain.	3.2	



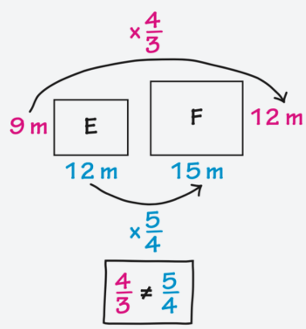
Evan's Reasoning

Rectangles E and F are similar because each shape has four right angles. Also, each rectangle has at least one side that is 12 meters long.



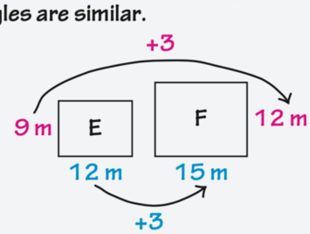
Melanie's Reasoning

The scale factor for the height from rectangle E to rectangle F is $\frac{12}{9}$, or $\frac{4}{3}$. The scale factor for the base is $\frac{15}{12}$, or $\frac{5}{4}$. $\frac{4}{3} \neq \frac{5}{4}$, so the rectangles are not similar.



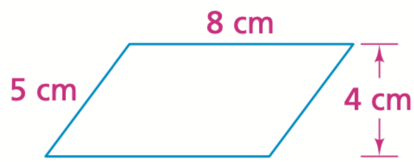
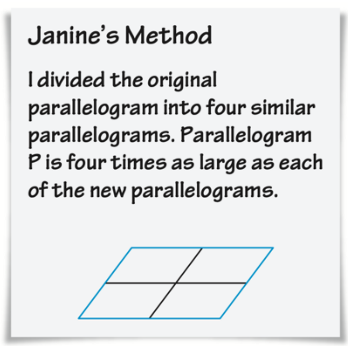
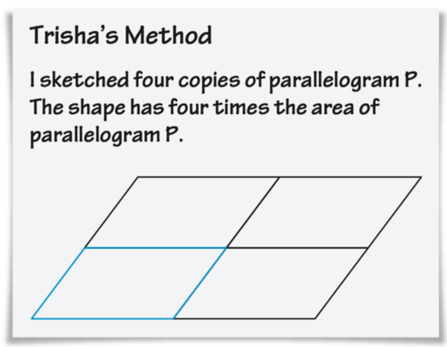
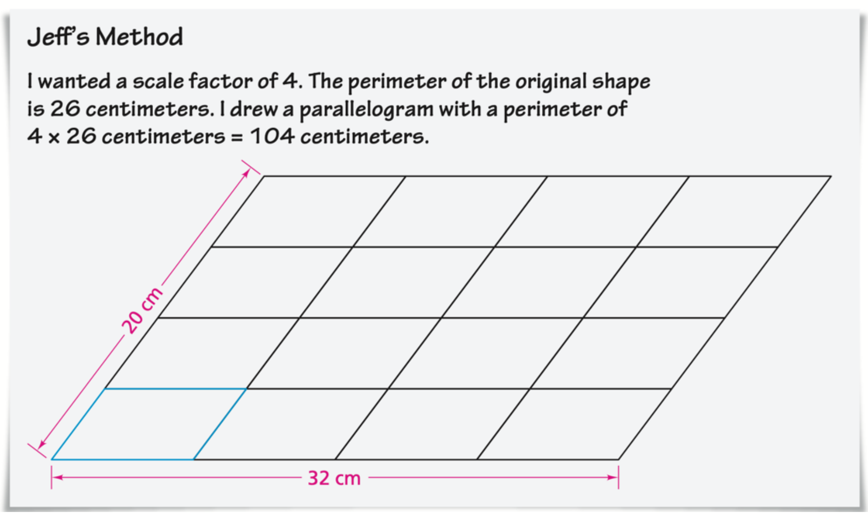
Wyatt's Reasoning

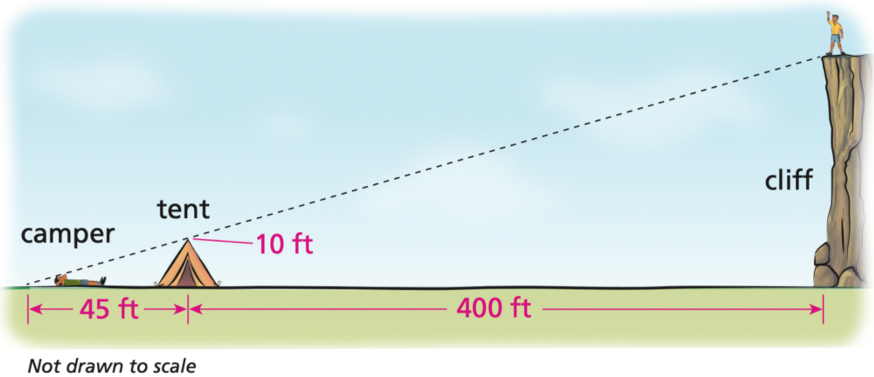
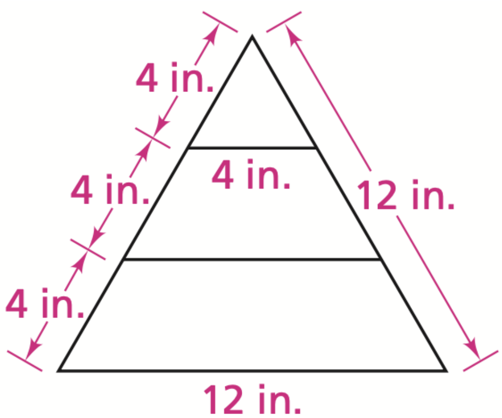
Rectangles E and F are similar. Rectangle F is 3 meters taller than Rectangle E since 9 meters + 3 meters = 12 meters. Rectangle F is also 3 meters wider than Rectangle E since 12 meters + 3 meters = 15 meters. Each dimension of Rectangle F is 3 meters greater than the corresponding dimension of Rectangle E, so the rectangles are similar.

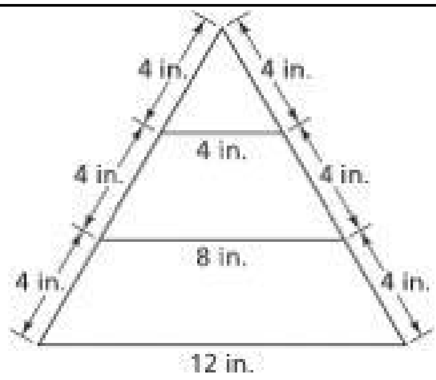
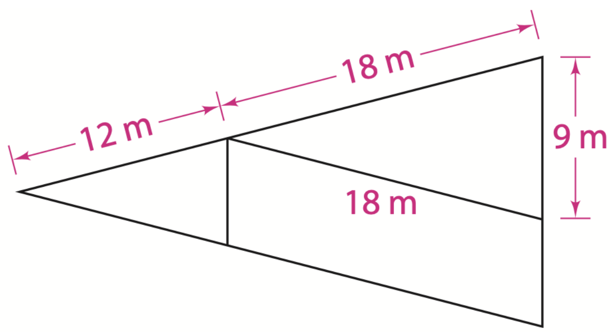
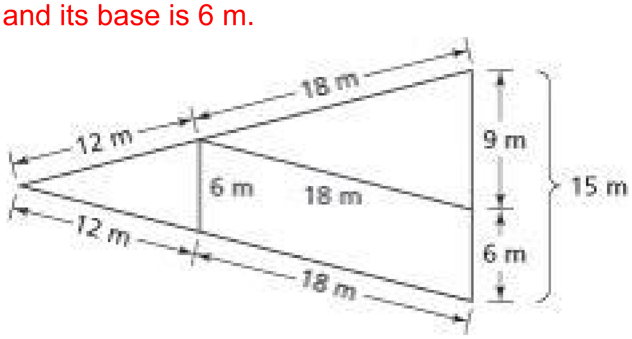


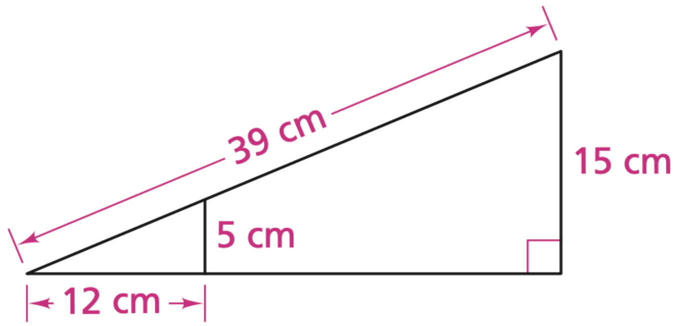
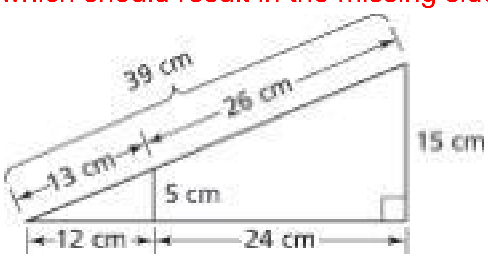
Melanie is correct. In order for the shapes to be similar, they must have consistent scale factors, which they do not.

Evan is incorrect. Evan's reasoning highlights the differences between polygons in general and the specific case of triangles. With triangles you can determine similarity just by comparing corresponding angles, but this does not always work with other figures.

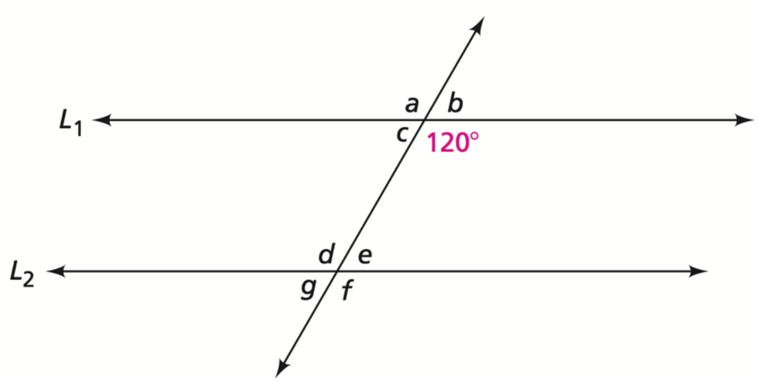
	Wyatt is incorrect. Wyatt's thinking is a common mistake for many students. Wyatt thinks that an additive relationship of the sides indicates similarity, rather than a multiplicative relationship.		
12	Janine, Trisha, and Jeff drew parallelograms that are similar to Parallelogram *P* below.  **Parallelogram *P*** Each student claims that the scale factor from *P* to the sketched parallelogram is 4. Are any of the students correct in their reasoning?    Jeff is correct in his reasoning. When a scale factor is applied in a figure, it is applied to lengths. Jeff's perimeter is 4 times as great as the perimeter of the original parallelogram.	3.2	

	Janie applied a scale factor of ½ and Trish applied a scale factor of 2.		
13	Judy lies on the ground 45 feet from her tent. Both the top of the tent and the top of a tall cliff are in her line of sight. Her tent is 10 feet tall. About how high is the cliff? Assume the two triangles are similar.  About 98.9 feet; Compare the corresponding side lengths of the similar triangles: 45 ft corresponds to 45 + 400 ft, which means the scale factor from the smaller triangle to the larger triangle is 445/45. The height of cliff is 10 x 445/45 = 4450/45 = 98.888…	3.3	
14	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can.  All the triangles are equilateral; the large triangle has all three sides measuring 12 inches. The measure of all three sides of the medium triangle is 8 inches, and the measure of all sides of the smallest triangle is 4 inches.	3.3	

			
15	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can.  All three triangles are isosceles, as can be seen by the medium triangle with sides 9 m, 18 m, and 18 m. The triangles are all similar because each smaller triangle is nested in the larger triangle. Note the base of each triangle is half the length of its legs. The large isosceles triangle has a leg of 30 m, so its base is 15 m. Also, the smallest triangle has one leg of 12 m, so its other leg is also 12 m and its base is 6 m. 	3.3	
16	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can.	3.3	

	 First, determine the length of the base of the large triangle. The length of the base should be 36 cm. You can determine the missing side of the small triangle by dividing 39 cm by the scale factor of 3, which should result in the missing side for the small triangle, 13 cm. 		

Connections

Problem #	Answer	CMP4 Problem #	Note
17	In the figure below, lines L_1 and L_2 are parallel.  a. Use what you know about parallel lines to find the measures of angles a through g. a = 120°, b = 60°, c = 60°, d = 120°, e = 60°, f = 120°, g = 60° b. List all pairs of *supplementary* angles in the diagram.	3.1	

<table>
<tr><td></td><td>Students may list any combination of angles as long as the pairs sum to 180°. Reference the answers in part (a). For example, angles a and b, a and c, a and g, and a and e are all pairs of supplementary angles.</td><td></td><td></td></tr>
<tr><td>18</td><td>For each of the following angle measures, find the measure of its supplementary angle.

a. 160°

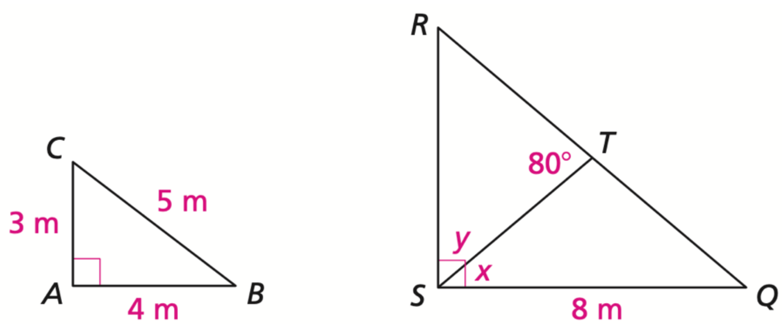
b. 90°

c. x°

a. 20°

b. 90°

c. 180°-x°</td><td>3.1</td><td></td></tr>
<tr><td>19</td><td>The right triangles below are similar.



a. Find the length of side RS.

6 m; Since the scale factor from the smaller triangle to the larger triangle is 2, side RS is 3 m x 2 = 6 m.

b. Find the length of side RQ.

10 m; The corresponding side of RQ is BC. 5 m x 2 = 10 m.

c. The measure of angle x is about 40°. If the measure of angle x were exactly 40°, what would be the measure of angle y?

50°; 90° - 40° = 50°</td><td>3.1</td><td></td></tr>
</table>

	d. Use your answer from part (c) to find the measure of angle R. Explain how you can find the measure of angle C. 50°; Since the sum of the angles in triangle STR is 180°, The measure of angle R is equal to 180° - 80° - 50° = 50°. Because the triangles are similar, the angle C is also 50°, since it corresponds to angle R. e. Angle x and angle y are *complementary angles*. Find two additional pairs of complementary angles in Triangles ABC and QRS. Angles R (50°) and Q (40°) are complementary, and angles C and B are complementary.		
20	For parts (a)-(f), find the number that makes the fractions equivalent. a. ½ = 3/• b. 5/6 = •/24 c. ¾ = 6/• d. 8/12 = 2/• e. 3/5 = •/100 f. 6/4 = •/10 Students may have a couple of ways of solving these problems. Below is one possible solution for part (d). Similar thinking can apply to all parts. The scale factor that takes 8 to 2 is ¼. Therefore, you need ¼ of 12, which is 3. a. 2 x 3 = 6 b. 5 x 4 = 20 c. 4 x 2 = 8 d. 12 x ¼ = 3 e. 3 x 20 = 60	3.1	

	f. 6 x 2.5 = 15		
21	For parts (a)-(f), suppose you copy a figure on a copier using the given scale factor. Find the scale factor from the original figure to the copy in decimal form. a. 200% b. 50% c. 150% d. 125% e. 75% f. 25% a. 200/100 = 2 b. 50/100 = 0.5 c. 150/100 = 1.5 d. 125/100 = 1.25 e. 75/100 = 0.75 f. 25/100 = 0.25	3.2	
22	For parts (a)-(d), tell whether the figures are mathematically similar. Explain your reasoning. If the figures are similar, give the scale factor from the left figure to the right figure.	3.2	

a.

b.

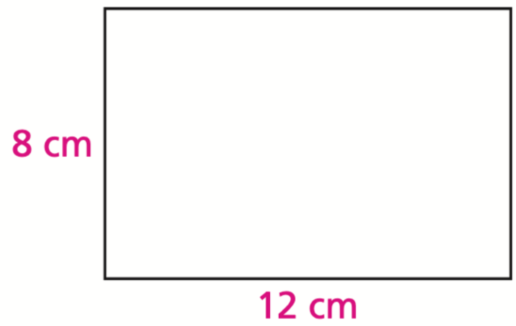
c.

d.

a. The birds are not similar since the ratio of base length of the larger figure to the base length of the smaller figure is not the same as the ratio of the height of the larger figure to the height of the smaller figure.

b. The figures are similar because the ratio of base length of the

	larger figure to the base length of the smaller figure is the same as the ratio of the height of the larger figure to the height of the smaller figure. Or, for both width and height the same reduction scale is applied; so, the figures are similar. The scale factor is about 0.7. c. The figures are not similar because the height of the first figure is reduced by about 56%, while the width is reduced by a smaller percent. d. The lighthouses are not similar because the height is enlarged but the width is reduced.		
For Exercises 23-25, decide whether the statement is true or false. Explain your reasoning.			
23	All squares are similar. True. The corresponding angles will always be equal to each other since they are all 90°, and the ratio of any two sides of a square is 1. Alternatively, students might notice that if they choose any side of one square and any side of the other square, the scale factor must be the same, regardless of which sides they choose.	3.2	
24	All rectangles are similar. False. While the angels of any two rectangles will be the same (90°), it is not the case that the ratios of the sides will be equal.	3.2	
25	If the scale factor between two similar shapes is 1, then the two shapes are the same size. True. The fact that there is a consistent scale factor implies that the shapes are similar, and so the corresponding angle measures are equal. The fact that the scale factor is 1 means that the side lengths are unchanged. Equal angle measures and equal side lengths yield congruent figures.	3.2	
26	a. Suppose the following rectangle is reduced by a scale factor of 50%. What are the dimensions of the reduced rectangle?	3.2	



4 cm by 6 cm; 50% is equal to ½. So, 8 cm x ½ = 4 cm and 12 cm x ½ = 6 cm.

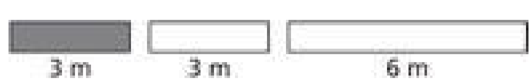
b. Suppose the reduced rectangle from part (a) is reduced again by a scale factor of 50%. What are the dimensions of the new rectangle? Explain your reasoning.

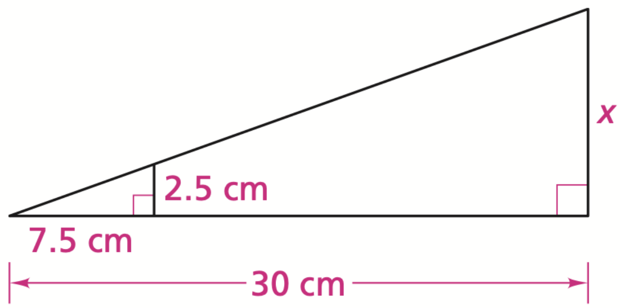
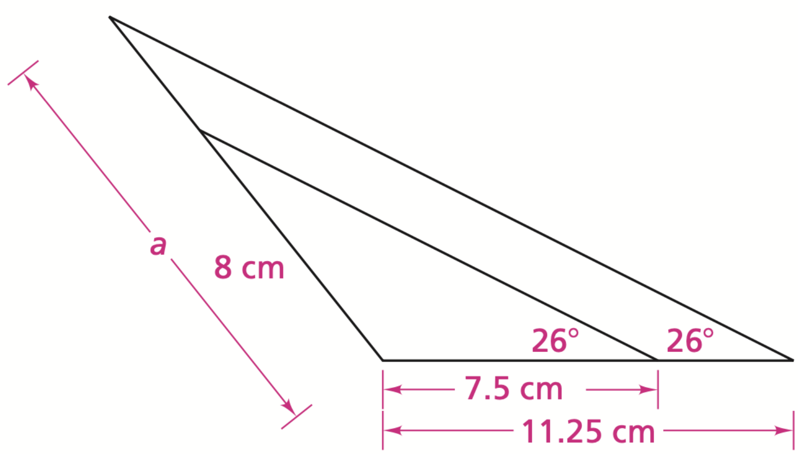
2 cm by 3 cm; When you reduce a figure by 50%, you need to make each side length half of the corresponding side length of the original. Since the first reduction of 50% resulted in a rectangle with dimensions of 4 centimeters and 6 centimeters, you need to find half of 4 centimeters and half of 6 centimeters. The dimensions should be 2 centimeters and 3 centimeters.

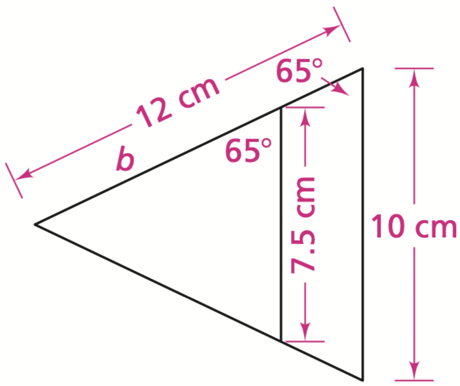
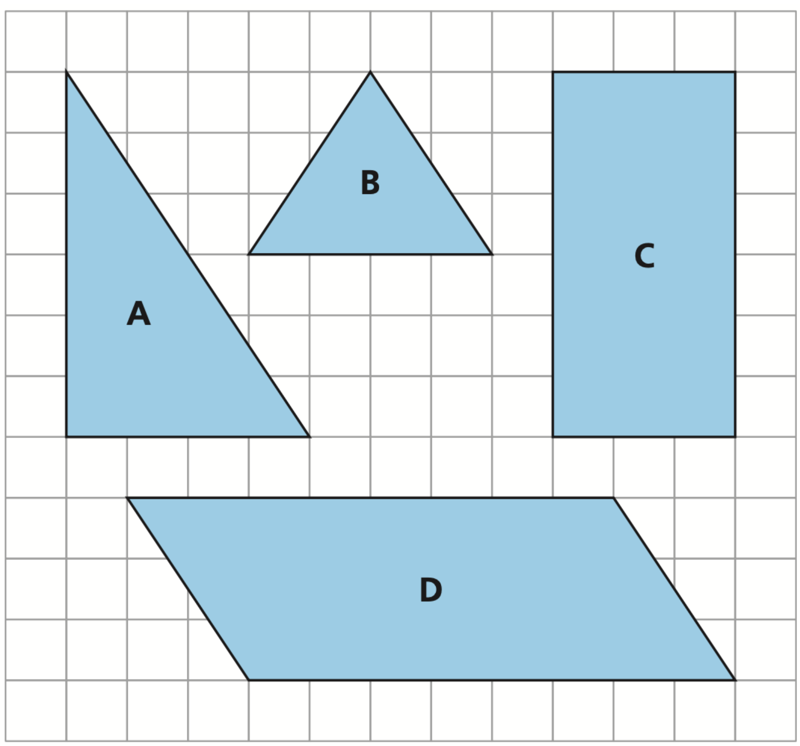
c. How does the reduced rectangle from part (b) compare to the original rectangle from part (a)?

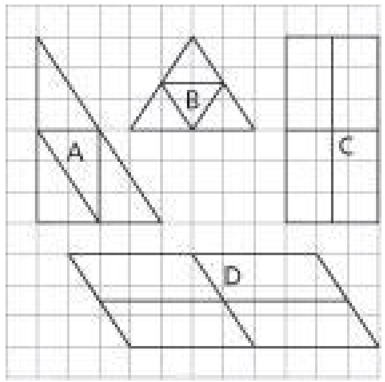
The rectangle is 1/16 the size of the original rectangle. The dimensions of the new rectangle are ¼ the lengths of the original rectangle.

Note: One thing students often have difficulty conceptually is that multiplying by a number smaller than 1 reduces the original. Multiplication is usually taught as a "makes larger" operation in the elementary grades. This concept makes the new world of rational numbers harder for students to enter. Suppose you take a piece of rope that is 12 m long and reduce its length by a factor of 0.5 (or ½). The new length of the rope is 6 m. Suppose you reduced the new length of the rope by a factor of 0.5 again. The length of the rope is 3 m. A physical model of what is happening to the rope is shown.

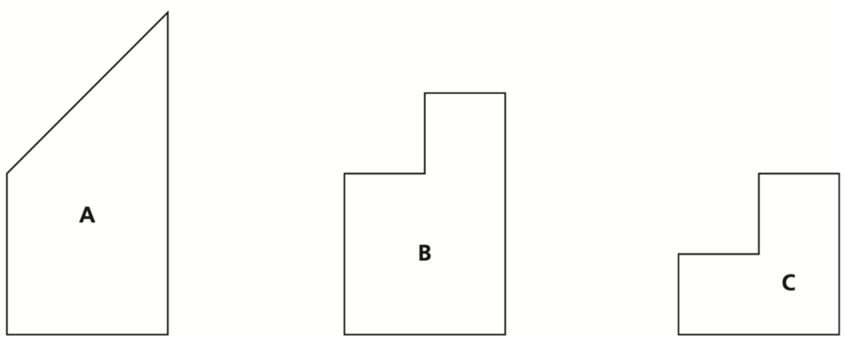
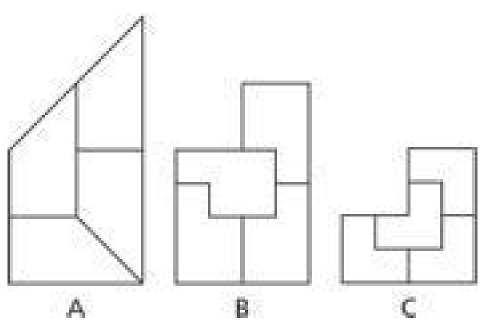


27	**Multiple Choice** What is the value of x? The diagram is not to scale.  A. 3 cm B. 10 cm C. 12 cm D. 90 cm B; Since 7.5 x 4 = 30, the scale factor from the smaller triangle to the larger triangle is 4. So, x = 2.5 x 4 = 10.	3.3	
28	Find the missing side length. The diagram is not to scale.  12 cm; Since 7.5 x 1.5 = 11.25, the scale factor from the smaller triangle to the larger triangle is 1.5 (or 3/2). So, a = 8 x 1.5 = 12.	3.3	
29	Find the missing side length. The diagram is not to scale.	3.3	

	 9 cm; Since 10 x ¾ = 7.5, the scale factor from the larger triangle to the smaller triangle is ¾. So, b = 12 x ¾ = 9.		
30	Copy polygons A-D onto grid paper. Draw line segments that divide each of the polygons into four congruent polygons that are similar to the original polygon. 	3.3	

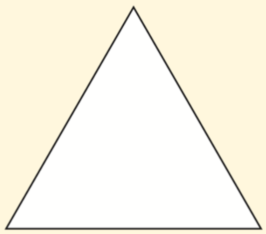
Extensions

Problem #	Answer	CMP4 Problem #	Note
31	Trace each shape. Divide each shape into four smaller, identical pieces that are similar to the original shape.  	3.1	
32	You can subdivide figures to get smaller figures that are mathematically similar to the original. The mathematician Benoit Mandelbrot called these figures fractals. A famous example is the	3.1	

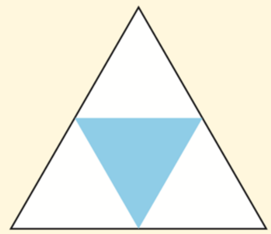
Sierpinski triangle.

Sierpinski Triangle

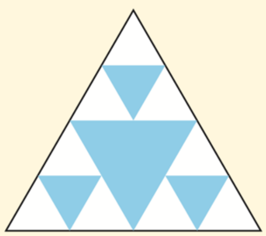
You can follow these steps to make the Sierpinski triangle.



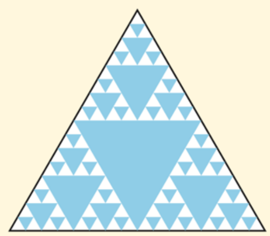
Step 1: Draw a triangle. (It does not have to be an equilateral triangle.)



Step 2: Mark the midpoint of each side. Connect the midpoints to form four identical triangles that are similar to the original. Shade the center triangle.

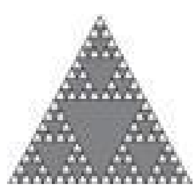


Step 3: For each unshaded triangle, mark the midpoints. Connect them in order to form four identical triangles. Shade the center triangle in each case.



Step 4: Repeat Steps 2 and 3 over and over. To make a real Sierpinski triangle, you need to repeat the process an infinite number of times! This triangle shows five subdivisions.

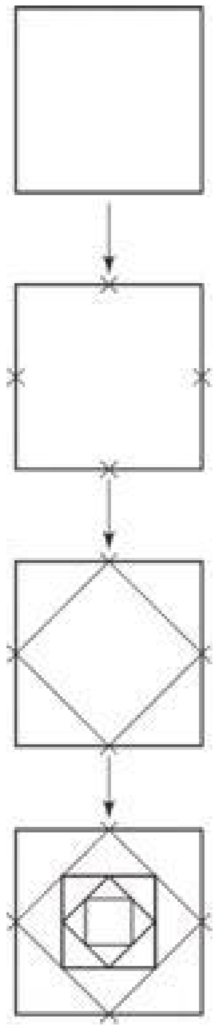
a. Follow the steps for making the Sierpinski triangle until you subdivide the original triangle three times.



b. Describe any patterns you observe in your figure.

Answers will vary. Possible answer: At each step, the side length of the new triangle is ½ the side length of the triangle from the previous step. The area of the new triangle is ¼ the area of the triangle of the previous step. The number of new shaded triangles obtained at each step follows the following pattern: 1, 3, 9, 27, … , 3^n (for the $n+1^{st}$ step).

	c. Mandelbrot used the term *self-similar* to describe fractals like the Sierpinski triangle. What do you think this term means? *Self-similar* means that the original figure is similar to a smaller part of itself. You can apply a reduction to the original figure and obtain a new figure that is the same as a part of the original figure.		
33	The **midpoint** of a line segment is a point that divides the segment into two segments of equal length. Draw a figure on grid paper by following these steps: **Step 1**: Draw a large square. **Step 2**: Mark the midpoint of each side. **Step 3**: Connect the midpoints, in order, with four line segments to form a new figure. (The line segments should not intersect inside the square.) **Step 4**: Repeat Step 2 and 3 three more times. Work with the newest figure each time. a. What kind of figure is formed when the midpoints of the sides of a square are connected? Another square. A sample drawing is below. The first, second, and fifth figures are shown.	3.2	



b. Find the area of the original square you drew in Step 1.

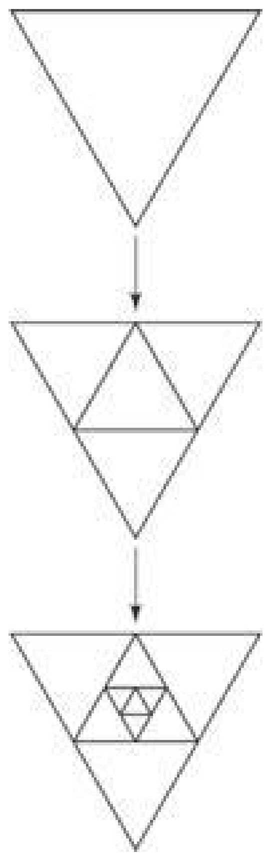
Answers will vary depending on how large a square they drew.

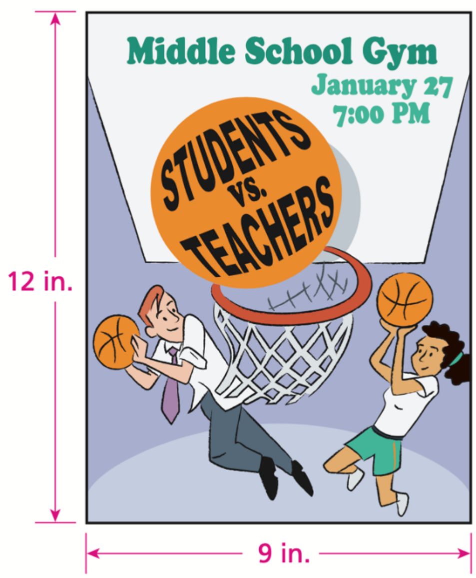
c. Find the area of each of the new figures that was formed.

Answers will vary, but each square should have ½ the area of the square before it.

d. How do the areas change between successive figures?

At each step, the area of the new square is ½ the area of the previous square.

	e. Are there any similar figures in your final drawing? Explain. All the squares are similar to each other. Also, all the triangles are similar to each other.		
34	Repeat Exercise 44 starting with an equilateral triangle, connecting three line segments to form a new triangle each time. a. Another equilateral triangle is formed. A sample drawing is below. The first, second, and third figures are shown.  b. Answers will vary depending on how large a triangle they draw. c. The answer should be the ¼ area of the original triangle. d. At each step, the area of the new triangle is ¼ the area of the previous triangle. e. All the triangles in the figures are similar to each other.	3.2	
35	Suppose Rectangle A is similar to Rectangle B and to Rectangle C. Can	3.2	

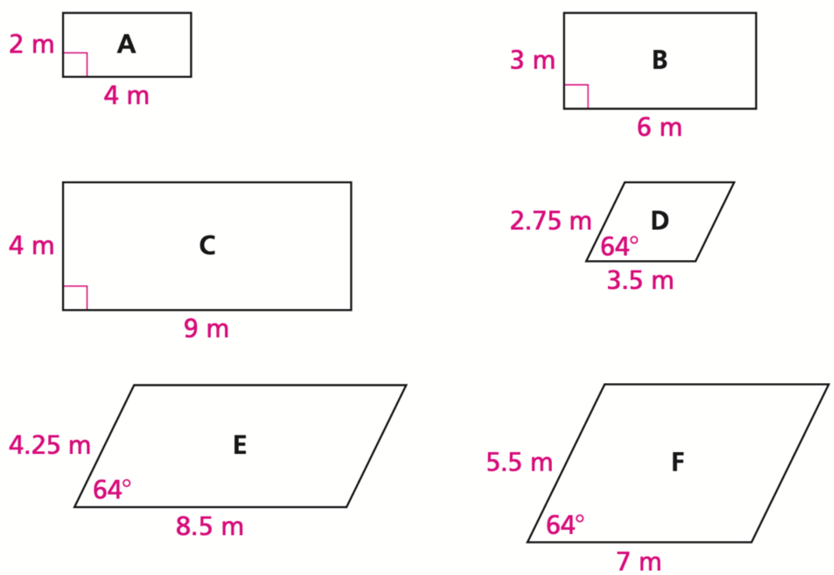
	you conclude that Rectangle B is similar to Rectangle C? Explain. Use drawings and examples to illustrate your answer. Yes. Rectangle B is similar to Rectangle C. Possible explanation: Because Rectangle A is similar to Rectangle B, the ratio of the short side of Rectangle A to the long side of Rectangle A is the same as the ratio of the short side of Rectangle B to the long side of Rectangle B. Because Rectangle A is similar to Rectangle C, the ratio of the short side of Rectangle C to the long side of the Rectangle C must equal to this same ratio. This means the ratio between sides in Rectangle C equals the ratio between sides in Rectangle B, making Rectangles C and B similar.		
36	Song makes a copy of the poster below.  a. She presses the 50% button on the copy machine. Now the length and width of the poster are each half of their original sizes. Song thinks that if she enlarges the copy by 150%, the new copy will be the same as the original. Is she correct? Song's conjecture is incorrect. After the first reduction her copy will be 4.5 in. x 6 in. When she enlarges it by 150%, it will be 6.75 in. x 9 in. This is 50% larger than the original copy. b. Suppose Song had done the opposite in part (a), first enlarging the poster by 150%, and then reducing the copy by 50%. Will the final copy be the same size as the original? Will it be the same size as the copy made in part (a)? After the enlargement, her copy will be 13.5 in. x 18 in. Reducing this by 50% produces a copy 6.75 in. x 9 in. This is the same as in part (a), but still smaller than the original.	3.3	

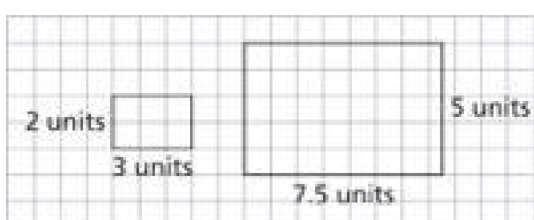
	c. Song uses the same process from parts (a) and (b) with a different-sized poster. Does she get similar results? Yes. The results are similar. Using the method in part (a), the scale factor is ½, then the copy is enlarged by a scale factor of 3/2. The sides of the final copy are ½ x 3/2, or ¾ the original size. If the process is reversed, the same result occurs because 3/2 x ½ = ¾. d. Song applied a scale factor of 25% to shrink the original poster. Now she wants to get the poster back to the original size. What scale factor should she use? Explain your reasoning. Song should choose to enlarge the original by 400%, or a scale factor of 4. This would "undo" the reduction by 25%, or scale factor ¼, because 4 x ¼ = 1. e. Suppose Song had used 75% and 125% in parts (a) and (b) instead of 50% and 150%. What would have happened? Answers will vary, but two likely responses include the second copy being smaller than the original. Also, similarly to part (c), reversing the order of a reduction and enlargement does not affect the final copy. f. What general statements can you make about applying any pair of two scale factors one after the other? Consider a pair of two enlargements, a pair of two reductions, and a pair consisting of one enlargement and one reduction. Answers will vary. Possible answers may include: Reversing the order of the scale factors results in the same final copy; Two reductions result in a copy smaller than either of the reductions would produce on its own; Two enlargements result in a copy larger than either of the enlargements would produce on its own; A reduction paired with an enlargement could result in a copy larger than, smaller than, or the same size as the original. You can make a copy the same size as the original if you apply an enlargement or reduction and its reciprocal.		

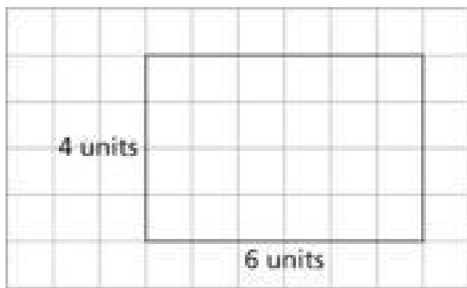
Investigation 4

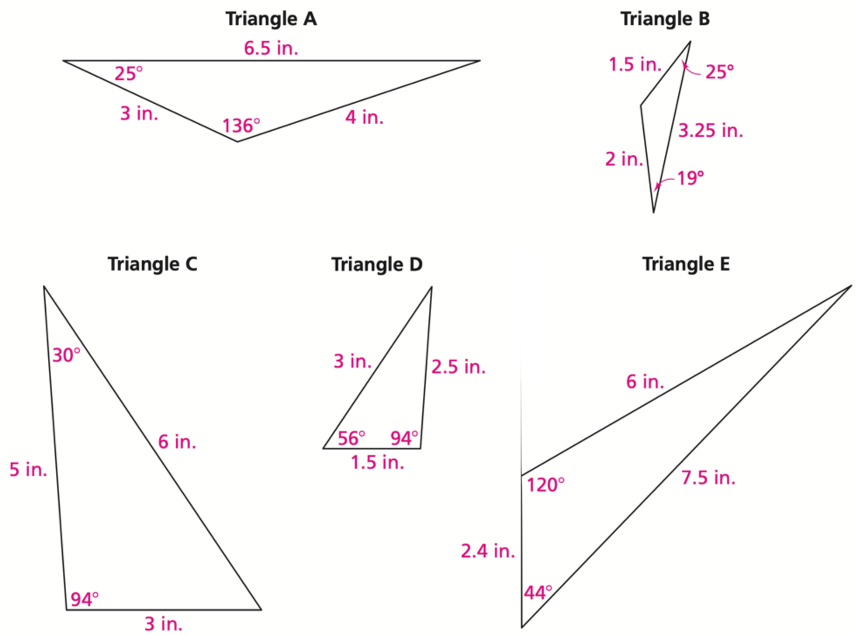
	Applications	Connections	Extensions	Total
4.1	3	4	4	11
4.2	4	5	5	14
4.3	3	4	4	11
Total	10	13	13	36

Applications

Problem #	Answer	CMP4 Problem #	Note
1	For parts (a)-(c), use the parallelograms below.  a. List all the pairs of similar parallelograms. Explain your reasoning. Rectangles A and B are similar because the ratio of 2 to 4 (short side to long side in A) is equal to the ratio of 3 to 6 (short side to long side in B). Also, the scale factor from A to B is constant: 1.5. Parallelograms D and F are similar because the ratio of 2.75 to 3.5 (short side to long side in D) is equal to the ratio of 5.5 to 7 (short side to long side in F), and the corresponding angles are the same measure. Also, the scale factor from D to F is constant: 2.	4.1	

	b. For each pair of similar parallelograms, find the ratio of two adjacent side lengths in one parallelogram. Find the ratio of the corresponding side lengths in the other parallelogram. How do these ratios compare? Rectangle A: 2/4 = 0.5; Rectangle B: 3/6 = 0.5 Parallelogram D: 2.75/3.5 ≈ 0.786; Parallelogram F: 5.5/7 ≈ 0.786 The ratios for A and B are equivalent; also, the ratios for D and F are equivalent. c. For each pair of similar parallelograms, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different from the information given by the ratios of adjacent side lengths. The scale factor from A to B is 1.5. The scale factor from D to F is 2. Each of these scale factors is different from the ratios found in part (b). The scale factor compares a side in one figure to its corresponding side in a similar figure. The scale factor has to be constant from one shape to the other for any pair of corresponding sides. The ratio of adjacent side lengths describes the relationship between two measures within one shape. The ratio in one shape has to be equivalent to the ratio of corresponding sides in the similar shape.		
2	a. On grid paper, draw two similar rectangles where the scale factor from one rectangle to the other is 2.5. Label the length and width of each rectangle. Answers will vary. Possible answer:  b. For each rectangle, find the ratio of the length to the width.	4.1	

	Answers will vary. For the sketch above, the smaller rectangle’s ratio is 3 to 2; the larger rectangle’s ratio is 7.5 to 5. **Note**: This assumes that the length is the longer dimension. The answer varies depending on the dimensions of the rectangles drawn. c. Draw a third rectangle that is similar to one of the rectangles in part (a). Find the scale factor from the new rectangle to the one from part (a). Answers will vary. Possible answer for the sketch above is drawn here. The scale factors from this rectangle to the rectangles in part (a) are ½ and 1 ¼.  d. Find the ratio of the length to the width for the new rectangle. Answers will vary. Ratios of length to width are equivalent in all similar rectangles, so the answer should be equivalent to the answer for part (b). For the sketch in part (c), the ratio of the length to the width is 6 to 4. e. What can you say about the length-to-width ratios of the three rectangles? Is this true for another rectangle that is similar to one of the three rectangles? Explain. The length-to-width ratios of the three rectangles are equivalent. If you were to sketch another rectangle similar to the first three, the length-to-width ratio of that rectangle would be equivalent as well. Since there is a common scale factor for all corresponding side lengths, both parts of the length-to-width ratio would grow by the same amount. This would make the new ratio equivalent to the previous three.		
3	For part (a)-(d), use the triangles below. The drawings are not to scale.	4.1	



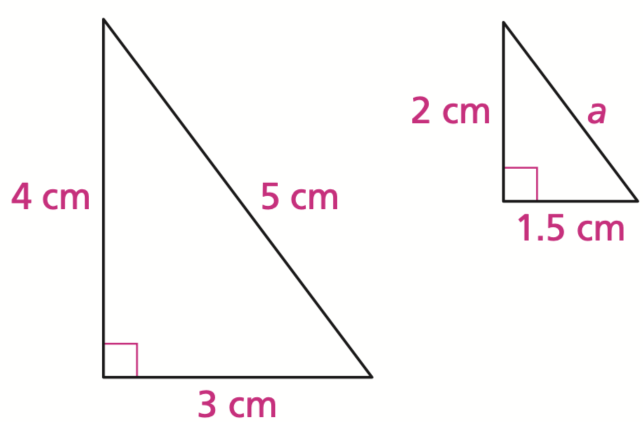
a. List all the pairs of similar triangles. Explain why they are similar.

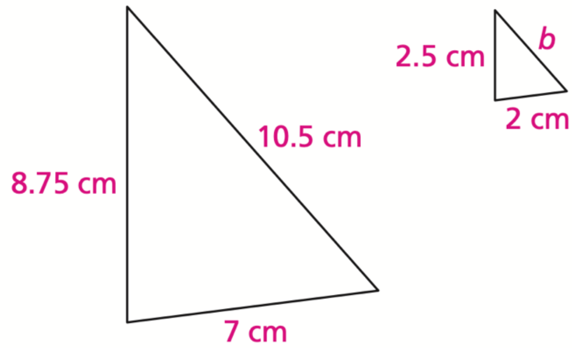
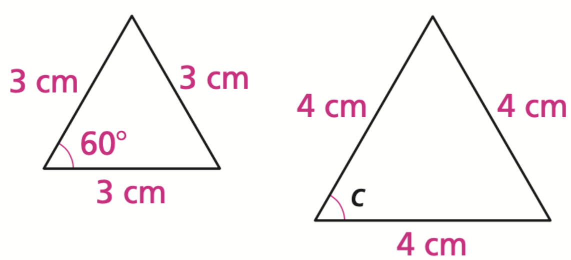
A and B are similar. C and D are similar. They are similar because their corresponding angle measures are congruent. Also, each ratio of adjacent side lengths within one figure is equivalent to the ratio of corresponding side lengths in the similar figure. Last, scale factors from each side length in one figure to the corresponding side length in the similar figure are constant.

b. For each pair of similar triangles, find the ratio of two side lengths in one triangle. Find the ratio of the corresponding side lengths in the other. How do these ratios compare?

The side-length ratios for Triangle A are 3 to 4, 3 to 6.5, and 4 to 6.5. The corresponding side-length ratios for Triangle B are 1.5 to 2, 1.5 to 3.25, and 2 to 3.25. When simplified, the ratios for Triangle A and Triangle B are equivalent: ¾ = 1.5/1, 3/6.5 = 6/13 = 1.5/3.25, and 4/6.5 = 8/13 = 2/3.25.

The side-length ratios for Triangle C are 3 to 5, 3 to 6, and 5 to 6. The corresponding side-length ratios for Triangle D are 1.5 to 2.5, 1.5 to 3, and 2.5 to 3. When simplified, the ratios for Triangle C and Triangle D are equivalent: 3/5 = 1.5/2.5, 3/6 = 1.5/3, and 5/6 = 2.5/3.

	c. For each pair of similar triangles, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different than the information given by the ratios of side lengths. The scale factor from A to B is ½. The scale factor from C to D is ½. The scale factors of these similar triangles tell how many times as great the corresponding side lengths or perimeter are from one figure to a similar figure. The ratio of adjacent side lengths within one triangle tells how many times as great one side length of the triangle is to another side length in the same triangle. d. How are corresponding angles related in similar triangles? Is it the same relationship as for corresponding side lengths? Explain. In similar triangles, corresponding angles are congruent. This is different from corresponding side lengths because corresponding side lengths vary by a consistent scale factor.		
4	**Each pair of figures is similar. Find the missing measurement. Explain your reasoning. (Note: The figures are not drawn to scale.)**  **a.** 2.5 cm; Possible explanation: The scale factor from large triangle to the small triangle is 0.5. So, a = 5 x 0.5 = 2.5.	4.2	

	b.  3 cm; Possible explanation: The ratio of 10.5 to 7 in decimal form is 1.5. Then, the ratio of b to 2 should also be 1.5. So, b = 2 x 1.5 = 3. c.  60°; Possible explanation: Angle c of the large triangle corresponds to the angle of the small triangle that measures 60°. Since corresponding angle measures of similar triangles are the same, c must be 60°. **Note:** All equilateral triangles are similar because the angles in an equilateral triangle are all 60°. Also, for all equilateral triangles, each ratio of adjacent sides is 1.		

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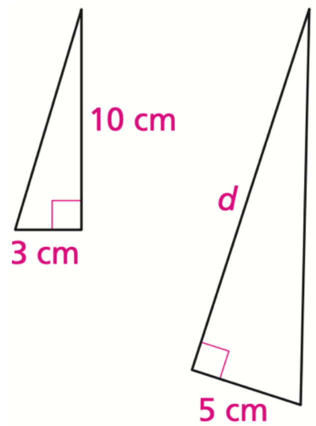
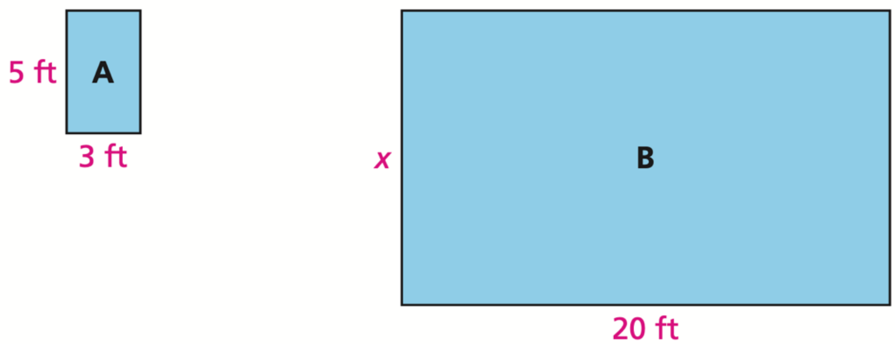
<tr><td></td><td>

d.
d ≈ 16.7; Possible explanation: 10/3 = d/5, so d = 50/3 ≈ 16.7.</td><td></td><td></td></tr>
<tr><td colspan="4">For Exercises 5-7, Rectangles A and B are similar.

</td></tr>
<tr><td>5</td><td>Multiple Choice What is the value of x?

A. 4 B. 12 C. 15 D 33 1/3

B; The scale factor from A to B is 4, since the side of 5 ft corresponds to the side of 20 ft. The side of x corresponds to the side of 3 ft. So, x = 3 x 4 = 12.</td><td>4.2</td><td></td></tr>
<tr><td>6</td><td>What is the scale factor from Rectangle B to Rectangle A?

0.25; 20 x ¼ = 5, and ¼ = 0.25.</td><td>4.2</td><td></td></tr>
<tr><td>7</td><td>Find the area of each rectangle. How are the areas related?

The area of A is 15 cm^2 and the area of B is 240 cm^2

The area of Rectangle B is 16, or 4^2 (the square of the scale factor), times the area of Rectangle A.</td><td>4.2</td><td></td></tr>
<tr><td>8</td><td>The Washington Monument is the tallest structure in Washington D.C. At a certain time, the monument casts a shadow that is about 500 feet</td><td>4.3</td><td></td></tr>
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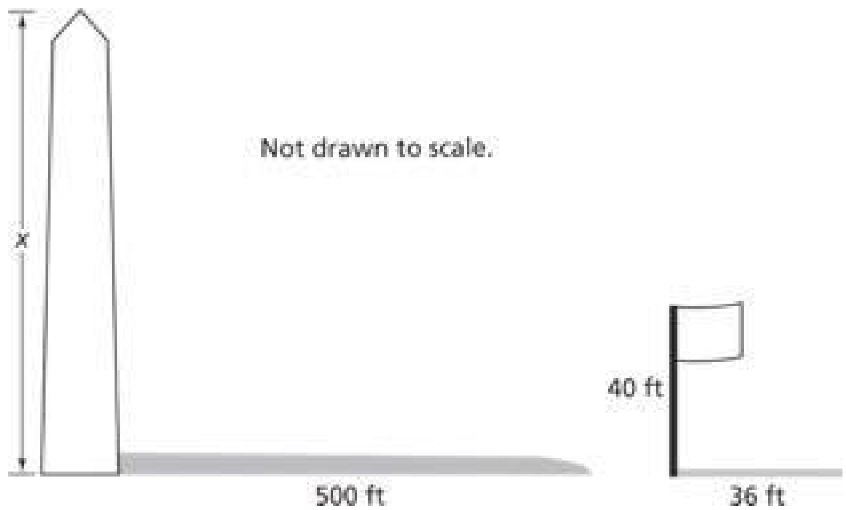
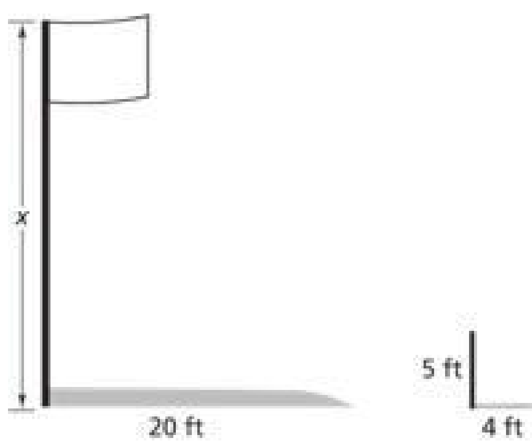
	long. At the same time, a 40-foot flagpole nearby casts a shadow that is about 36 feet long. About how tall is the monument? Sketch a diagram. About 556 feet; sample sketch is shown below.  The scale factor is 500/36. So, x = 40 x 500/36.		
9	Darius uses the shadow method to estimate the height of a flagpole. He finds that a 5-foot stick casts a 4-foot shadow. At the same time, he finds that the flagpole casts a 20-foot shadow. What is the height of the flagpole? Sketch a diagram. 25 feet; sample sketch is shown below.  The scale factor from the stick to the flagpole is 5. So, x = 5 x 5 = 25.	4.3	
10	a. Greg and Zola are trying to find the height of their school building. Zola takes a picture of Greg standing next to the	4.3	

	building. How might this picture help them determine the height of the building? One way is to determine the scale factor between Greg's image in the picture and Greg's actual height. For example, if Greg is 1 inch tall in the picture and is 5 feet (or 60 inches) tall in real life, the scale factor from picture to real life is 60/1. You can then measure the height of the building in the picture and multiply that height by 60 to find the actual height of the building. b. Greg is 5 feet tall. The picture Zola took shows Greg as ¼ inch tall. If the building is 25 feet tall in real life, how tall should the building be in the picture? Explain. 1.25 inches; Since Greg is 5 feet tall in real life and is ¼ (or 0.25) inch tall on the screen, the scale factor from real life to the picture is 0.25/60. The building is 25 feet, or 300 inches. If you multiply that height by 0.25/60, the result is the height of the building on the screen: 1.25 inches. c. In part (a), you thought of ways to use a picture to find the height of an object. Think of an object in your school that is different to measure directly, such as a high wall, bookshelf, or trophy case. Describe how you might find the height of the object. You need to take a picture with the tall object and another object that you know the actual height of. Then, you can determine the scale factor from the picture to the real object. To determine the scale factor, you need to divide one object's real height by the object's height in the picture. Then, measure the height of the tall object in the picture and multiply it by the scale factor.		

Connections

Problem #	Answer	CMP4 Problem #	Note
11	**Tell whether each pair of ratios is equivalent.** a. 3 to 2 and 5 to 4 Not equivalent	4.1	

	b. 8 to 4 and 12 to 8 Not equivalent c. 7 to 5 and 21 to 15 Equivalent **d.** 1.5 to 0.5 and 6 to 2 Equivalent		
12	Use a pair of equivalent ratios from Exercise 11. Write a similarity problem using the ratios. Explain how to solve the problem. Answers will vary.	4.1	
13	**For each ratio write two other equivalent ratios.** a. 5 to 3 Answers will vary. Sample answer: 10 to 6 and 15 to 9. b. 4 to 1 Answers will vary. Sample answer: 8 to 2 and 20 to 5. c. 3 to 7 Answers will vary. Sample answer: 12 to 28 and 7.5 and 17.5. d. 1.5 to 1 Answers will vary. Sample answer: 3 to 2 and 0.75 to 0.5. **Note:** For each answer the division of the first number by the second number should give the same result as the division of the first number in the question by the second number in the question.	4.1	
14	Rectangle C and D are similar.	4.1	

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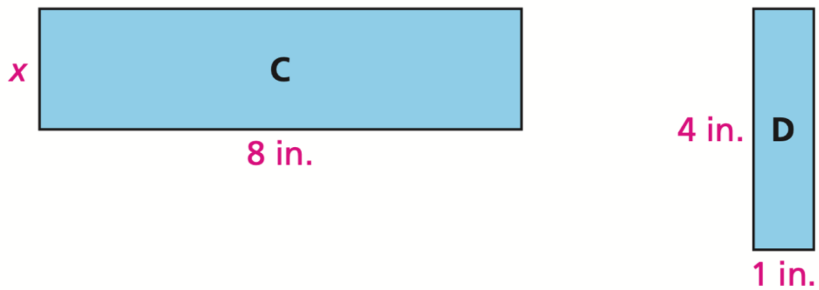

a. What is the value of x?

x = 2 in.

b. What is the scale factor from Rectangle C to Rectangle D?

0.5

c. Find the area of each rectangle. How are the areas related?

The area of C is 16 square inches. The area of D is 4 square inches. The area of D is ¼ the area of C. The factor ¼ is found by taking the square of the scale factor, ½. $\frac{1}{4} = \left(\frac{1}{2}\right)^2$
</td>
<td></td>
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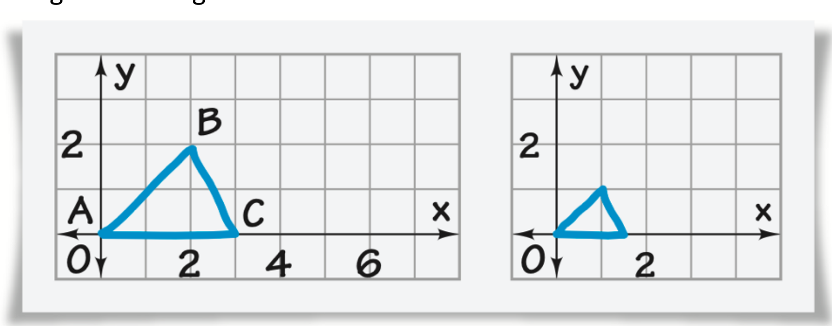
<tr>
<td>15</td>
<td>
Here is a picture of Duke. The scale factor from Duke to the picture is 12.5%. Use an inch ruler to make any measurements.

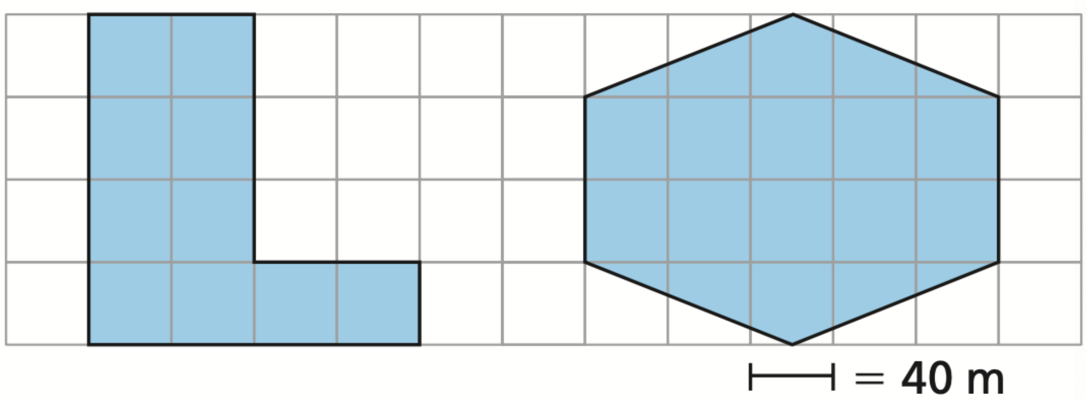
a. How long is Duke from his nose to the tip of his tail? Explain
</td>
<td>4.2</td>
<td>The picture must be 5 inch wide when it’s printed.</td>
</tr>
</table>

	how you used the picture to find your answer. 40 in. Explanations will vary. Possible answer: The dog in the picture is 5 inches long. Since the picture of the dog is 12.5% of the real dog's size, then 0.125 x the real length = the picture's length, or 5 inches. You can rewrite this as L = 5 ÷ 0.125, or 40 in. b. To build a doghouse for Duke, you need to know his height. How tall is Duke? Explain. 23 in. The dog in the picture is 2 7/8 in. tall. 2 7/8 is 0.125 times the real dog's height, so you can divide 2 7/8 by the scale factor, 0.125, to find the real height of the dog. c. A copy center has a machine that prints on poster-size paper. You can resize an image from 50% to 200%. How can you use the machine to make a life-size picture of Duke? Duke is 8 times as large as the picture. Using 200, enlargement, you can double the size of the picture. Use the 200, enlargement three times in a row: 2 x 2 x 2 = 8 times as large a picture.		
16	Movie screens often have an *aspect ratio* of 16 by 9. This means that for every 16 feet of width along the base of the screen there are 9 feet of height. The width of the screen at a local drive-in theater is about 115 feet wide. The screen has a 16:9 aspect ratio. About how tall is the screen? Approximately 65 feet; solve for h in the proportion 16/9 = 115/h; or, multiply 115 feet by the scale factor 9/16.	4.2	
17	Paloma draws triangle ABC on a grid. She applies a rule to make the triangle on the right.  	4.2	

	a. What rule did Paloma apply to make the new triangle? (0.5x, 0.5y) b. Is the new triangle similar to triangle ABC? Explain your reasoning. If the triangles are similar, give the scale factor from triangle ABC to the new triangle. Yes, they are similar. The scale factor is 0.5 from triangle ABC to the new triangle.		

For Exercises 18 and 19, use the paragraph below.

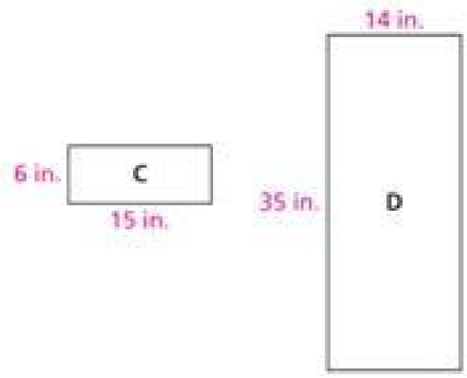
The Rosavilla School District wants to build a new middle school building. They ask architects to make scale drawings of possible layouts for the building. Two possibilities are shown below.



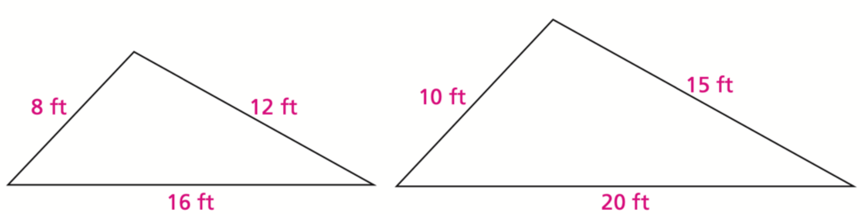
18	a. What is the area of each scale drawing in square units? 10 square units; 15 square units b. What would the area of the ground floor of each building be? 16,000 m^2; 24,000 m^2	4.2	
19	**Multiple Choice** The school board likes the L-shaped layout but wants a building with more space. They increase the L-shaped layout by a scale factor of 2. For the new layout, choose the correct statement. F. The area is two times the original.	4.2	

	G. The area is four times the original. H. The area is eight times the original. J. None of the statements above are correct. G		
20	For each angle measure, find the measure of its complement and the measure of its supplement. **Sample**: 30° complement: 60°; supplement: 150° a. 20° complement: 70°; supplement: 160° b. 70° complement: 20°; supplement: 110° c. 45° complement: 45°; supplement: 135°	4.3	
21	Rectangles A and B are similar.  a. What is the scale factor from Rectangle A to Rectangle B? 1/3; 12 x 1/3 = 4 b. Complete the following sentence in two different ways. Use the side lengths of Rectangle A and B.	4.3	

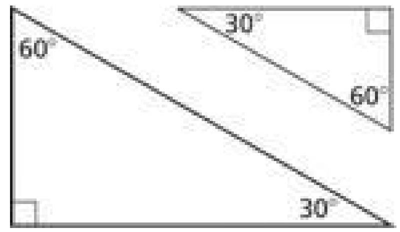
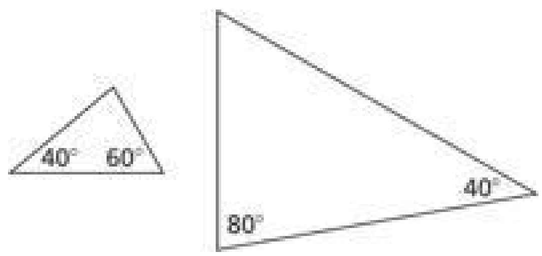
	The ratio of ■ to ■ is equivalent to the ratio of ■ to ■. Possible answer: The ratio of 6 to 12 is equivalent to the ratio of x to 4. The ratio of 6 to x is equivalent to the ratio of 12 to 4. c. What is the value of x? Explain your reasoning. x = 2 cm; the length of the short side of Rectangle A is half the length of the long side of Rectangle A. So, x must be half the length of 4 cm, which is 2 cm. d. What is the ratio of the area of Rectangle A to the area of Rectangle B? 9 to 1		
22	Triangle A has sides that measure 4 inches, 5 inches, and 6 inches. Triangle B has sides that measure 8 feet, 10 feet, and 12 feet. Taylor and Landon are discussing whether the two triangles are similar. Do you agree with Taylor or with Landon? Explain. **Taylor's Explanation** The triangles are similar. If you double each of the side lengths of Triangle A, you get the side lengths for Triangle B. **Landon's Explanation** The triangles are not similar. Taylor's method works when two measures have the same units. However, the sides of Triangle A are measured in inches, and the sides of Triangle B are measured in feet. So, they cannot be similar. Taylor is right. The two triangles ar similar, because if you convert the measures of Triangle B to inches, the scale factor from A to B is 24 for each side.	4.3	
23	Anya and Jalen disagree about whether the two figures below are similar. Do you agree with Anya or with Jalen? Explain.	4.3	

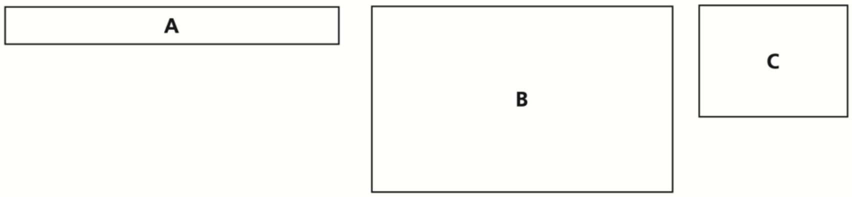
	 **Anya's Reasoning** The two rectangles are not similar. The height of Rectangle D is almost 6 times the height of Rectangle C, but the widths are almost the same. Similar rectangles must have the same scale factor for the base and the height. **Jalen's Reasoning** The two rectangles are similar. The scale factor from C to D is 7/3. You can multiply the short side of C (the height) by to get 14 inches, which is the short side of D (the base). This scale factor also works for the long sides of the rectangles since 15 x 7/3 = 35. Jalen's reasoning is correct. Similarity between shapes does not depend on orientation. If you increase the size of the smaller rectangle by 7/3, then rotate it 90°, the result would be the larger rectangle.		

Extensions

Problem #	Answer	CMP4 Problem #	Note
24	For parts (a)-(e), use the similar triangles below.  a. What is the scale factor from the smaller triangle to the larger triangle? Write your answer as a fraction and a decimal.	4.1	

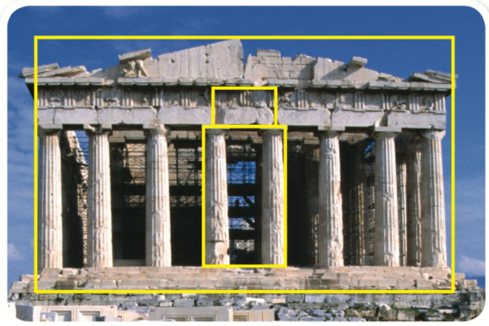
	10/8 = 1.25. b. Choose any side of the larger triangle. Find the ratio of this side length to the corresponding side length in the smaller triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (a)? The ratio using the longest sides is 20/16 = 1.25. The same ratio is obtained using other sides as well. This ratio is the same as the scale factor in part (a). c. What is the scale factor from the larger triangle to the smaller triangle? Write your answer as a fraction and a decimal. 8/10 = 0.8 d. Choose any side of the smaller triangle. Find the ratio of this side length to the corresponding side length in the larger triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (c)? The ratio using the longest sides is 16/20 = 0.8. The same ratio is obtained using other sides as well. This ratio is the same as the scale factor in part (c). e. What patterns do you notice in parts (a)-(d)? Are these patterns the same for any pair of similar figures? Explain. The scale factor tells by what factor each side is enlarged or reduced. The ratio of the corresponding sides is measuring the same quantity. Yes, the pattern will be true in general.		
25	For part (a) and (b), use a straightedge and an angle ruler or protractor. a. Draw two different triangles that each have angle measures of 30°, 60°, and 90°. Do the triangles appear to be similar? The drawings vary. However, all triangles with the given angles will be similar to each other.	4.1	

	 b. Draw two different triangles that each have angle measures of 40°, 80°, and 60°. Do the triangles appear to be similar? The drawings vary. However, all triangles with the given angles will be similar to each other.  c. Based on your findings for parts (a) and (b), make a conjecture about triangles with congruent angle measures. Conjecture: If the interior angle measures of a triangle are the same as those of another triangle, then the triangles are similar.		
26	One of these rectangles is “most pleasing to the eye.”	4.1	



The question of what shapes are most attractive has interested builders, artists, and craftspeople for thousands of years.

The ancient Greeks were particularly attracted to rectangular shapes similar to Rectangle B above. They referred to such shapes as "golden rectangles." They used golden rectangles frequently in buildings and monuments. The ratio of the length to the width in a golden rectangle is called the "golden ratio."



This photograph of the Parthenon (a temple in Athens, Greece) shows several golden rectangles.

a. Measure the length and width of Rectangles A, B, and C above in centimeters. For each rectangle, estimate the ratio of the length to the width as accurately as possible. The ratio for Rectangle B is an approximation of the golden ratio.

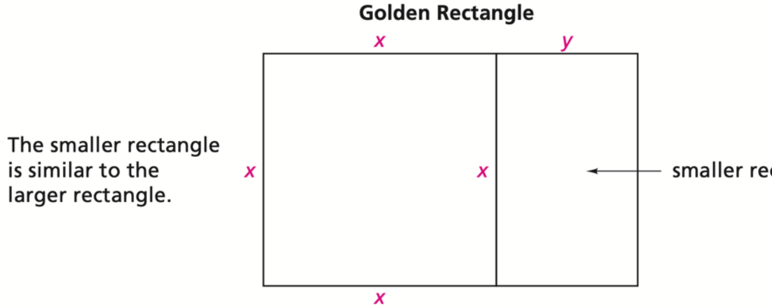
Rectangle A: ratio is 2.25 to 0.25

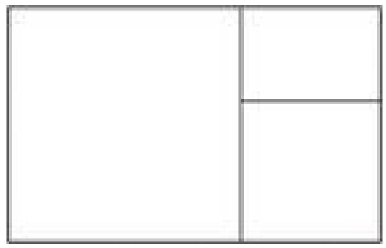
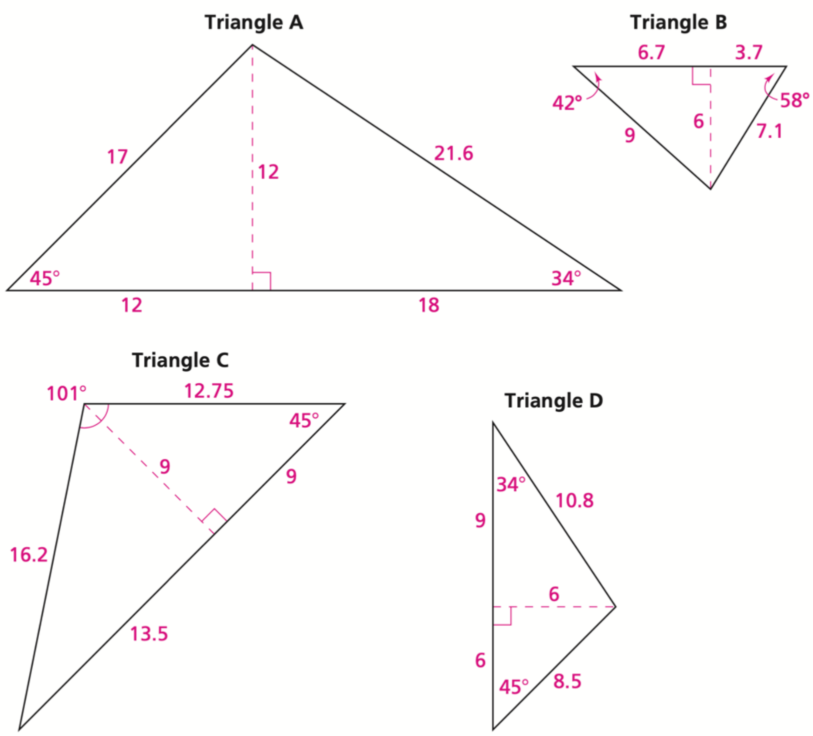
Rectangle B: ratio is 2 to 1.25, which gives 1.6 as a decimal number.

Rectangle C: ratio is 1 to 0.75

b. You can divide a golden rectangle into a square and a smaller rectangle similar to the original rectangle.

Golden Rectangle



	Copy Rectangle B above. Divide this golden rectangle into a square and a rectangle. Is the smaller rectangle a golden rectangle? Explain.  The smaller rectangle is a golden rectangle. The ratio of long side to short side is about 1.6.		
27	For parts (a) and (b), use the triangles below.  a. Identify the triangles that are similar to each other. Explain your reasoning. Triangle A, C, and D are similar. The corresponding angle measures are congruent. Also, the ratios between the	4.1	

	corresponding adjacent sides are the same. For example: Triangle A's longest side to shortest side: 17/30 ≈ 0.57; Triangle D's longest side to shortest side: 8.5/15 ≈ 0.57. They are the same. Triangle A's shortest side to middle side: 17/21.6 ≈ 0.79; Triangle C's shortest side to middle side: 12.75/16.2 ≈ 0.79. They are again the same. **Note**: Students have to use the fact that the sum of the angles in a triangle is 180°. b. For each triangle, find the ratio of the base to the height. How do these ratios compare for the similar triangles? How do these ratios compare for the non-similar triangles? Triangle A: 30/12 = 2.5 Triangle B: 10.4/6 ≈ 1.73 Triangle C: 22.5/9 = 2.5 Triangle D: 15/6 = 2.5 Similar triangles have the same base-to-height ratio; the non-similar triangle does not share that ratio.		

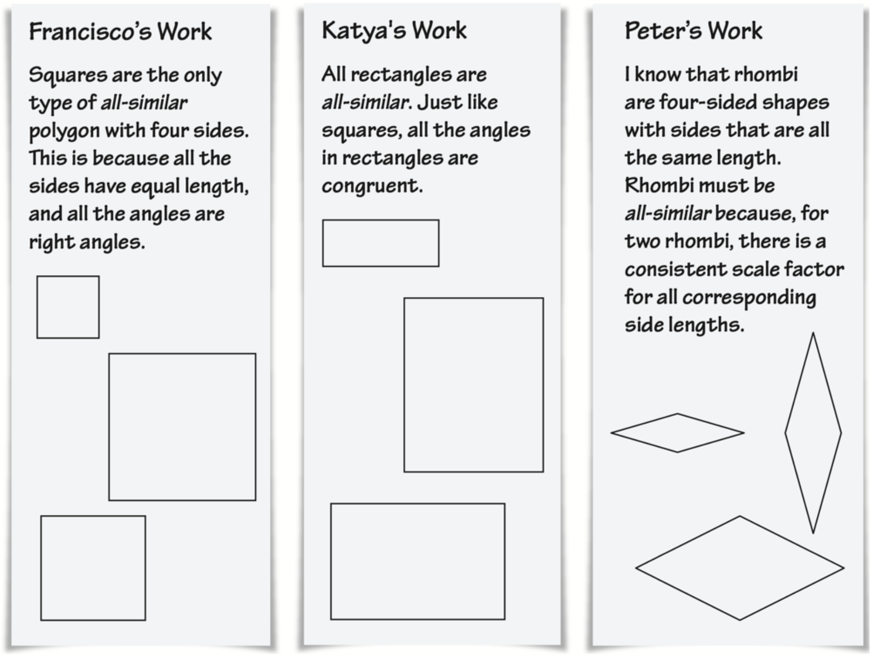
For Exercises 28-32, suppose a photographer for the school newspaper took this picture. The editors want to resize the photo to fit in a specific space on a page.

28	Can the original photo be changed to a similar rectangle with the given	4.2	

	measurements (in inches)? a. 8 by 12 Yes. 4/6 = 8/12 b. 9 by 11 No. 4/6 ≠ 9/11 c. 6 by 9 Yes. 4/6 = 6/9 d. 3 by 4.5 Yes. 4/6 = 3/4.5		
29	Suppose that the school copier only has three paper sizes (in inches): 8 ½ by 11, 11 by 14, 11 by 17. You can enlarge or reduce documents by specifying a percent from 50% to 200%. Can you make copies of the photo that fit exactly on any of the three paper sizes? Explain your reasoning. No. None of the given paper sizes have the same base-to-height ratio as the photo.	4.2	
30	A copy machine accepts scale factors from 50% to 200%. How can you use the copy machine to produce a copy that is 25% of the original photo's size? How does the area of the copy relate to the area of the original photo? Use the 50% reduction two times in a row (i.e., copy the photo once, then take the image and make its copy again.) Each time the drawing will be reduced to half its size. So, after two reductions it will be ½ x ½ = ¼ of its original size.	4.2	
31	How can you use the copy machine to reduce the photo to a copy that is 12.5% of the original photo's size? 36% of the original photo's size? How does the area of the reduced figure compare to the area of the original in each case? For a copy of 12.5%, applying a 50% reduction three times in a row using the image each time will reduce the size to 12/5% of its original dimensions. The area would be 1/64 the area of the original.	4.2	

	For a copy of 36%, apply a 60% reduction two times in a row. The area would be 81/625 the area of the original.		
32	What is the greatest enlargement of the photo that will fit on paper that is 11 inches by 17 inches? You can enlarge the original 4 inch by 6 inch picture until the side that is originally 4 inches is enlarged to 11 inches. The 6 inch side is then enlarged to 16.5 inches. This is the largest possible image to fit paper that is 11 inches by 17 inches. This requires an enlargement of 275%, because 11/4 x 100 = 1100/4 = 275. **Note**: To accomplish this, first enlarge the original photograph to 200%, then enlarge the result a second time using 125% (the new copy is now 2.5 times larger than the original). Finally take this copy and enlarge it 110%. Because the scale factors are multiplicative (and therefore commutative) the order in which these enlargements are done does not matter.	4.2	
33	Suppose you want to buy new carpeting for your bedroom. The bedroom floor is a 9-foot-by-12-foot rectangle. Carpeting is sold by the square yard. a. How much carpeting do you need to buy? 108 square feet, or 12 square yards. b. Carpeting costs $22 per square yard. How much will the carpet cost? $264; 12 x 22 = 264. Suppose you want to buy the carpet for a library. The library floor is similar to the floor of the 9-foot-by-12-foot bedroom. The scale factor from the bedroom to the library is 2.5. c. What are the dimensions of the library? Explain. 22.5 feet by 3o feet (or 7.5 yards by 10 yards); The dimensions of the library are 2.5 times the corresponding dimensions of the bedroom. d. How much carpeting do you need for the library? 675 square feet, or 75 square yards.	4.3	

	e. How much will the carpet for the library cost? $1,650		
34	The following sequence of numbers is called the *Fibonacci sequence*. It is named after an Italian mathematician from the 14th century who contributed to the early development of algebra. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 … a. Look for patterns in this sequence. How are the numbers found? Use your ideas to find the next four terms. Each number in the sequence is found by adding the previous two numbers. The following four numbers in the sequence will be: 610, 987, 1597, 2584. b. Find the ratio of each term to the term before it. For example, 1 to 1, 2 to 1, 3 to 2, and so on. Write each of the ratios as a fraction and as an equivalent decimal. Compare the results to the golden ratios you found in Exercise 44. Describe similarities and differences. The approximate decimal equivalents of the fractions in order are: 1, 2, 1.5, 1.67, 1.6, 1.625, 1.615, 1.619, 1.618, 1.618, 1.618, … (1.618 repeats). The sequence approaches a number that is very close to the estimation of the golden ratio in Exercise 44. (In fact, the "limit" of this sequence will be equal to the golden ratio.)	4.3	
35	Francisco, Katya, and Peter notice that all squares are similar. They wonder if other shapes that have four sides are *all-similar*. Who is correct?	4.3	

	 Francisco is right. All angles of a regular polygon are equal in measure, as are the sides. Since the measures of corresponding sides for any regular polygons with the same number of sides have the same ratio, a regular polygon of any size is similar to all other sizes of that regular polygon. Thus, as a regular polygon, all squares are similar. The ratio of side lengths of any square is always equal to 1:1:1:1, and all angels are 90°. Not all rectangles are similar because the ratios of the lengths of adjacent sides can be anything. So, some pairs of rectangles are not simple magnifications of each other. Not all rhombi are similar because their angles can be different, even for rhombi with the same side lengths as each other.		
36	Ernie and Vernon are having a discussion about *all-similar* shapes. Ernie says that regular polygons and circles are the only types of *all-similar* shapes. Vernon claims isosceles right triangles are *all-similar*, but they are not regular polygons. Who is correct? Explain. Vernon is right. In an isosceles right triangle, the equal sides make the right angle and they have the ratio 1:1. All isosceles right triangles are similar because any isosceles right triangle has same 45°, 45°, and 90° angles, and the sides have the ratio $1:1:\sqrt{2}$.	4.3	