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"Developing and Algorithm for Multiplying Decimals:
***Bits and Pieces III*, 2.2"**

LAUNCH:
Before viewing the video of students doing *Bits and Pieces III*, 2.2.

Before viewing the video participants need to do *Bits and Pieces III*, 2.2. I can launch 2.2 as I would for students (see TE). Participants should try to explore and summarize 2.2 as if they were students. My goal in talking to people in the exploration phase and in the summary phase will be to focus on the "strategies" asked for in A and B part 4, trying to have participants connect the idea of the denominators or place values of the digits given, to the denominator or place value of the missing factor. After the "student" summary I will ask "teacher" questions such as, did my launch give you enough to get into the problem? Did I give too much away? (It might be a good idea to look at the TE together.) How does this problem help students understand the algorithm that they were beginning to formulate at the end of 2.1? Did our summary get us closer to making sense of an algorithm? (Again we can look at the TE together.)

After the summary of 2.2, I need to prepare participants to watch the video. The following discussion questions should help.

Getting Ready to View the Video	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
After participants have done 2.2	• What would be your goal for 2.2? (We can refer to the TE.) • What ideas and strategies do you expect students to have to use to find the missing factor?	• *Use place value to compute a decimal product* • *Reason about factors* • *Write as fractions* • *Think of whole numbers to get the digits in the missing factor, then reason proportionally*	- *What do you mean "reason proportionally? "*[Ω]

[Ω] Suppose we are trying to find the missing factor in 0.03 x ? = 0.00036, we know 3x12 = 36, but the required product is smaller than 36 by a factor of 100,000 or 10^5. We have a factor of 10^2 in the denominator of 0.03, so we need another 10^3 in the second factor; we need 12/1000 or 0.012.

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• What difficulties might you predict for your students in 2.2?	• *Might find it difficult to set up the problem as a number sentence*	
• How close do you expect students to be to making sense of an algorithm by the end of 2.2?	• *I think students will be there, though they are not asked to write an algorithm until 2.4*	- *How important is it that students can explain their algorithm,as opposed to just observing it always works?*[Ω]
• What else do students have to understand to be able to make sense of an algorithm?	• *It would be good if they understood that dividing by 100 is the same as moving the decimal point 2 places etc.*	

[Ω] It's an important mathematical habit of mind to ask *why* something works, as opposed to just repeating a tried and true procedure. If we don't ask students to explain why, then we seem to devalue reasoning, and overvalue memorization.

VIDEO: "Developing a Multiplication Algorithm for Decimals"
(*Bits and Pieces III*, 2.2, 15 chapters. 29 mins)

Note: This part of the video has been edited to focus on the progress made by students towards understanding and using an efficient algorithm for multiplying decimals. At the end of 2.1at least two different students have proposed algorithms, but it is not clear that everyone has understood why these work. Real time is 1.5 class periods.

EXPLORE: When Principals are viewing the video they should be asking themselves one or more of the following :

While watching the video

Focus Questions for Principals

* What is the responsibility of teachers in knowing what students bring with them from prior curricular experiences?
* How can I help teachers gain this knowledge? What is my role in supporting this program after implementation?
* Since this way of teaching is not familiar to most parents what is my role in helping parents and teachers communicate effectively?
* What are the advantages and disadvantages of taking the time to have students develop an algorithm, rather than being shown one?
* What is the evidence that concentrating on making sense and developing concepts leads to understanding?
* What is the evidence that concentrating on making sense leads to retention of skills and success on assessments that measure both skills and concepts?

Focus Questions for Teachers and Teacher Leaders

Teachers will benefit from having available a copy of a transcript, perhaps 1 per pair of participants, and copies of 2.2. To Launch into viewing the video I should remind teachers of their own experience with 2.2 and their stated expectations for students.

Each person should choose a focus question from the list below, and imagine Teri re-viewing the video from this perspective.

This list is similar to the list of focus questions for 2.1, but the responses from participants are necessarily different.

- What moments appear to be important mathematically? What was the teacher's role?
- What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? In advancing the goal of the lesson? Of the Investigation? Of the Unit?
- What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?
- What prior skills do you see students using? Are they using these correctly? Effectively?
- What is the teacher's role in the Summarize phase? How does she pull the mathematics out of the student work? How does her selection and discussion of student work advance the goal of the lesson?

- What is the evidence that the students are learning?
- What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these expectations?
- Where are students on the path towards making sense of an efficient algorithm for multiplying decimals?

Form Focus Groups of Teachers and Teacher Leaders

Following my usual pattern I will re-arrange participants into focus groups before viewing the video and allow a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video.

	Focus Questions (as on previous page):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE **Focus Group Discussion after viewing the video**	• What moments appear to be important mathematically? What was the teacher's role?	• *Several times students state an algorithm. Sometimes the teacher has asked for a summary of the procedure, but sometimes these statements are spontaneous.*	- *What does the teacher do with these student statements of algorithms?*
Note: Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? In advancing the goal of the lesson? Of the Investigation? Of the Unit?	• *She asks questions to clarify what the problem is – some students don't seem to grasp the goal* • *She reminds student of the fraction connection*	- *Teri chooses a group to answer each part of the problem. How does she make her selection? Can you plan ahead for what to look for in the Explore phase?*

Focus Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?	• *They help each other. They independently explain strategies to each other* • *They question each others strategies*	- *Hannah comes up with a way to "get plain 9" which is incorrect. (Ch.5) What is her method? Why does Becca looks perplexed?*[Ω]
• What prior skills do you see students using? Are they using these correctly? Effectively?	• *Multiplying whole numbers, multiplying fractions, equivalent fractions, switching decimals to fractions. Most of this they do correctly.*	- *Ellie and Maddie get tangled in an error (Ch. 6). What does the teacher do about the error?*[Ω]

[Ω] I think Becca looks perplexed because Hannah has arrived at a product of 9/10 and says that is "the closest" to 9. Becca isn't buying this, but can't see what factor she needs to get "plain 9."

[Ω] The teacher asks the class what they think, and lets the correction come from other students. This seems to be a norm in the class. Errors occur and are corrected and become the occasions for review or re-teaching or re-thinking. In this case Ellie has become entangled in her representation.

Focus Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What is the teacher's role in the Summarize phase? How does she pull the mathematics out of the student work? How does her selection and discussion of student work advance the goal of the lesson?	• *She revoices what students say, to focus on the main goal, in this case, not just getting an answer that works, but having a strategy that works.* • *She connects student solutions.* • *She refocuses the student solutions for part B.*	- *In Chapter 7 Teri says "We are going to come to something today." This implies she just saw something that gave the class a way to approach this goal. What did she hear or see?*[Ω] - *Why was it important that the teacher refocus students on part B solutions?*[Ω]
• What is the evidence that the students are learning?	• *They continue to offer conjectures about an algorithm, and about specific solutions.*	
• What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these	• *They question each other in small group exploration and large group summaries. They contribute to the large group summary. They seem invested in making sense.*	- *Hayden voices an algorithm involving counting place values in the factors. What does the teacher do with this opportunity? What might you*

[Ω] Teri heard something in the language the 2 boys used that connected multiplying whole numbers, 9 x 3 = 27, to multiplying smaller factors, 0.9 x ? = 2.7. That's the connection she's going for.

[Ω] Because students did not use the same digits in their solutions the need for proportional reasoning with factors of 10 disappeared.

expectations?		*have done?*
Focus Questions (cont'd): • Where are students on the path towards making sense of an efficient algorithm for multiplying decimals?	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:** - *Is it acceptable if they stop at "multiply the digits as if whole numbers, multiply the implied denominators, make sure the answer has the right denominator." How does our "count the decimal places and move the point" relate to this?*[Ω]

[Ω] The standard "count the decimal places in the factors and make the same number of places in the product" works, but it doesn't make obvious sense. The students' "multiply the digits as if whole numbers, multiply the denominators, make sure the answer has the right denominator" connects directly to fraction multiplication. The first step is just multiplying the numerators. It also connects to the standard algorithm, in that the first steps are the same (multiply as if whole numbers), and the second step of multiplying the denominators (which are all powers of 10) will give another power of 10, which is exactly the factor by which the whole number product has to be decreased to get the correct answer. For example, the student algorithm at this point in the investigation is $0.03 \times 0.015 = \frac{3}{10^2} \times \frac{15}{10^3} = \frac{(3 \bullet 15)}{10^{2+3}} = 0.00045$. (They would not use exponents to express this, but the meaning is the same.) The standard algorithm would start with $3 \times 15 = 45$, and "count" 2 decimal places for 0.03, and 3 places for 0.015, make the answer 5"places" smaller --- actually dividing by 10^5. At this point the student algorithm works, and makes sense, but is a bit clumsy.

	Focus Questions:	In Previous Workshops Principals Have Said:	Follow Up Questions:
Focus Questions for Principals	* What is the responsibility of teachers in knowing what students bring with them from prior curricular experiences? * How can I help teachers gain this knowledge? What is my role in supporting this program after implementation?	• *I think this is important, but I can see that without someone to help organize this communication it's not going to happen, particularly if different text series are used.*	
	* Since this way of teaching is not familiar to most parents what is my role in helping parents and teachers communicate effectively?	• *We have had parent meetings where teachers teach some mathematics and then outline the goals of the current unit. This seems to calm parent fears.* • *As a Principal I need to be aware of the goals of the mathematics program, and of research about student achievement.*	- *See CMP websites, one of which is specifically for parents.*

Focus Question (cont'd):	**In Previous Workshops Principals Have Said:**	**Follow Up Questions:**
* What are the advantages and disadvantages of taking the time to have students develop an algorithm, rather than being shown one?	• *In theory, understanding something should lead to a different kind of learning from rote learning, using different brain activities. Things we learn and understand can be connected: things we memorize without meaning cannot be connected. Is there some research I could refer to on this?*	
* What is the evidence that concentrating on making sense and developing concepts leads to understanding?	• *It seems clear that rote learning does not attempt to build understanding at all, though students may build understanding in spite of this. However, I would like some research I could refer to on this.*	- *Note: A good article to read is "Mathematical Understanding: An Introduction," (Fuson, 2005)* See Appendix
* What is the evidence that concentrating on making sense leads to retention of skills and success on assessments that measure both skills and concepts?		

FINAL SUMMARY Large Group Discussion after viewing the video	After participants have had an opportunity to talk in their small groups we should have a large group discussion. This is another opportunity to ask follow up questions. See comments for 2.1. If no one else raises the issue of the value of taking all this time to allow students to develop an algorithm that makes sense to them, then I should be sure to include this in the large group discussion.