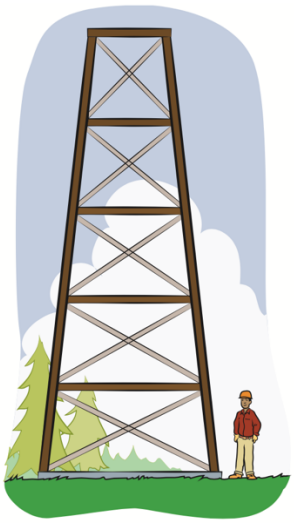
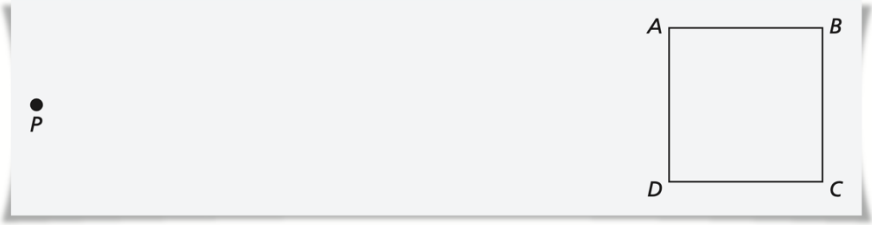
STRETCHING AND SHRINKING
Applications-Connections-Extensions
With Answers & Problem Correlations

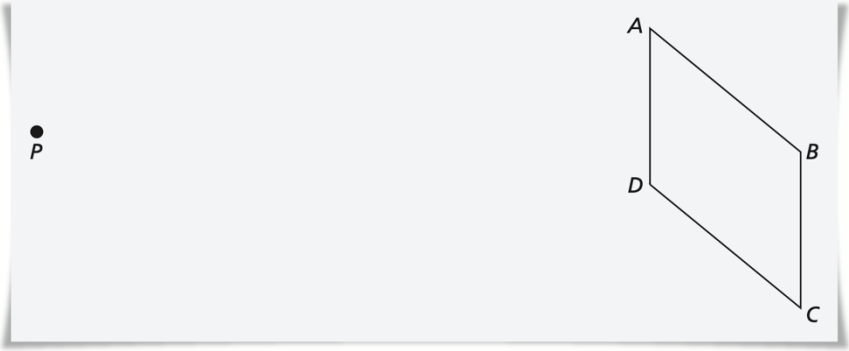
Investigation1

	Applications	Connections	Extensions	Total
1.1	2	4	1	7
1.2	2	2	2	6
1.3	3	2	1	6
Total	7	8	4	19

Applications

Problem #	Answer	CMP4 Problem #	Note
For Exercises 1 and 2, use the drawing below, which shows a person standing next to a ranger's outlook tower. 			
1	Find the approximate height of the tower if the person is a. 6 feet tall b. 5 feet 6 inches tall	1.1	
2	Find the approximate height of the person if the tower is	1.1	

	a. 28 feet tall b. 36 feet tall		
3	Copy square ABCD and anchor point P onto a sheet of paper. Use the rubber-band method to enlarge the figure. Then, answer parts (a)-(e) below.  a. How do the side lengths of the original figure compare to the side lengths of the image? b. How does the perimeter of the original figure compare to the perimeter of the image? c. How do the angle measures of the original compare to the angle measures of the image? d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image? e. How does the distance between each point in the original figure and P compare to the corresponding distances in the image?	1.2	
4	Copy parallelogram ABCD and anchor point P onto a sheet of paper. Use the rubber-band method to enlarge the figure. Then, answer parts (a)-(e) from Exercise 3 for your diagram.	1.2	



a. How do the side lengths of the original figure compare to the side lengths of the image?

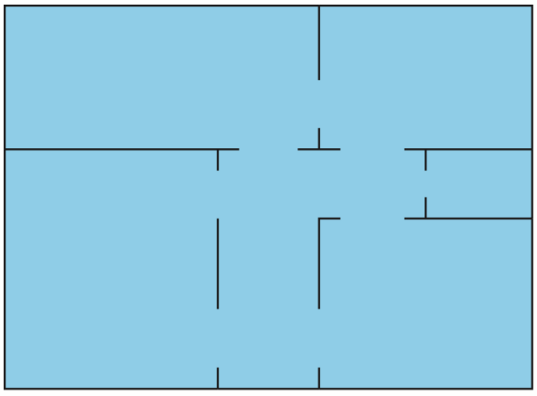
b. How does the perimeter of the original figure compare to the perimeter of the image?

c. How do the angle measures of the original compare to the angle measures of the image?

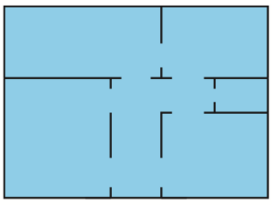
d. How does the area of the original figure compare to the area of the image? How many copies of the original figure would it take to cover the image?

e. How does the distance between each point in the original figure and P compare to the corresponding distances in the image?

5	The diagram on the left is the floor plan for a model house. The diagram on the right is a scale drawing of the floor plan. The scale drawing was made by reducing the original on a copy machine.	1.3	



Original

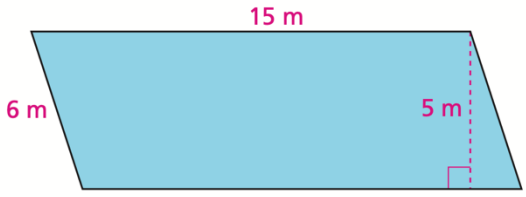


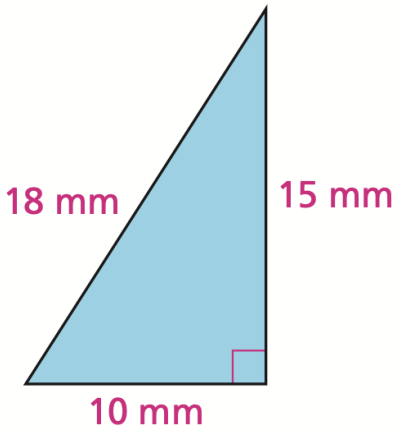
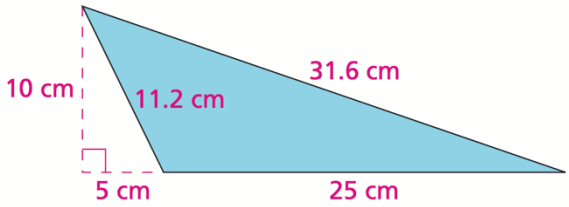
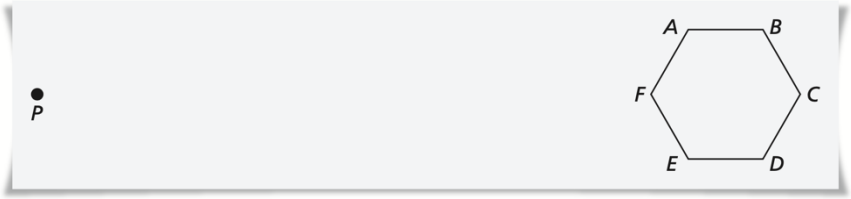
Reduced Image

	a. Estimate the copier size factor used. Give your answer as a percent. b. How do the segment lengths in the original plan compare to the corresponding segment lengths in the reduced image? c. Compare the area of the entire original floor plan to the area of the entire reduced image. Then, do the same with one room in the plan. Is the relationship between the areas of the rooms the same as the relationship between the areas of the whole plans? Explain. d. The scale on the original plan in 1 inch = 1 foot. This means that 1 inch on the floor plan represents 1 foot on the model house. What is the scale on the reduced plan?		
6	**Multiple Choice** Suppose you reduce the design below with a copy machine. Which of the following can be the image?  A.  B.  C.  D. 	1.3	
7	Suppose you copy a drawing of a polygon using the given size factor. How will the side lengths, angle measures, and perimeter of the image compare to those of the original? a. 200% b. 150%	1.3	

	c. 50% d. 75%		

Connections

Problem #	Answer	CMP4 Problem #	Note
For Exercises 8-11, find the perimeter and the area of each figure. In Exercises 10 and 11, the measurements are rounded.			
8		1.1	
9		1.1	

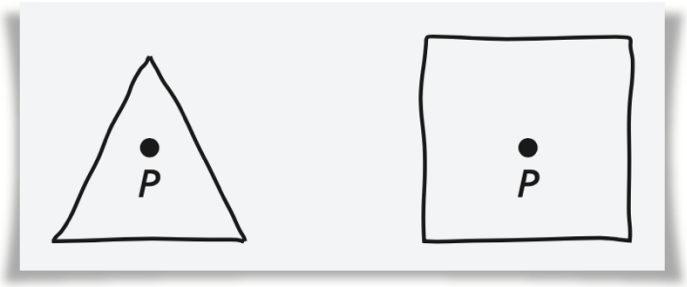
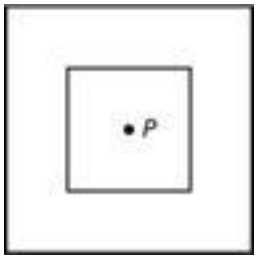
10		1.1	
11		1.1	
12	Copy hexagon ABCDEF and anchor point P onto a sheet of paper. Make an enlargement of the hexagon using your two-band stretcher.  a. How do the side lengths of the two hexagons compare? b. How do the angles of the hexagons compare? c. How do the areas of the hexagons compare? d. How do the perimeters of the hexagons compare?	1.2	

13	Make a three-band stretcher by tying three rubber bands together. Use this stretcher to enlarge the "Super Sleuth" drawing from Problem 1.1. a. How does the shape of the image compare to the shape of the original figure? b. How do the lengths of the segments in the two figures compare? c. How do the areas of the two figures compare? d. How do the distances from *P* compare?	1.2	
14	**Multiple Choice** What is the 28% tax on a $600,000 cash prize? A. $16,800 B. $21.429 C. $168,000 D. $214,290	1.3	

15	While shopping for sneakers, Ling finds two pairs she likes. One pair costs $55 and the other costs $165. She makes the following statements about the prices. “The expensive sneakers cost $110 more than the cheaper sneakers.” “The cost of the expensive sneakers is 300% of the cost of the cheaper sneakers.” “The cheaper sneakers are 1/3 the cost of the expensive sneakers.” a. Are all statements accurate? Explain. b. How are the comparison methods Ling uses like the methods you use to compare the sizes and shapes of similar figures? c. Which statements are appropriate for comparing the size and shape of an image to the original figure? Explain.	1.3	

Extensions

Problem #	Answer	CMP4 Problem #	Note
16	A movie projector that is 6 feet away from a large screen shows a rectangular picture that is 3 feet wide and 2 feet high.	1.1	

	a. Suppose the projector is moved to a point 12 feet from the screen. What size will the picture be (width, height, and area)? b. Suppose the projector is moved to a point 9 feet from the screen. What size will the picture be (width, height, and area)?		
17	Suppose you enlarge some triangles and squares with a two-band stretcher. You use an anchor point inside the original figure, as shown in the sketches below.  a. In each case, how do the shape and position of the image compare to the shape and position of the original? b. What relationships do you expect to find among the side lengths, angle measures, perimeters, and areas of the figures, and the distances from P? c. Test your ideas with larger copies of the given shapes. Make sure the shortest distance from the anchor point to any side of a shape is at least one band length.  	1.2	
18	Suppose you make a stretcher with two different-sized rubber	1.2	

	bands. Suppose the band attached to the anchor point is twice as long as the band attached to the pencil. a. If you used the stretcher to enlarge polygons, what relationships would you expect to find among the side lengths, angle measures, perimeters, and areas of the figures? b. Test your ideas with copies of some basic geometric shapes.		
19	Amy's friend gave her a picture from Field Day. The picture is 3 in. by 2 in. Amy has a picture frame that is 6 in. by 4 in. She wants the photo to fit in the frame exactly. What percent enlargement does she need to make? 	1.3	

Investigation 2

	Applications	Connections	Extensions	Total
2.1	3	3	1	7
2.2	4	3	3	10
2.3	6	4	3	13
Total	13	10	7	30

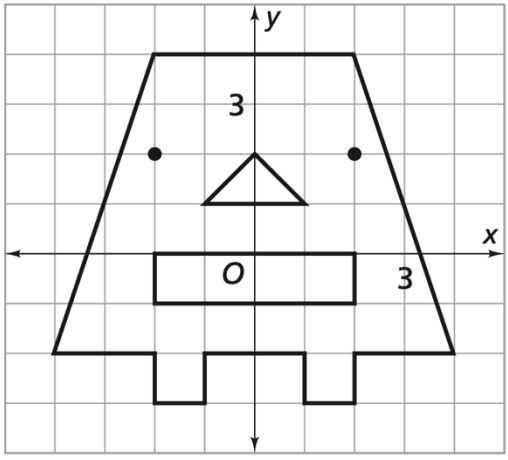
Applications

<table>
<tr><th>Problem #</th><th>Answer</th><th>CMP4 Problem #</th><th>Note</th></tr>
<tr><td>1</td><td>
The table below gives key coordinates for drawing Mug Wump's mouth and nose. It also gives rules for finding the corresponding points for four other characters-some members of the Wump family and some imposters.

Coordinates of Characters
<table>
<tr><th></th><th>Mug Wump</th><th>Glum</th><th>Sum</th><th>Tum</th><th>Crum</th></tr>
<tr><th>Rule</th><th>(x, y)</th><th>(1.5x, 1.5y)</th><th>(3x, 2y)</th><th>(4x, 4y)</th><th>(2x, y)</th></tr>
<tr><th>Point</th><th colspan="5">Mouth</th></tr>
<tr><td>M</td><td>(2, 2)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>N</td><td>(6, 2)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>O</td><td>(6, 3)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>P</td><td>(2, 3)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q</td><td>(2, 2) (connect Q to M)</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="5">Nose (Start Over)</td></tr>
<tr><td>R</td><td>(3, 4)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>S</td><td>(4, 5)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>T</td><td>(5, 4)</td><td></td><td></td><td></td><td></td></tr>
<tr><td>U</td><td>(3, 4) (connect U to R)</td><td></td><td></td><td></td><td></td></tr>
</table>
a. Before you find the coordinates or plot points, predict which characters are the imposters.

b. Copy and complete the table. Then, plot the figures on grid paper. Label each figure.

c. Which of the new characters (Glum, Sum, Tum, and Crum) are members of the Wump family? Which are
</td><td>2.1</td><td></td></tr>
</table>

	imposters? d. Choose one of the new Wumps. How do the mouth and nose measurements (side lengths, perimeter, area, angle measures) compare with those of Mug Wump? e. Choose one of the imposters. What are the dimensions of this imposter's mouth and nose? How do the mouth and nose measurements compare with those of Mug Wump? f. Do your findings in parts (b)-(e) support your prediction from part (a)? Explain.		
2	a. Design a Mug-like character of your own grid paper. Give your character eyes, a nose, and a mouth. b. Give coordinates so that someone else could draw your character. c. Write a rule for finding coordinates of a member of your character's family. Check your rule by plotting the figure. d. Write a rule for finding the coordinates of an imposter. Check your rule by plotting the figure.	2.1	
3	The diagram below shows Mug Wump drawn on a coordinate grid. Use this diagram to answer the questions.  a. Use the diagram, complete the first column of a table like	2.1	

the one shown to record coordinates of key points needed to draw Mug. (You will need to determine the number of points needed for each body part.)

Coordinates for Mug and Variations

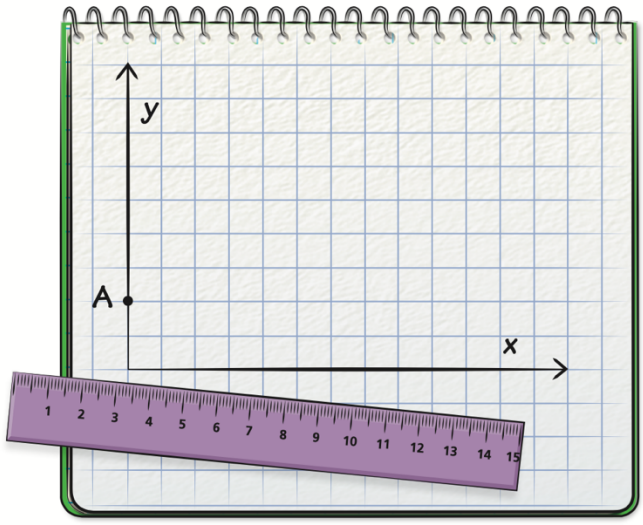
Rule	(x, y)	$(2x, 2y)$	$(-2x, -2y)$
Head Outline	$(-4, -2)$	■	■
	$(-2, -2)$	■	■
	$(-2, 3)$	■	■
	■	■	■
	■	■	■
Nose	$(-1, 1)$	■	■
	■	■	■
	■	■	■
Mouth	$(-2, -1)$	■	■
	■	■	■
	■	■	■
	■	■	■
Eyes	$(-2, 2)$	■	■
	■	■	■

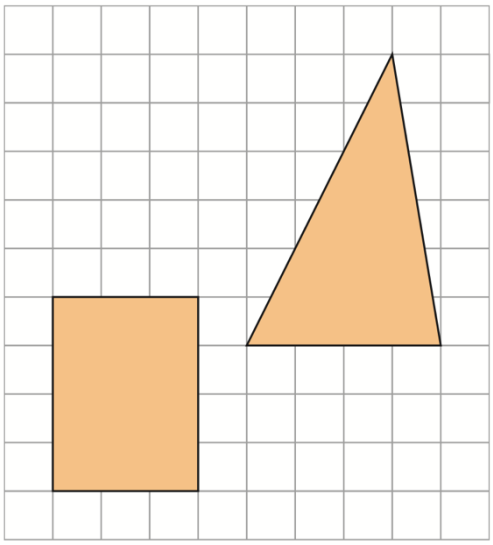
b. Suppose you make scale drawings with rules (2x, 2y) and (-2x, -2y)

c. On graph paper, plot the images of Mug Wump produced by the new sets of coordinates in part (b).

d. Compare the length, width, and area of Mug's mouth to those of the figures drawn in part (c). Explain how you could have predicted those results by studying the coordinate rules for the drawings.

4	a. On grid paper, draw triangle ABC with vertex coordinates A(0, 2), B(6, 2), and C(4, 4).	2.2	

	 b. Apply the rule (1.5x, 1.5y) to the vertices of triangle ABC to get triangle PQR. Compare the corresponding measurements (side lengths, perimeters, areas, angle measures) of the two triangles. c. Apply the rule (2x, 0.5y) to the vertices of triangle ABC to get triangle FGH. Compare the corresponding measurements (side lengths, perimeters, areas, angle measures) of the two triangles. d. Which triangle, PQR or FGH, seems similar to triangle ABC? Why?		
5	a. On grid paper, draw parallelogram ABCD with vertex coordinates A(0, 2), B(6, 2), C(8, 6), and D(2, 6). b. Write a rule to find the vertex coordinates of a parallelogram PQRS that is smaller than, but similar to, ABCD. Test your rule to see if it works. c. Write a rule to find the vertex coordinates of a parallelogram TUVW that is the same size as parallelogram ABCD but is in a different position on the grid?	2.2	
6	Copy the figures below accurately onto your own grid paper.	2.2	



a. Draw a figure similar, but not identical, to the rectangle.

b. Draw a figure similar, but not identical, to the triangle.

c. How do you know your scale drawings are similar to the given figures?

7

The diagram below shows two similar polygons.

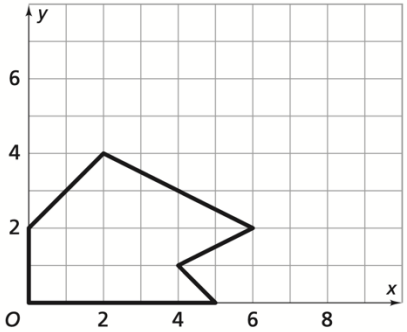


Figure A

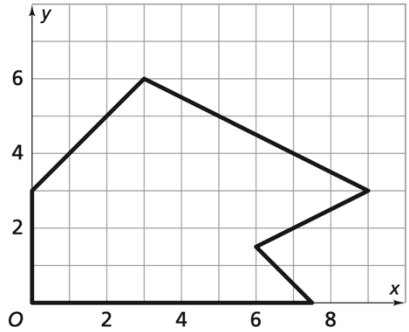


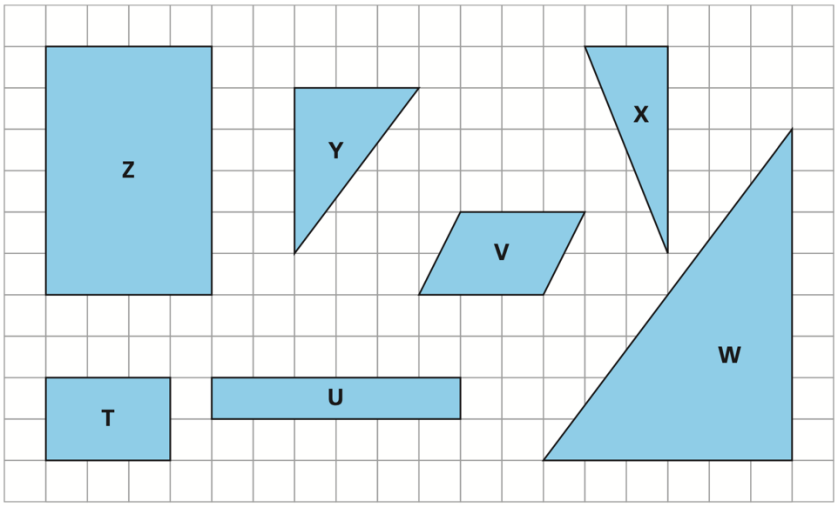
Figure B

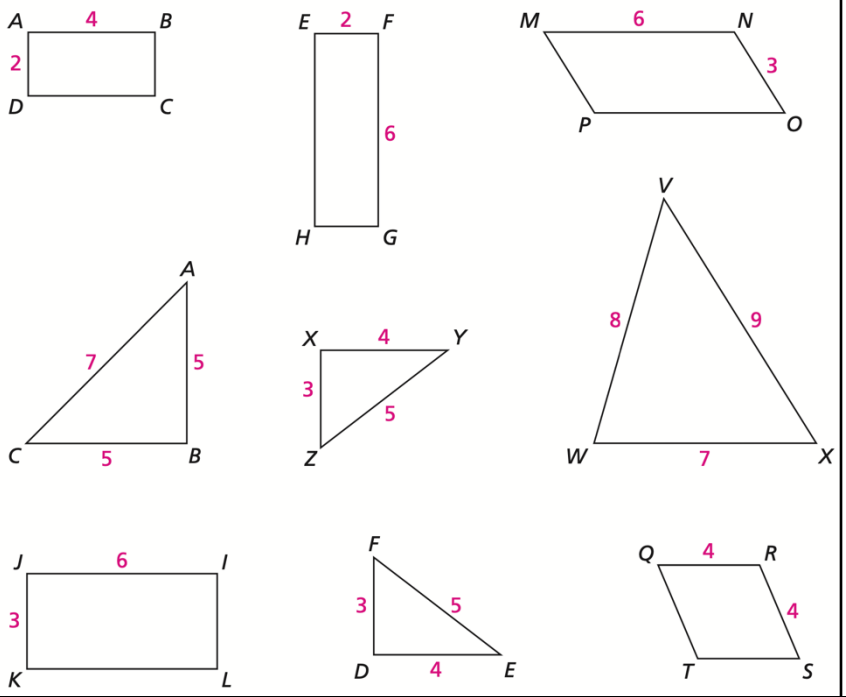
a. Write a rule for finding the coordinates of a point on Figure B from the corresponding point on Figure A.

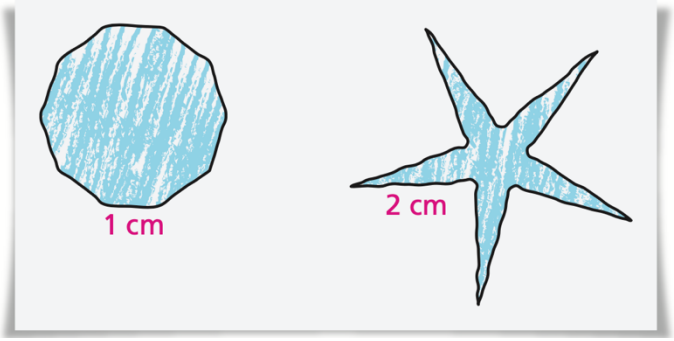
b. Write a rule for finding the coordinates of a point on Figure A from the corresponding point of Figure B.

c. i. What is the scale factor from Figure A to Figure B?

2.2

	ii. Use the scale factor to describe how the perimeter and area of Figure B are related to the perimeter and area of Figure A. d. i. What is the scale factor from Figure B to Figure A? ii. Use the scale factor to describe how the perimeter and area of Figure A are related to the perimeter and area of Figure B.		
For Exercises 8 and 9, study the size and shape of the polygons below.  			
8	**Multiple Choice** Choose the pair of similar figures. A. Z and Y B. V and T C. X and Y D. Y and W	2.3	
9	Find another pair of similar figures Explain your reasoning.	2.3	
10	What is the scale factor from an original figure to its image if the image is made using the given method? a. a two-rubber-band stretcher b. a copy machine with size factor 150% c. a copy machine with size factor 250%	2.3	

	d. the coordinate rule (0.75x, 0.75y)		
11	a. Use the polygons below. Which pairs of polygons are similar figures? b. For each pair of similar figures, list corresponding sides and angles. c. For each pair of similar figures, find the scale factor that relates side lengths of the larger figure to the corresponding side lengths of the smaller figure. 	2.3	
12	On grid paper, draw a rectangle with an area of 14 square centimeters. Label it ABCD. a. Write and use a coordinate rule that will make a rectangle similar to rectangle ABCD that is three times as long and three times as wide. Label it EFGH. b. How does the perimeter of rectangle EFGH compare to the perimeter of rectangle ABCD? c. How does the area of rectangle EFGH compare to the area of rectangle ABCD? d. How do your answers to parts (b) and (c) relate to the	2.3	

	scale factor from rectangle ABCD to rectangle EFGH?		
13	A student drew the figures below. The student says the two shapes are similar because there is a common scale factor for all of the sides. The sides of the figure on the right are twice as long as those of the figure on the left. What do you say to the student to explain why the figures are *not* similar?  	2.3	

Connections

Problem #	Answer	CMP4 Problem #	Note
For Exercises 14 and 15, the rule $(x, \frac{3}{4}y)$ is applied to a polygon.			
14	Is the image similar to the original polygon? Explain.	2.1	
15	Each of the following points is on the original polygon. Find the coordinates of each corresponding point on the image. a. (6, 8) b. (9, 8) c. (3/2, 4/3)	2.1	
16	One angle measure is given for each of the parallelograms below.	2.1	

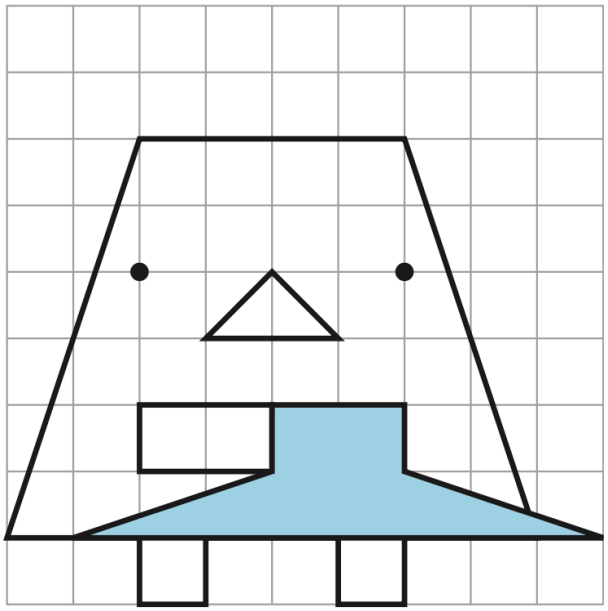
	● Find the measure of the other three angles in the parallelogram. ● List all pairs of supplementary angles in the diagram. Then, classify each angle as *acute, right*, or *obtuse*. a.  b. 		
17	Multiple Choice: What is the percent reduction or enlargement that will result if the rule (1.5x, 1.5y) is applied to a figure or an coordinate gird? A. 150% B. 15% C. 1.5% D. None of these	2.2	
18	Multiple Choice: What is the percent reduction or enlargement that will result if the rule (0.7x, 0.7y) is applied to a figure or an coordinate gird? F. 700% G. 7% H. 0.7% J. None of these	2.2	
19	The rule $(x + \frac{2}{3}, y - \frac{3}{4})$ is applied to a polygon. For each vertex below of the polygon, find the coordinates of the corresponding vertex on the image. a. (5, 3) b. (1/6, 11/12) c. (9/12, 4/5)	2.2	
20	An accurate map is a scale drawing of the place it represents. Below is a map of South Africa.	2.3	

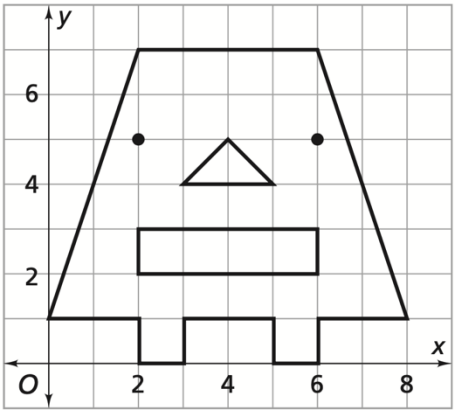
	 a. Use the scale to estimate the distance from Cape Town to Port Elizabeth. b. Use the scale to estimate the distance form Johannesburg to East London. c. What is the relationship between the scale for the map and a "scale factor"?		
Find each quotient.			
21	Find each quotient a. $\frac{1}{2}$ $\frac{1}{4}$ b. $\frac{1}{4}$ $\frac{1}{2}$ c. $\frac{3}{7}$ $\frac{4}{7}$ d. $1\frac{1}{2}$ $\frac{3}{8}$	2.3	
22	At a bake sale, 0.72 of a pan of corn bread has not been sold. A serving is 0.04 of a pan. a. How many serving are left?	2.3	

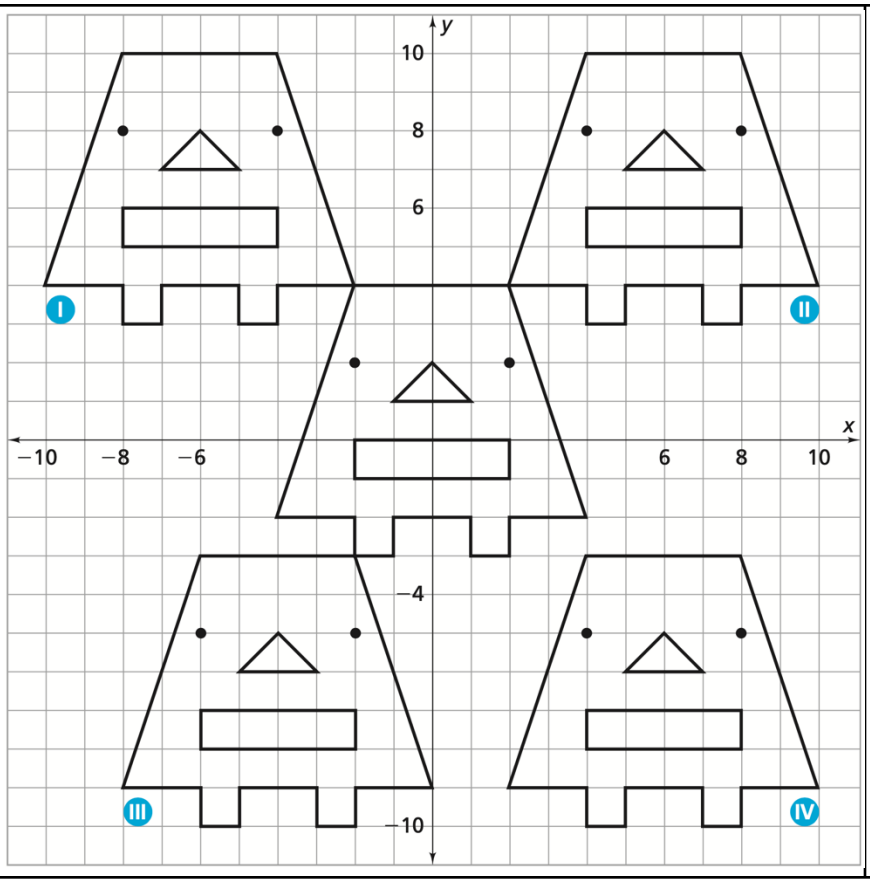
	b. Use a hundredths grid to show your reasoning.		
23	Each pizza takes 0.3 of a large block of cheese. Charlie has 0.8 of a block of cheese left. a. How many pizzas can he make? b. Use a diagram to show your reasoning.	2.3	

Extensions

Problem #	Answer	CMP4 Problem #	Note
24	Select a drawing of a comic strip character from a newspaper or magazine. Draw a grid over the figure or tape a transparent grid on top of the figure. Identify key points on the figure and then enlarge it using each of these rules. Which figures are similar? Explain. a. (2x, 2y) b. (x, 2y) c. (2x, y)	2.1	
25	Suppose you use the rule (3x+1, 3y-4) to transform Mug Wump into a new figure. a. How will the angle measures in the new figure compare to corresponding angle measures in Mug? b. How will the side lengths of the new figure compare to corresponding side lengths of Mug? c. How will the area and perimeter of this new figure compare to the area and perimeter of Mug?	2.2	
26	The vertices of three similar triangles are given. ● triangle ABC: A(1, 2), B(4, 3), C(2, 5) ● triangle DEF: D(3, 6), E(12, 9), F(6, 15) ● triangle GHI: G(5, 9), H(14, 12), I(8, 18) a. Find a rule that changes the vertices of triangle ABC to the vertices of triangle DEF. b. Find a rule that changes the vertices of triangle DEF to the vertices of triangle GHI. c. Find a rule that changes the vertices of triangle ABC to	2.2	

	the vertices of triangle GHI.		
27	If you drew Mug and his hat on the same grid, his hat would be at his feet instead of on his head.  a. Write a rule that puts Mug's hat centered on his head. b. Write a rule that changes Mug's hat to fit Zug and puts the hat on Zug's head. c. Write a rule that changes Mug's hat to fit Lug and puts the hat on Lug's head.	2.2	
28	Films are sometimes modified to fit a TV screen. Find out what that means. What exactly is modified? If Mug is in a movie, is he still a Wump when you see the video on TV?	2.3	
29	Explain how each rule changes the original shape, size, and location of Mug Wump.	2.3	

	 a. (-x, y) b. (x, -y) c. (-0.5x, -0.5y) d. (-0.5x, y) e. (-3x, -3y) f. (3x+5, -3x-4)		
30	The diagram below shows Mug Wump drawn at the center of a coordinate grid and in four other positions.	2.3	



Coordinates for Mug and Variations

Rule	(x, y)	$(2x, 2y)$	$(-2x, -2y)$
Head Outline	$(-4, -2)$	■	■
	$(-2, -2)$	■	■
	$(-2, 3)$	■	■
	■	■	■
	■	■	■
Nose	$(-1, 1)$	■	■
	■	■	■
	■	■	■
Mouth	$(-2, -1)$	■	■
	■	■	■
	■	■	■
	■	■	■
Eyes	$(-2, 2)$	■	■
	■	■	■

a. Find a sequence of coordinates to draw Mug's body at the center of the grid. Make a table to keep track of the points. For parts (b) and (c) below, use this Mug as the original Mug.

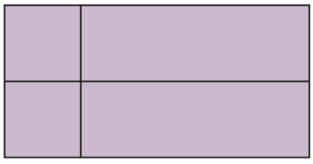
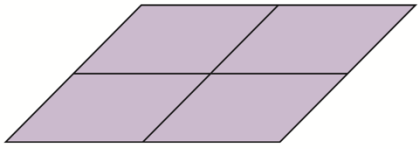
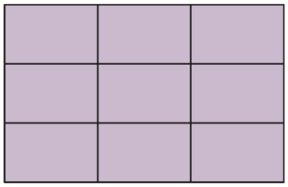
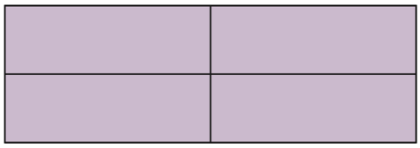
b. You can write a coordinate rule to describe the movement of points from one location to another. For example, the coordinate rule (x-2, y+3) moves a point (x, y) to the left 2 units and up 3 units from its original location. Which of the other drawings is produced by the coordinate rule (x+6, y-7)?

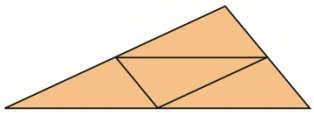
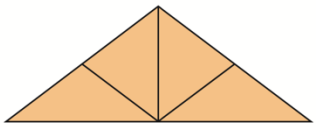
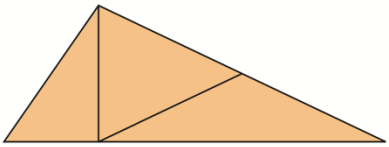
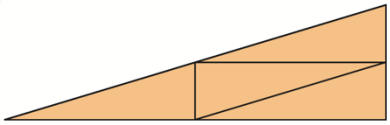
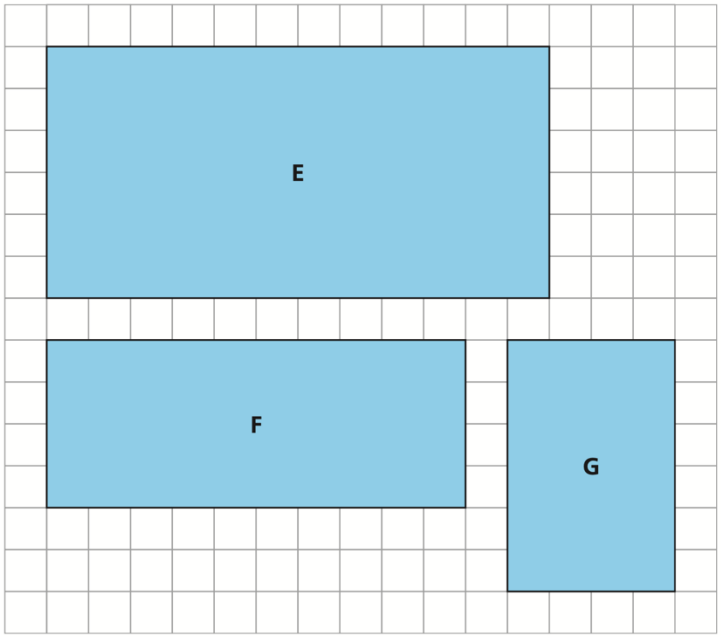
c. Find coordinate rules for moving the original Mug to the other positions on the grid.

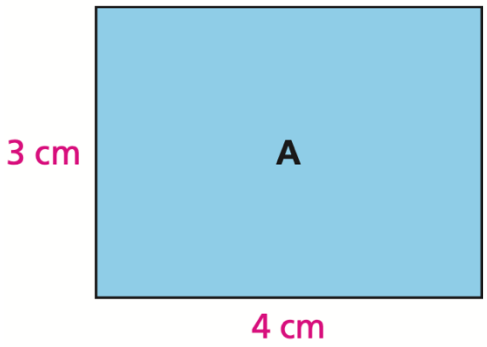
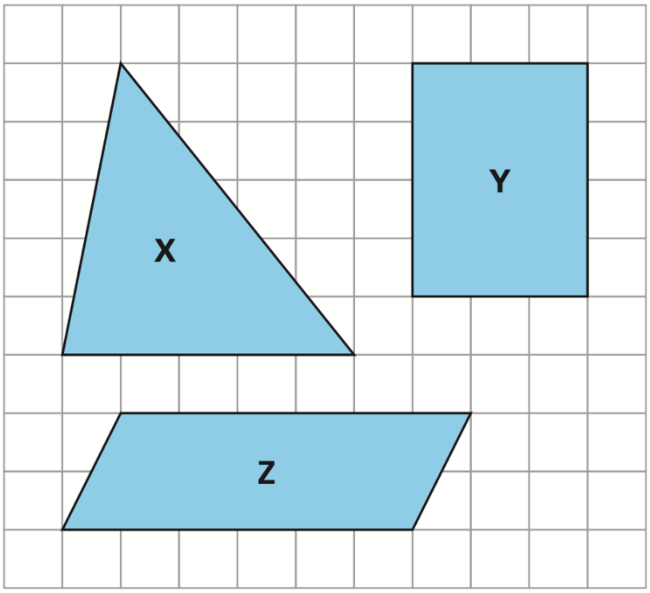
Investigation 3

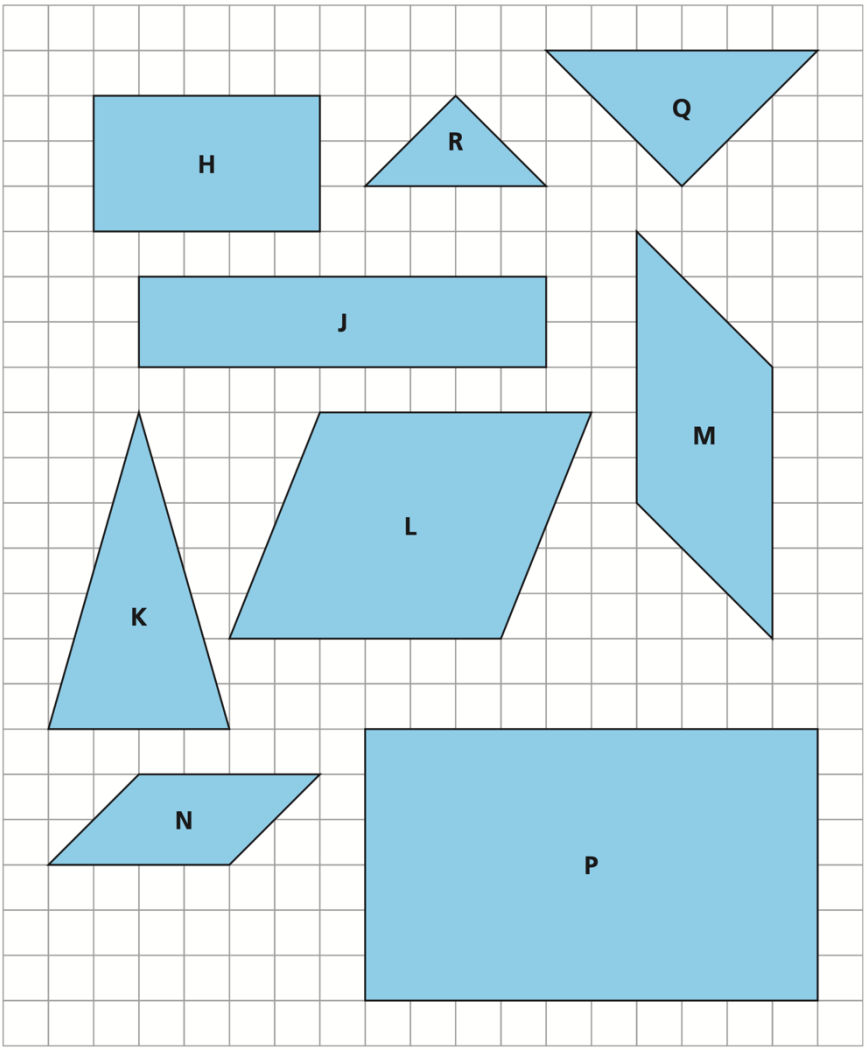
	Applications	Connections	Extensions	Total
3.1	6	4	2	12
3.2	6	6	3	15
3.3	4	4	1	9
Total	16	14	6	36

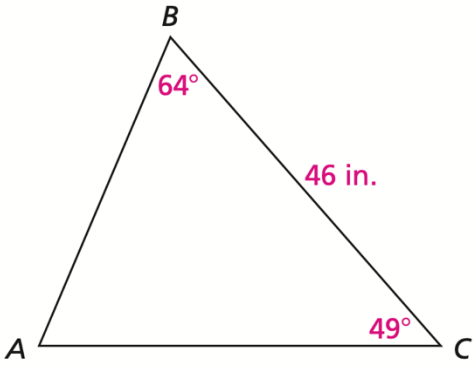
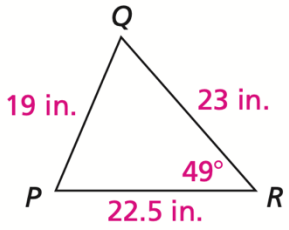
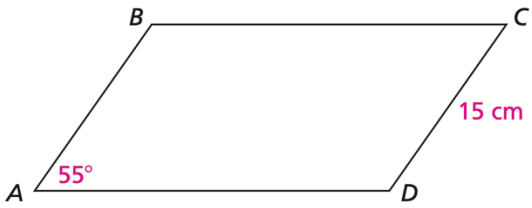
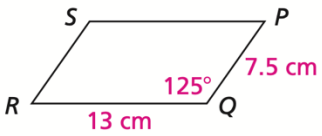
Applications

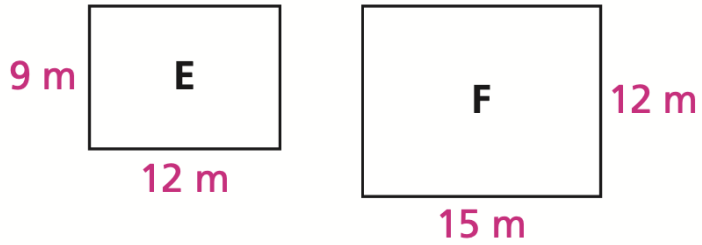
Problem #	Answer	CMP4 Problem #	Note
1	Look for rep-tile patterns in the designs below. For each design, • Decide whether the small quadrilaterals are similar to the large quadrilateral. Explain. • If the quadrilaterals are similar, give the scale factor from each small quadrilateral to the large quadrilateral. a.  b.  c.  d. 	3.1	
2	Suppose you divide a rectangle into 25 smaller rectangles such that each rectangle is similar to the original rectangle. a. How is the area of each of the smaller rectangles related to the area of the original rectangle? b. What is the scale factor from the original rectangle to each of the smaller rectangles?	3.1	
3	Look for rep-tile patterns in the figures below.	3.1	

	● Tell whether the small triangles are similar to the large triangle. Explain. ● If the triangles are similar, give the scale factor from each small triangle to the large triangle. **a.**  **b.**  **c.**  **d.** 		
4	a. For rectangles E-G, give the length and width of a different, similar rectangle. Explain how you know the new rectangles are similar.  b. Give the scale factor from each original rectangle in part (a) to the similar rectangles you described. Explain what the scale factor tells you about the corresponding lengths, perimeters, and areas.	3.1	
5	Suppose Rectangle B is similar to Rectangle A below. The scale factor from Rectangle A to Rectangle B is 4. What is the area of Rectangle B?	3.1	

			
6	Suppose Rectangle E has an area of 9 square centimeters and Rectangle F has an area of 900 square centimeters. The two rectangles are similar. What is the scale factor from Rectangle E to Rectangle F?	3.1	
7	For parts (a)-(c), use grid paper.  a. Sketch a triangle similar to Triangle X with an area that is ¼ the area of Triangle X. b. Sketch a rectangle similar to Rectangle Y with a perimeter that is 0.5 times the perimeter of Rectangle Y. c. Sketch a parallelogram similar to Parallelogram Z with side lengths that are 1.5 times the side lengths of Parallelogram Z.	3.2	

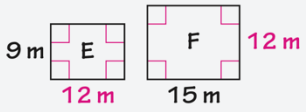
8	Use the polygons below.  a. List pairs of similar shapes. b. For each pair of similar shapes, find the scale factor from the smaller shape to the larger shape.	3.2	

9	Triangle ABC is similar to triangle PQR. Find the indicated angle measure or side length.   a. angle A b. angle Q c. angle P d. length of side AB e. length of side AC f. perimeter of triangle ABC	3.2	
10	**Multiple Choice:** Use the similar parallelograms below.   a. What is the measure of angle D? A. 55° B. 97.5° C. 125° D. 135° b. What is the measure of angle R? F. 55° G. 97.5° H. 125° J. 135° c. What is the measure of angle S?	3.2	

	A. 55° B. 97.5° C. 125° D. 135° d. What is length of side AB? F. 3.75cm G. 13 cm H. 15 cm J. 26 cm		
11	Evan, Melanie, and Wyatt discuss whether the two figures E and F are similar. Do you agree with Evan, Melanie, and Wyatt? Explain. 	3.2	

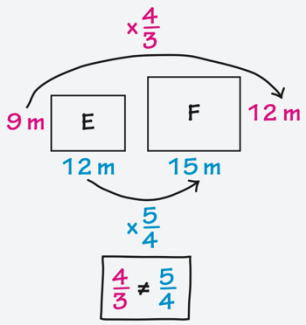
Evan's Reasoning

Rectangles E and F are similar because each shape has four right angles. Also, each rectangle has at least one side that is 12 meters long.



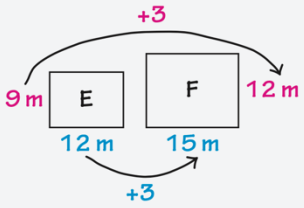
Melanie's Reasoning

The scale factor for the height from rectangle E to rectangle F is $\frac{12}{9}$, or $\frac{4}{3}$. The scale factor for the base is $\frac{15}{12}$, or $\frac{5}{4}$. $\frac{4}{3} \neq \frac{5}{4}$, so the rectangles are not similar.

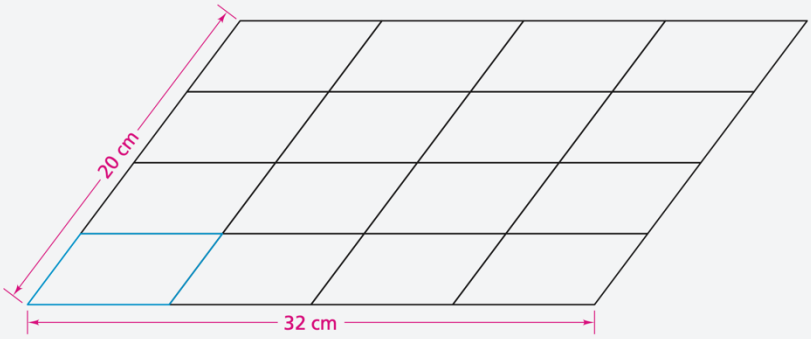
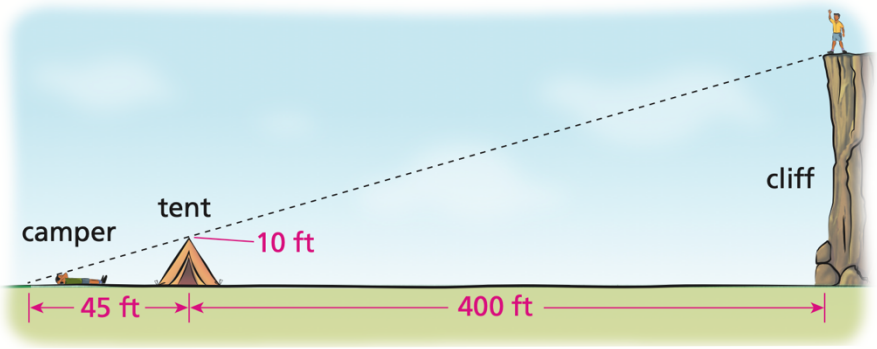


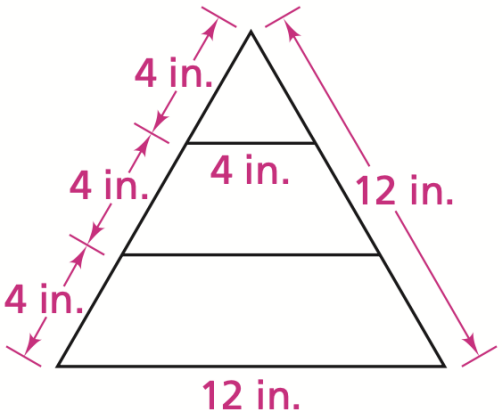
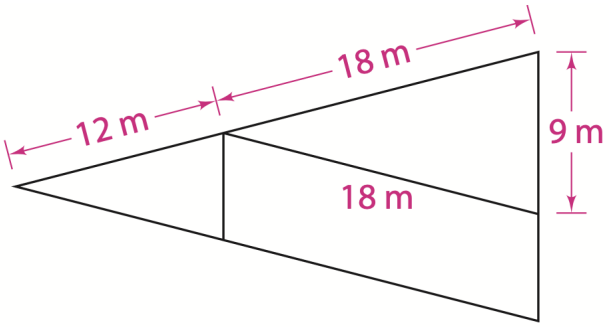
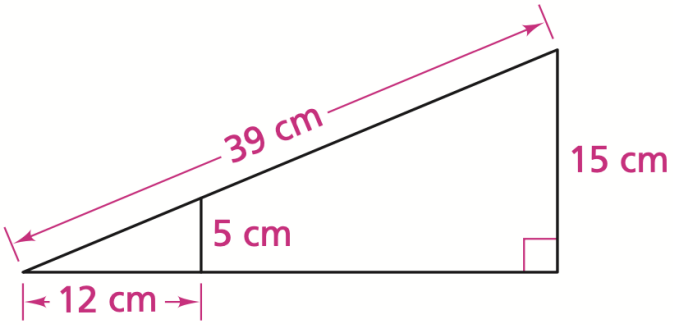
Wyatt's Reasoning

Rectangles E and F are similar. Rectangle F is 3 meters taller than Rectangle E since 9 meters + 3 meters = 12 meters. Rectangle F is also 3 meters wider than Rectangle E since 12 meters + 3 meters = 15 meters. Each dimension of Rectangle F is 3 meters greater than the corresponding dimension of Rectangle E, so the rectangles are similar.



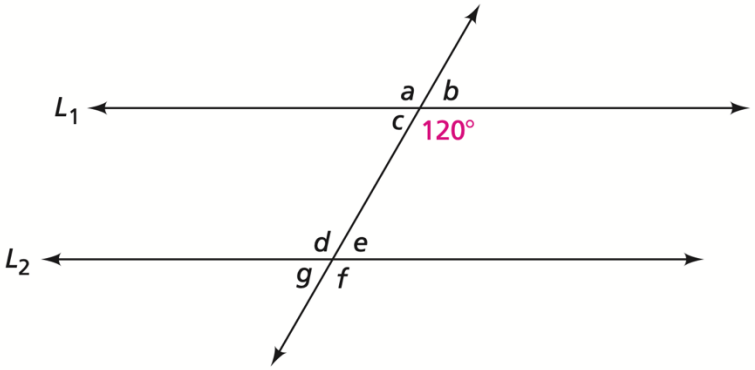
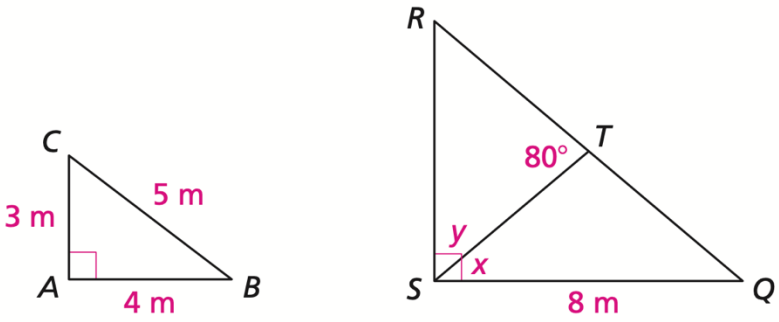
12	Janine, Trisha, and Jeff drew parallelograms that are similar to Parallelogram *P* below.  **Parallelogram *P*** Each student claims that the scale factor from *P* to the sketched parallelogram is 4. Are any of the students correct in their reasoning?	3.2	

	Janine's Method I divided the original parallelogram into four similar parallelograms. Parallelogram P is four times as large as each of the new parallelograms.  **Trisha's Method** I sketched four copies of parallelogram P. The shape has four times the area of parallelogram P.  **Jeff's Method** I wanted a scale factor of 4. The perimeter of the original shape is 26 centimeters. I drew a parallelogram with a perimeter of 4 x 26 centimeters = 104 centimeters. 		
13	Judy lies on the ground 45 feet from her tent. Both the top of the tent and the top of a tall cliff are in her line of sight. Her tent is 10 feet tall. About how high is the cliff? Assume the two triangles are similar.  *Not drawn to scale*	3.3	
14	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can.	3.3	

			
15	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can. 	3.3	
16	The triangle has been subdivided into triangles that are similar to the original triangle. Copy each triangle and label as many side lengths as you can. 	3.3	

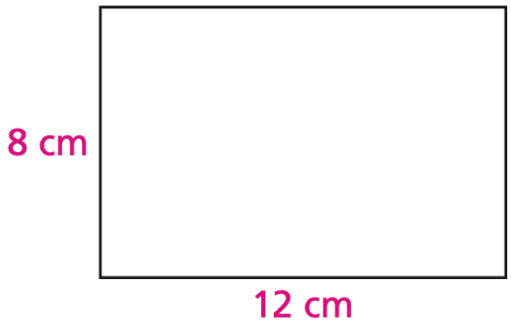
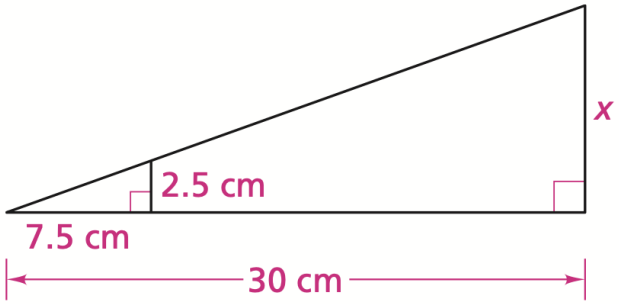
Connections

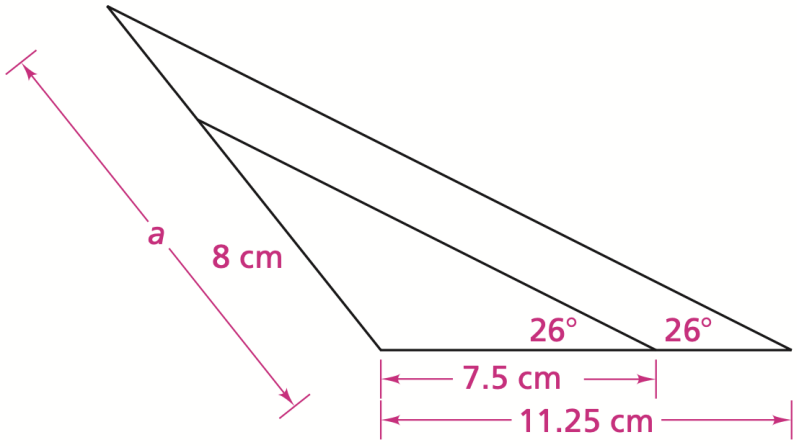
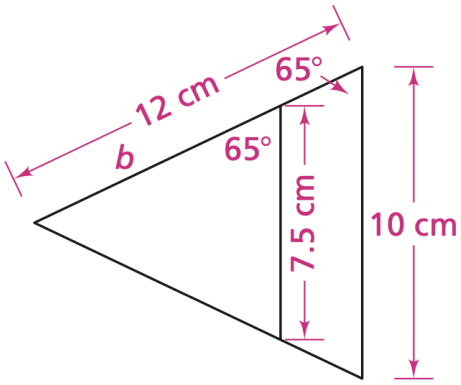
Problem #	Answer	CMP4	Note

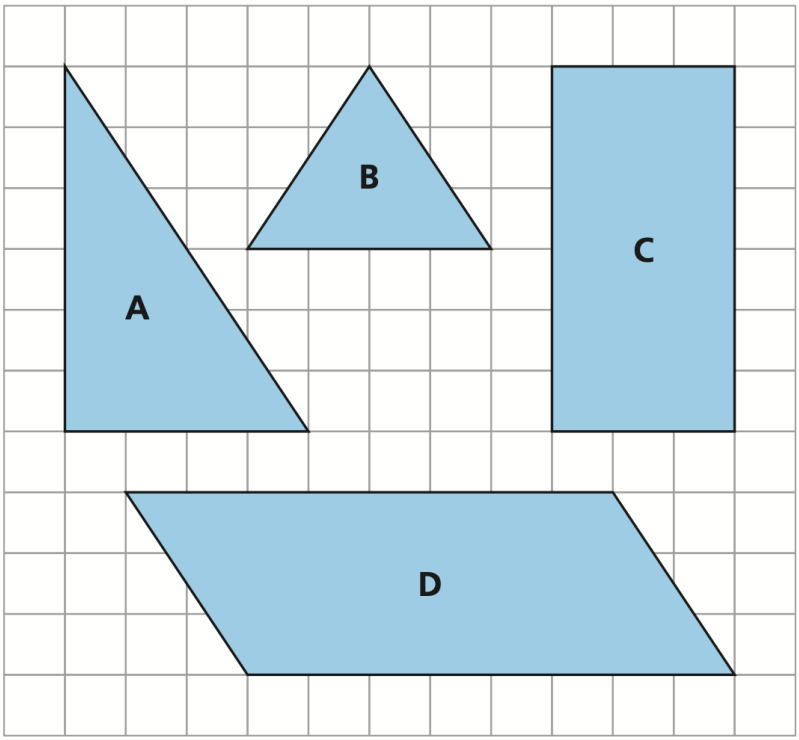
		Problem #	
17	In the figure below, lines L1 and L2 are parallel.  a. Use what you know about parallel lines to find the measures of angles a through g. b. List all pairs of *supplementary* angles in the diagram.	3.1	
18	For each of the following angle measures, find the measure of its supplementary angle. a. 160° b. 90° c. x°	3.1	
19	The right triangles below are similar.  a. Find the length of side RS. b. Find the length of side RQ. c. The measure of angle x is about 40°. If the measure of angle x were exactly 40°, what would be the measure of angle y? d. Use your answer from part (c) to find the measure of	3.1	

	angle R. Explain how you can find the measure of angle C. e. Angle x and angle y are *complementary angles*. Find two additional pairs of complementary angles in Triangles ABC and QRS.		
20	For parts (a)-(f), find the number that makes the fractions equivalent. a. ½ = 3/• b. 5/6 = •/24 c. ¾ = 6/• d. 8/12 = 2/• e. 3/5 = •/100 f. 6/4 = •/10	3.1	
21	For parts (a)-(f), suppose you copy a figure on a copier using the given scale factor. Find the scale factor from the original figure to the copy in decimal form. a. 200% b. 50% c. 150% d. 125% e. 75% f. 25%	3.2	
22	For parts (a)-(d), tell whether the figures are mathematically similar. Explain your reasoning. If the figures are similar, give the scale factor from the left figure to the right figure.	3.2	

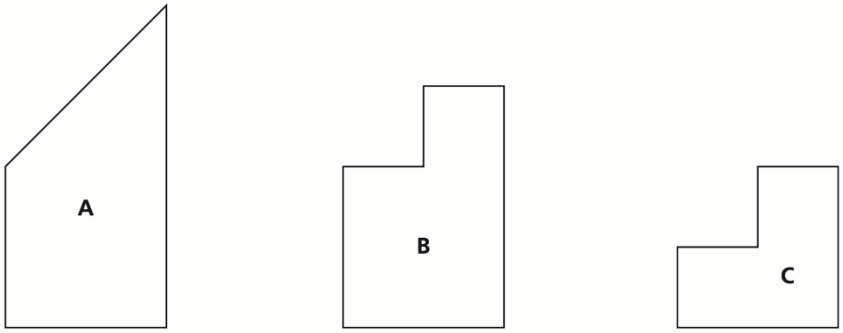
	a.   b.   c.   d.  		
23	True or False: All squares are similar. Explain.	3.2	
24	True or False: All rectangles are similar. Explain.	3.2	

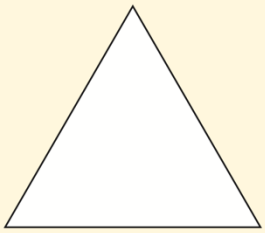
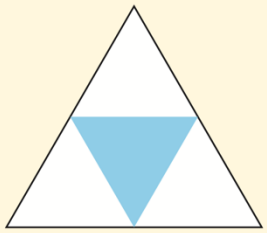
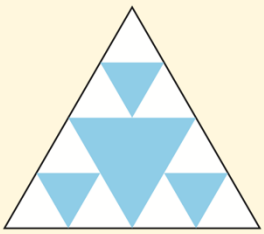
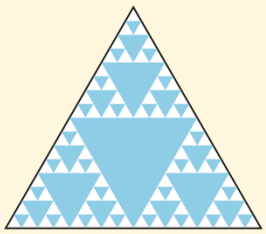
25	True or False: If the scale factor between two similar shapes is 1, then the two shapes are the same size. Explain.	3.2	
26	a. Suppose the following rectangle is reduced by a scale factor of 50%. What are the dimensions of the reduced rectangle?  b. Suppose the reduced rectangle from part (a) is reduced again by a scale factor of 50%. What are the dimensions of the new rectangle? Explain your reasoning. c. How does the reduced rectangle from part (b) compare to the original rectangle from part (a)?	3.2	
27	**Multiple Choice** What is the value of x? The diagram is not to scale.  A. 3 cm B. 10 cm C. 12 cm D. 90 cm	3.3	

28	Find the missing side length. The diagram is not to scale.  	3.3	
29	Find the missing side length. The diagram is not to scale.  	3.3	
30	Copy polygons A-D onto grid paper. Draw line segments that divide each of the polygons into four congruent polygons that are	3.3	

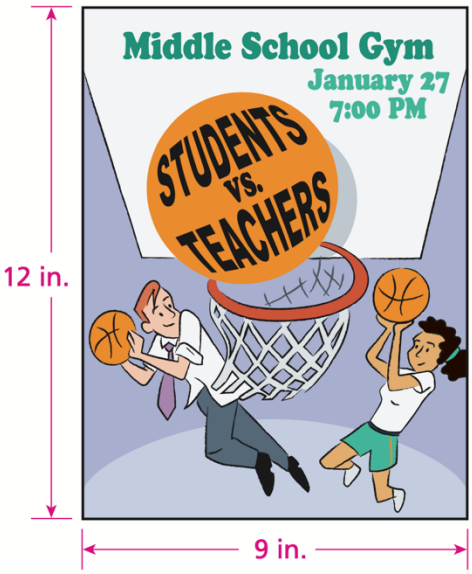
	similar to the original polygon.  		

Extensions

Problem #	Answer	CMP4 Problem #	Note
31	Trace each shape. Divide each shape into four smaller, identical pieces that are similar to the original shape.  	3.1	
32	You can subdivide figures to get smaller figures that are mathematically similar to the original. The mathematician Benoit	3.1	

	Mandelbrot called these figures fractals. A famous example is the Sierpinski triangle. **Sierpinski Triangle** You can follow these steps to make the Sierpinski triangle.  **Step 1:** Draw a triangle. (It does not have to be an equilateral triangle.)  **Step 2:** Mark the midpoint of each side. Connect the midpoints to form four identical triangles that are similar to the original. Shade the center triangle.  **Step 3:** For each unshaded triangle, mark the midpoints. Connect them in order to form four identical triangles. Shade the center triangle in each case.  **Step 4:** Repeat Steps 2 and 3 over and over. To make a real Sierpinski triangle, you need to repeat the process an infinite number of times! This triangle shows five subdivisions. a. Follow the steps for making the Sierpinski triangle until you subdivide the original triangle three times. b. Describe any patterns you observe in your figure. c. Mandelbrot used the term *self-similar* to describe fractals like the Sierpinski triangle. What do you think this term means?		
33	The midpoint of a line segment is a point that divides the segment into two segments of equal length. Draw a figure on grid paper by following these steps: Step 1: Draw a large square.	3.2	

	Step 2: Mark the midpoint of each side. **Step 3**: Connect the midpoints, in order, with four line segments to form a new figure. (The line segments should not intersect inside the square.) **Step 4**: Repeat Step 2 and 3 three more times. Work with the newest figure each time. a. What kind of figure is formed when the midpoints of the sides of a square are connected? b. Find the area of the original square you drew in Step 1. c. Find the area of each of the new figures that was formed. d. How do the areas change between successive figures? e. Are there any similar figures in your final drawing? Explain.		
34	Repeat Exercise 44 starting with an equilateral triangle, connecting three line segments to form a new triangle each time.	3.2	
35	Suppose Rectangle A is similar to Rectangle B and to Rectangle C. Can you conclude that Rectangle B is similar to Rectangle C? Explain. Use drawings and examples to illustrate your answer.	3.2	
36	Song makes a copy of the poster below.	3.3	



a. She presses the 50% button on the copy machine. Now the length and width of the poster are each half of their original sizes. Song thinks that if she enlarges the copy by 150%, the new copy will be the same as the original. Is she correct?

b. Suppose Song had done the opposite in part (a), first enlarging the poster by 150%, and then reducing the copy by 50%. Will the final copy be the same size as the original? Will it be the same size as the copy made in part (a)?

c. Song uses the same process from parts (a) and (b) with a different-sized poster. Does she get similar results?

d. Song applied a scale factor of 25% to shrink the original poster. Now she wants to get the poster back to the original size. What scale factor should she use? Explain your reasoning.

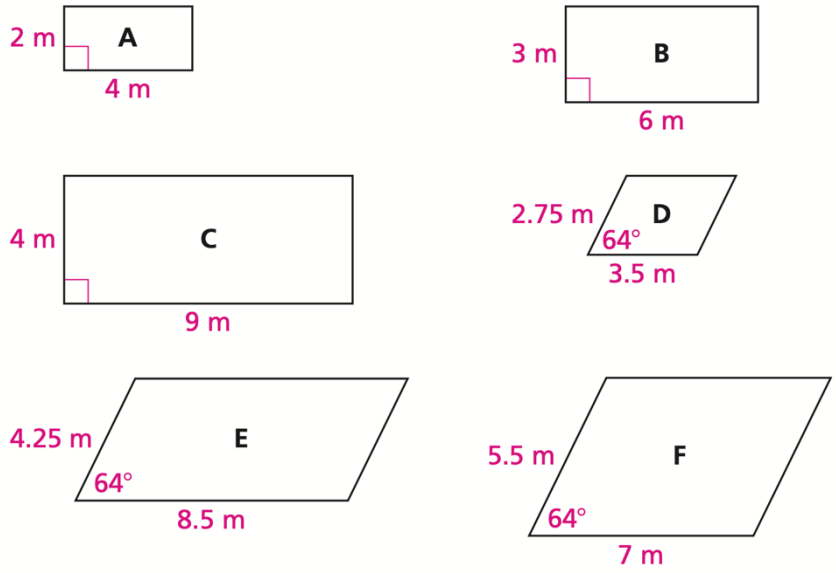
e. Suppose Song had used 75% and 125% in parts (a) and (b) instead of 50% and 150%. What would have happened?

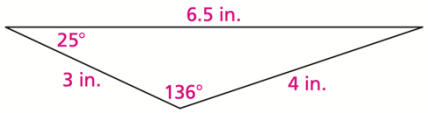
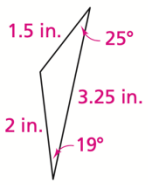
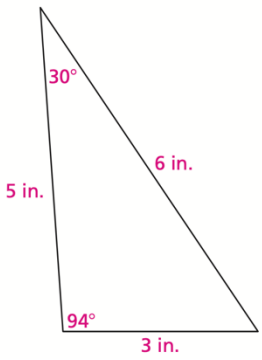
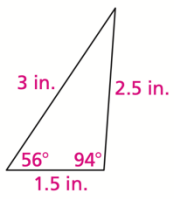
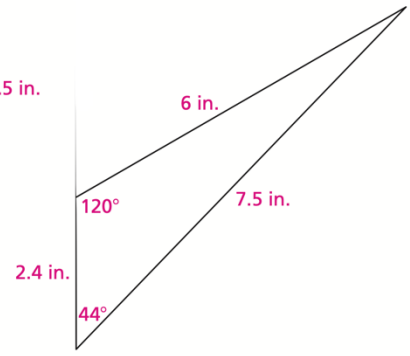
f. What general statements can you make about applying any pair of two scale factors one after the other? Consider a pair of two enlargements, a pair of two reductions, and a pair consisting of one enlargement and one reduction.

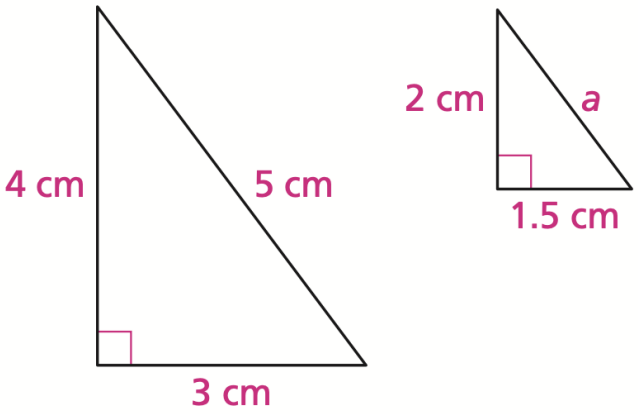
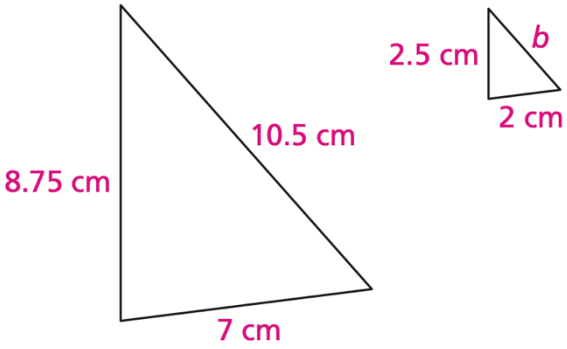
Investigation 4

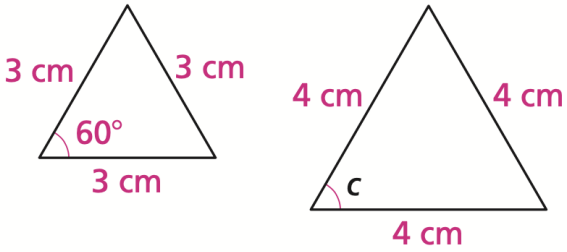
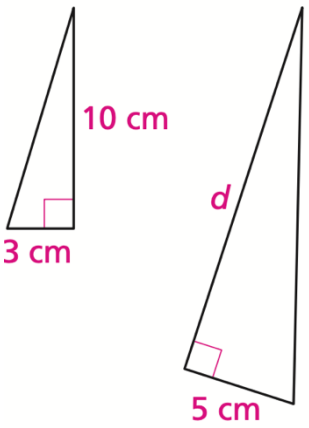
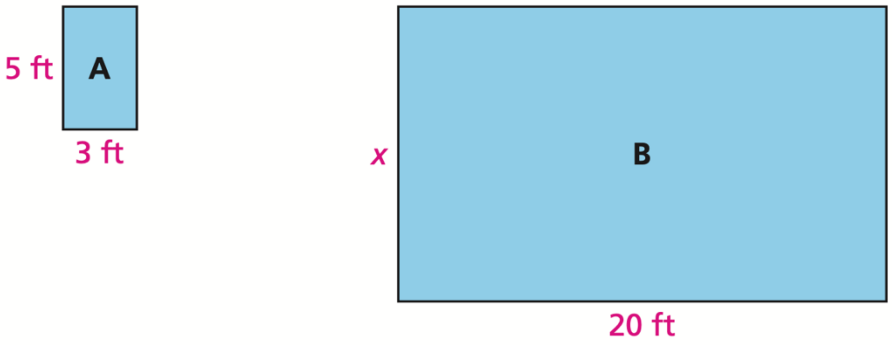
	Applications	Connections	Extensions	Total
4.1	3	4	4	11
4.2	4	5	5	14
4.3	3	4	4	11
Total	10	13	13	36

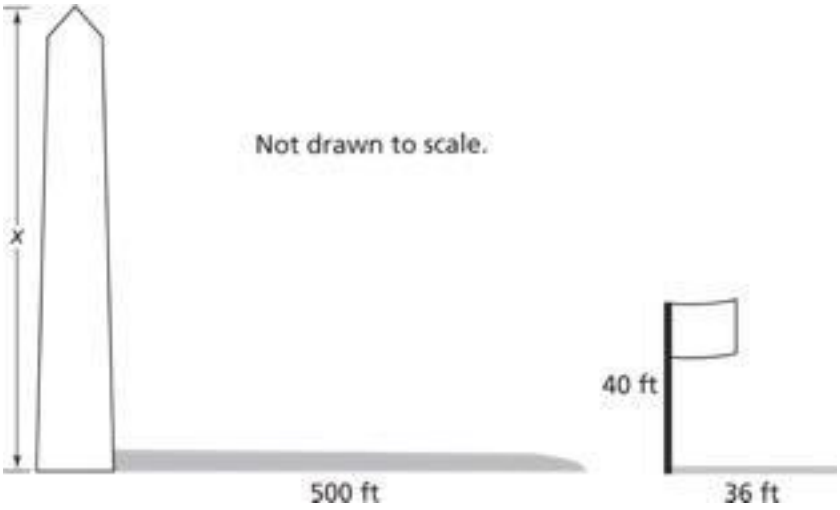
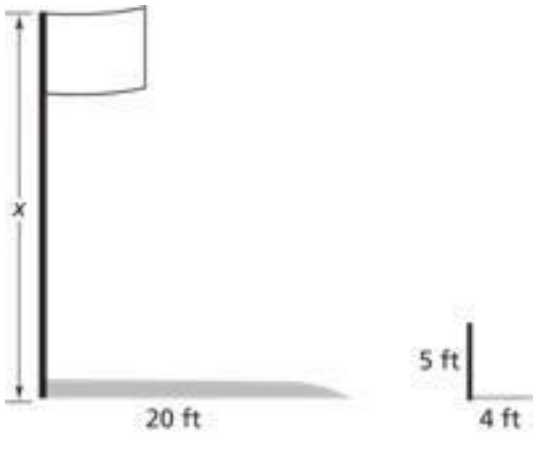
Applications

Problem #	Answer	CMP4 Problem #	Note
1	For parts (a)-(c), use the parallelograms below.  a. List all the pairs of similar parallelograms. Explain your reasoning. b. For each pair of similar parallelograms, find the ratio of two adjacent side lengths in one parallelogram. Find the ratio of the corresponding side lengths in the other parallelogram. How do these ratios compare? c. For each pair of similar parallelograms, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different from the information given by the ratios of adjacent side lengths.	4.1	

2	a. On grid paper, draw two similar rectangles where the scale factor from one rectangle to the other is 2.5. Label the length and width of each rectangle. b. For each rectangle, find the ratio of the length to the width. c. Draw a third rectangle that is similar to one of the rectangles in part (a). Find the scale factor from the new rectangle to the one from part (a). d. Find the ratio of the length to the width for the new rectangle. e. What can you say about the length-to-width ratios of the three rectangles? Is this true for another rectangle that is similar to one of the three rectangles? Explain.	4.1	
3	For part (a)-(d), use the triangles below. The drawings are not to scale. **Triangle A**  **Triangle B**  **Triangle C**  **Triangle D**  **Triangle E**  a. List all the pairs of similar triangles. Explain why they are similar. b. For each pair of similar triangles, find the ratio of two side lengths in one triangle. Find the ratio of the corresponding side lengths in the other. How do these ratios compare? c. For each pair of similar triangles, find the scale factor from one shape to the other. Explain how the information given	4.1	

	by the scale factors is different than the information given by the ratios of side lengths. d. How are corresponding angles related in similar triangles? Is it the same relationship as for corresponding side lengths? Explain.		
4	Each pair of figures is similar. Find the missing measurement. Explain your reasoning. (Note: The figures are not drawn to scale.) **a.**  b. 	4.2	

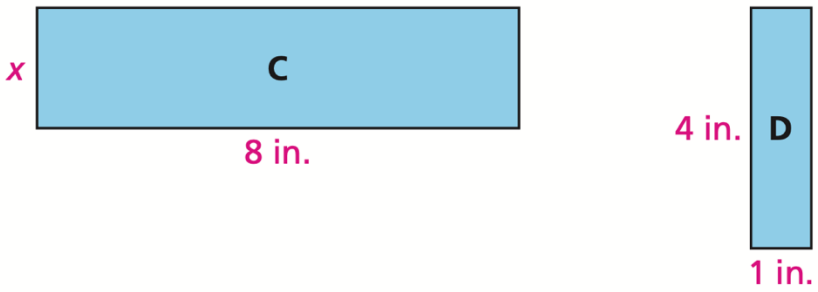
	c.  d.  		
For Exercises 5-7, Rectangles A and B are similar.  			
5	Multiple Choice What is the value of x? A. 4 B. 12 C. 15 D $33\ ^1/_3$	4.2	
6	What is the scale factor from Rectangle B to Rectangle A?	4.2	
7	Find the area of each rectangle. How are the areas related?	4.2	

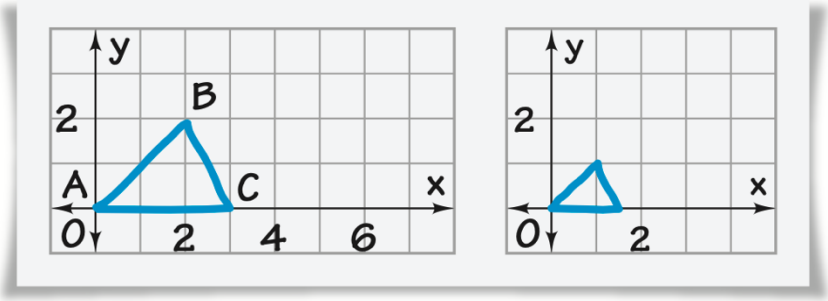
8	The Washington Monument is the tallest structure in Washington D.C. At a certain time, the monument casts a shadow that is about 500 feet long. At the same time, a 40-foot flagpole nearby casts a shadow that is about 36 feet long. About how tall is the monument? Sketch a diagram.  	4.3	
9	Darius uses the shadow method to estimate the height of a flagpole. He finds that a 5-foot stick casts a 4-foot shadow. At the same time, he finds that the flagpole casts a 20-foot shadow. What is the height of the flagpole? Sketch a diagram.  	4.3	
10	**a.** Greg and Zola are trying to find the height of their school building. Zola takes a picture of Greg standing next to the building. How might this picture help them determine the height of the building?	4.3	

	b. Greg is 5 feet tall. The picture Zola took shows Greg as ¼ inch tall. If the building is 25 feet tall in real life, how tall should the building be in the picture? Explain. **c.** In part (a), you thought of ways to use a picture to find the height of an object. Think of an object in your school that is different to measure directly, such as a high wall, bookshelf, or trophy case. Describe how you might find the height of the object.		

Connections

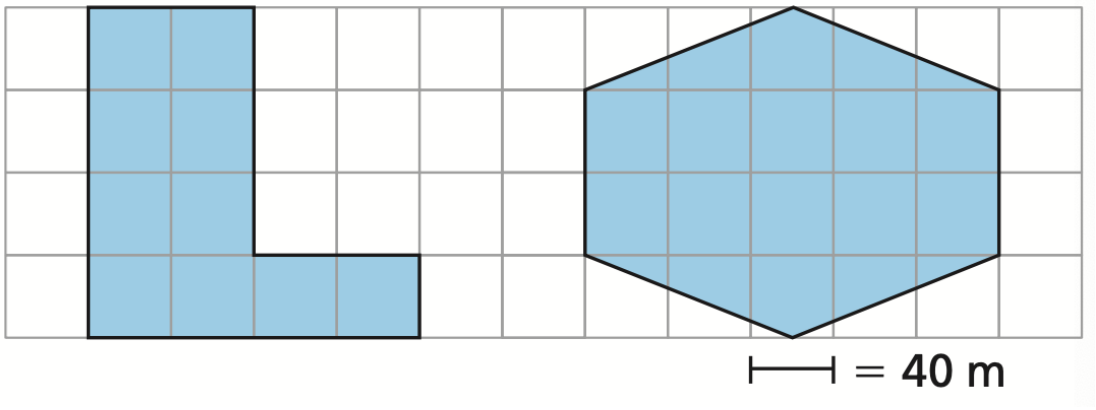
Problem #	Answer	CMP 4 Problem #	Note
11	Tell whether each pair of ratios is equivalent. a. 3 to 2 and 5 to 4 b. 8 to 4 and 12 to 8 c. 7 to 5 and 21 to 15 d. 1.5 to 0.5 and 6 to 2	4.1	
12	Use a pair of equivalent ratios from Exercise 11. Write a similarity problem using the ratios. Explain how to solve the problem.	4.1	
13	For each ratio write two other equivalent ratios. a. 5 to 3 b. 4 to 1 c. 3 to 7 d. 1.5 to 1	4.1	

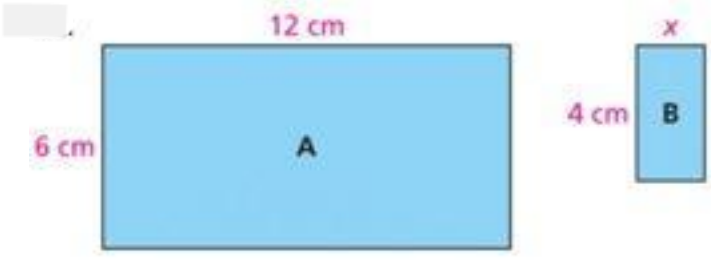
14	Rectangle C and D are similar.  a. What is the value of x? b. What is the scale factor from Rectangle C to Rectangle D? c. Find the area of each rectangle. How are the areas related?	4.1	
15	Here is a picture of Duke. The scale factor from Duke to the picture is 12.5%. Use an inch ruler to make any measurements.  a. How long is Duke from his nose to the tip of his tail? Explain how you used the picture to find your answer. b. To build a doghouse for Duke, you need to know his height. How tall is Duke? Explain. c. A copy center has a machine that prints on poster-size paper. You can resize an image from 50% to 200%. How can you use the machine to make a life-size picture of Duke?	4.2	The picture must be 5 inch wide when it's printed.

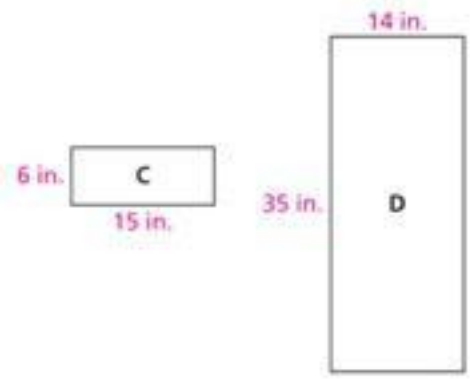
16	Movie screens often have an *aspect ratio* of 16 by 9. This means that for every 16 feet of width along the base of the screen there are 9 feet of height. The width of the screen at a local drive-in theater is about 115 feet wide. The screen has a 16:9 aspect ratio. About how tall is the screen?	4.2	
17	Paloma draws triangle ABC on a grid. She applies a rule to make the triangle on the right.  a. What rule did Paloma apply to make the new triangle? b. Is the new triangle similar to triangle ABC? Explain your reasoning. If the triangles are similar, give the scale factor from triangle ABC to the new triangle.	4.2	

For Exercises 18 and 19, use the paragraph below.

The Rosavilla School District wants to build a new middle school building. They ask architects to make scale drawings of possible layouts for the building. Two possibilities are shown below.

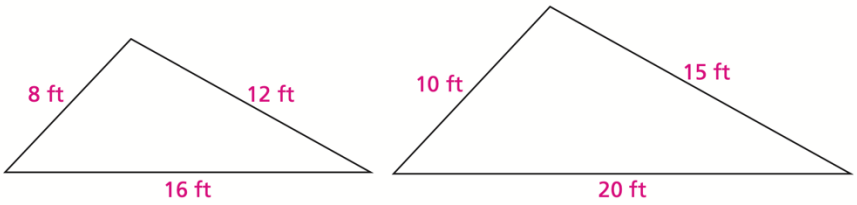


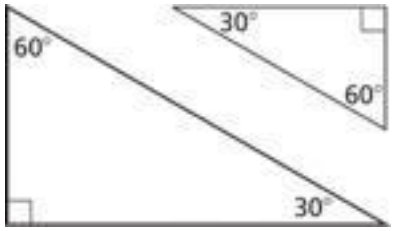
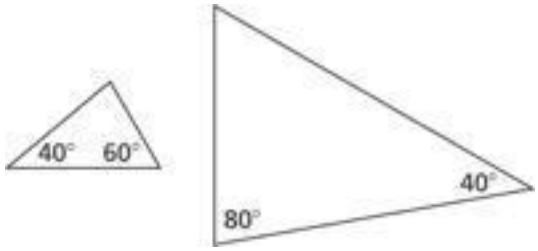
18	a. What is the area of each scale drawing in square units? 10 square units; 15 square units b. What would the area of the ground floor of each building be? 16,000 m^2; 24,000 m^2	4.2	
19	Multiple Choice The school board likes the L-shaped layout but wants a building with more space. They increase the L-shaped layout by a scale factor of 2. For the new layout, choose the correct statement. F. The area is two times the original. G. The area is four times the original. H. The area is eight times the original. J. None of the statements above are correct.	4.2	
20	For each angle measure, find the measure of its complement and the measure of its supplement. **Sample**: 30° complement: 60°; supplement: 150° a. 20° b. 70° c. 45°	4.3	
21	Rectangles A and B are similar. 	4.3	

	a. What is the scale factor from Rectangle A to Rectangle B? b. Complete the following sentence in two different ways. Use the side lengths of Rectangle A and B. *The ratio of ■ to ■ is equivalent to the ratio of ■ to ■.* c. What is the value of x? Explain your reasoning. d. What is the ratio of the area of Rectangle A to the area of Rectangle B?		
22	Triangle A has sides that measure 4 inches, 5 inches, and 6 inches. Triangle B has sides that measure 8 feet, 10 feet, and 12 feet. Taylor and Landon are discussing whether the two triangles are similar. Do you agree with Taylor or with Landon? Explain. **Taylor's Explanation** The triangles are similar. If you double each of the side lengths of Triangle A, you get the side lengths for Triangle B. **Landon's Explanation** The triangles are not similar. Taylor's method works when two measures have the same units. However, the sides of Triangle A are measured in inches, and the sides of Triangle B are measured in feet. So, they cannot be similar.	4.3	
23	Anya and Jalen disagree about whether the two figures below are similar. Do you agree with Anya or with Jalen? Explain. 	4.3	

	Anya's Reasoning The two rectangles are not similar. The height of Rectangle D is almost 6 times the height of Rectangle C, but the widths are almost the same. Similar rectangles must have the same scale factor for the base and the height. **Jalen's Reasoning** The two rectangles are similar. The scale factor from C to D is 7/3. You can multiply the short side of C (the height) by to get 14 inches, which is the short side of D (the base). This scale factor also works for the long sides of the rectangles since 15 x 7/3 = 35.		

Extensions

Problem #	Answer	CMP4 Problem #	Note
24	For parts (a)-(e), use the similar triangles below.  a. What is the scale factor from the smaller triangle to the larger triangle? Write your answer as a fraction and a decimal. b. Choose any side of the larger triangle. Find the ratio of this side length to the corresponding side length in the smaller triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (a)? c. What is the scale factor from the larger triangle to the smaller triangle? Write your answer as a fraction and a decimal. d. Choose any side of the smaller triangle. Find the ratio of this side length to the corresponding side length in the	4.1	

	larger triangle. Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor from part (c)? e. What patterns do you notice in parts (a)-(d)? Are these patterns the same for any pair of similar figures? Explain.		
25	For part (a) and (b), use a straightedge and an angle ruler or protractor. a. Draw two different triangles that each have angle measures of 30°, 60°, and 90°. Do the triangles appear to be similar?  b. Draw two different triangles that each have angle measures of 40°, 80°, and 60°. Do the triangles appear to be similar?  c. Based on your findings for parts (a) and (b), make a conjecture about triangles with congruent angle measures.	4.1	
26	One of these rectangles is “most pleasing to the eye.”	4.1	

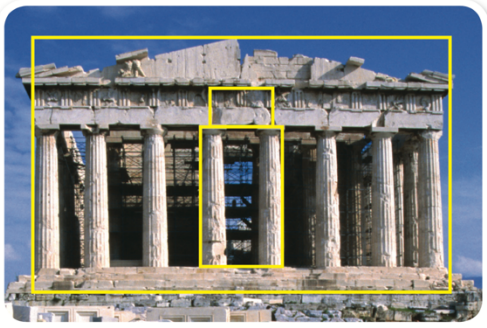
A

B

C

The question of what shapes are most attractive has interested builders, artists, and craftspeople for thousands of years.

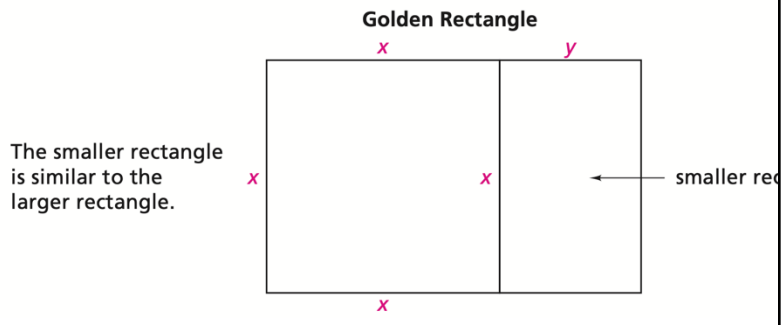
The ancient Greeks were particularly attracted to rectangular shapes similar to Rectangle B above. They referred to such shapes as "golden rectangles." They used golden rectangles frequently in buildings and monuments. The ratio of the length to the width in a golden rectangle is called the "golden ratio."



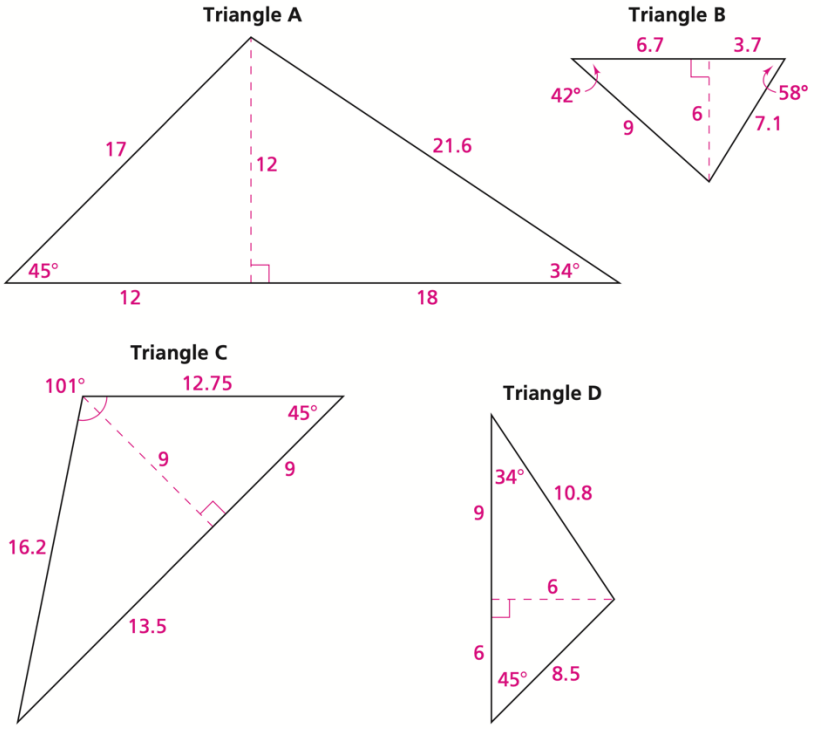
This photograph of the Parthenon (a temple in Athens, Greece) shows several golden rectangles.

a. Measure the length and width of Rectangles A, B, and C above in centimeters. For each rectangle, estimate the ratio of the length to the width as accurately as possible. The ratio for Rectangle B is an approximation of the golden ratio.

b. You can divide a golden rectangle into a square and a smaller rectangle similar to the original rectangle.

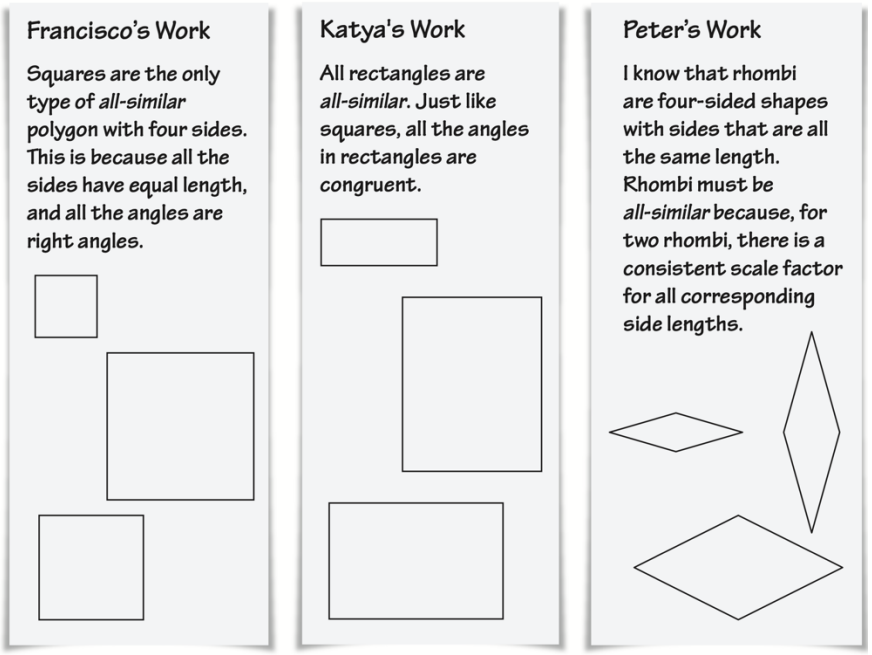


Copy Rectangle B above. Divide this golden rectangle into a square and a rectangle. Is the smaller rectangle a golden rectangle? Explain.

27	For parts (a) and (b), use the triangles below.  a. Identify the triangles that are similar to each other. Explain your reasoning. b. For each triangle, find the ratio of the base to the height. How do these ratios compare for the similar triangles? How do these ratios compare for the non-similar triangles?	4.1	
For Exercises 28-32, suppose a photographer for the school newspaper took this picture. The editors want to resize the photo to fit in a specific space on a page.			

28	Can the original photo be changed to a similar rectangle with the given measurements (in inches)? a. 8 by 12 b. 9 by 11 c. 6 by 9 d. 3 by 4.5	4.2	
29	Suppose that the school copier only has three paper sizes (in inches): 8 ½ by 11, 11 by 14, 11 by 17. You can enlarge or reduce documents by specifying a percent from 50% to 200%. Can you make copies of the photo that fit exactly on any of the three paper sizes? Explain your reasoning.	4.2	
30	A copy machine accepts scale factors from 50% to 200%. How can you use the copy machine to produce a copy that is 25% of the original photo's size? How does the area of the copy relate to the area of the original photo?	4.2	
31	How can you use the copy machine to reduce the photo to a copy that is 12.5% of the original photo's size? 36% of the original photo's size? How does the area of the reduced figure compare to the area of the original in each case?	4.2	
32	What is the greatest enlargement of the photo that will fit on paper that is 11 inches by 17 inches?	4.2	

33	Suppose you want to buy new carpeting for your bedroom. The bedroom floor is a 9-foot-by-12-foot rectangle. Carpeting is sold by the square yard. a. How much carpeting do you need to buy? b. Carpeting costs $22 per square yard. How much will the carpet cost? Suppose you want to buy the carpet for a library. The library floor is similar to the floor of the 9-foot-by-12-foot bedroom. The scale factor from the bedroom to the library is 2.5. c. What are the dimensions of the library? Explain. d. How much carpeting do you need for the library? e. How much will the carpet for the library cost?	4.3	
34	The following sequence of numbers is called the *Fibonacci sequence*. It is named after an Italian mathematician from the 14th century who contributed to the early development of algebra. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 … a. Look for patterns in this sequence. How are the numbers found? Use your ideas to find the next four terms. b. Find the ratio of each term to the term before it. For example, 1 to 1, 2 to 1, 3 to 2, and so on. Write each of the ratios as a fraction and as an equivalent decimal. Compare the results to the golden ratios you found in Exercise 44. Describe similarities and differences.	4.3	
35	Francisco, Katya, and Peter notice that all squares are similar. They wonder if other shapes that have four sides are *all-similar*. Who is correct?	4.3	

			
36	Ernie and Vernon are having a discussion about *all-similar* shapes. Ernie says that regular polygons and circles are the only types of *all-similar* shapes. Vernon claims isosceles right triangles are *all-similar*, but they are not regular polygons. Who is correct? Explain.	4.3	