Jacqueline Stewart and Elizabeth Phillips, Connected Mathematics Project, Michigan State University

"Developing an Algorithm for Multiplying Fractions: *Bits and Pieces II*, Investigation 3"

LAUNCH:
Before viewing the video of students doing *Bits and Pieces II*, 3.1 and 3.2.

(See Journal for "Teacher Reflections" for an alternative use of this video.)

Before viewing the video participants need to do *Bits and Pieces II*, 3.1 and 3.2. This is the always the first step in planning to teach a lesson. I think I will launch 3.1 by having a "teacher" discussion about what multiplication means, about models students may have in their heads (whole number) prior to this activity, and about whether student ideas about whole number multiplication will connect to, provide a basis for, or interfere with making sense of multiplying 2 fractions less than 1. Then we will do Inv 3.1 in its entirety, Launch- Explore and Summarize, just as if the participants are students. Small group discussions of 3.2 should suffice.

After the "student" summary of 3.1 and small group discussions of 3.2, I have, in the past, used "teacher" questions which help participants prepare their mindsets to watch the video of students doing the same two problems. (If my audience is mostly principals then I need to focus on evidence of student engagement and the demands placed on the teacher by the CMP approach.) I can select appropriate questions from this list.

Getting Ready to View the Video: After participants have done 3.1	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• What do you expect your students will understand just by doing Problem 3.1?	- *The answer gets smaller* - *They may see a pattern and guess at an algorithm*	- *Why does the answer get smaller?* - *How does the model "show" that the answer gets smaller?*
	• What difficulties might you predict for your students in 3.1?	- *They will not see that this is multiplication* - *They will not see why the algorithm works* - *They will mix up division and multiplication because "part of" sounds "divide."*	- *How does the model develop the meaning of "part of?"* - *How does this model resemble whole number multiplication models?*
	• What do they still have to understand after doing 3.1?	- *They still have to understand why the algorithm works.*	- *Why does the algorithm work?* - *Does the model prove fraction multiplication is commutative?*

After 3.1. (Cont’d)	**Possible “Teacher” Discussion Questions: (Cont’d)**	**In Previous Workshops Teachers Have Said: (Cont’d)**	**Follow Up Questions: (Cont’d)**
	• What do you want to come out of Problem 3.1?	- *I want them to see a pattern* - *I want them to understand why it works.* - *I want them to connect the pattern to the model. I am concerned that the subdivision into vertical and then horizontal parts can become an unthinking routine itself.* - *I want students to see that the BP model explains why a part of a part is always smaller than either part that they started with.* - *I want Students to understand the switch from “part of” to “multiplying.”*	- *Will you tell students why the algorithm works?* - *How does the pattern connect to the model? Can you explain with one of the pieces of “Student” work that came out of our exploration of 3.1?* - *How do you know that this pattern will always work?* - *What if no one mentions the pattern?* - *Is there any advantage in having a model to remember rather than a symbolic rule?*
	• What will your role be in the Summary?	- *I will orchestrate the summary*	- *What do you mean “orchestrate?” How will you choose student work to share?*[Ω] *(See Student Work on DVD)*

[Ω] Since we have the student work on the DVD it is possible to challenge experienced teachers to make their own decisions about how they would select and sequence this student work in the summary, before watching the video. However, beginning teachers may not be ready for this challenge.

	Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
	• Does whole number multiplication connect/interfere with student understanding of a part of a part?		- *Can we connect multiplication of whole numbers to this model?*
After participants have discussed 3.2	• What answers do you expect for part C?	- *Students will very likely spot the "pattern" that the denominators are multiplied and so are the numerators, if they have not already done so in 3.1.*	
	• What will you do with a student statement of an algorithm?	- *I will have them note their algorithm in their notebooks.* - *I will try to connect the Brownie pan model and the linear model to the algorithm.*	- *How does the algorithm connect to the model?* [Ω]

[Ω] I don't have to push participants to explain the connection between the algorithm and the model *completely*, but I think they should at least try at this stage. Teri and her students do a fine job of laying this out on the video.

Possible "Teacher" Discussion Questions (cont'd):	In Previous Workshops Teachers Have Said:	Follow Up Questions:
• What answers do you expect for part D? What kind of reasoning or evidence will you accept for the "why" part of the question in part D? • What prior knowledge are students using?	- *Students are likely to argue from examples for part D. They may say that both are right because they can think of whole number examples to confirm Ian's statement, and part of a part examples to confirm Libby's statement.*	- *Is an example, or set of examples, the same thing as a proof? When is Ian's statement correct? Libby's?* - *What is the characteristic of* $\frac{2}{3} x \frac{3}{4}$ *that makes it possible to find the answer without subdividing into twelfths?*[Ω]

[Ω] Suppose the problem is $\frac{t}{s}$ x $\frac{y}{x}$. The first division and shading of the whole brownie pan produces *y* parts shaded out of *x*. Instead of routinely doing the second division in the opposite direction, producing a total of *sx* parts of which *ty* are double shaded, we could look for an alternative. For example, if *s* is 3 and *y* is a multiple of 3 then we will be able to find thirds of the *y* shaded parts without any subdividing. Or if *s* is a multiple of 3, say 6, and *y* is also a multiple of 3 say 9, then we only need to subdivide the 9 parts into 18 parts, so that we can find sixths of that. In general, if y and s have a common factor then it will not be necessary to end up with a denominator of *sx* parts; you can accomplish the second subdivision in fewer parts, by looking for a common multiple of s and y. For example, here is the symbolic version of such a subdivision:

$\frac{5}{6} \text{ x } \frac{9}{14} = \frac{5}{6} \text{ x } \frac{18}{28} = 5(\frac{1}{6} \text{ x } \frac{18}{28}) = 5(\frac{3}{28}) = \frac{15}{28}$.

VIDEO: "Developing an Algorithm for Multiplying Fractions"
(*Bits and Pieces II*, 3.1 & 3.2, 8 chapters. 24 mins)

Note: This video has been edited to focus on the learning trajectory from first exposure to a model for representing a part of a part to making sense of an algorithm for the computation. Real time is 3 class periods; a lot of Exploration was cut in the edit.

EXPLORE: While watching the video
(See "Teacher Reflections" for an alternative way to use this video)

Focus Questions for Principals

Principals benefit from doing the mathematics in 3.1 and 3.2, but I need to remember that their responsibility includes supporting teachers. When they are viewing the video they should consider one or more of the following :

* What evidence is there of student engagement? Of students taking responsibility for learning?
* How is this way of teaching different from what teachers are currently doing? How prepared are the teachers in my school to teach like this?
* What can I do to help teachers move towards inquiry based teaching?
* How can I help teachers form collaborative planning groups?
* How is this view of learning different from a traditional view?
* What is my role in supporting this program?

Focus Questions for Teachers and Teacher Leaders

Teachers will benefit from having available a copy of a transcript, perhaps1 per pair of participants, and copies of 3.1 and 3.2. To introduce the video I should remind teachers of what they said they expected from their students and what difficulties they anticipated. *These predictions as the first step in planning a lesson.*

Each person should choose a focus question from the list below, and imagine Teri re-viewing the video from this perspective.

- What moments appear to be important mathematically? Was this a student interchange? What was the teacher's role?
- What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? What are the students doing during the "explore" phase? Are their actions and conversations effective in addressing the mathematics?
- What is the teacher's role in the Summarize phase? How does she pull the mathematics out of the student work?
- What is the evidence that the students are learning?
- What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these expectations?
- What evidence is there that the model is important in communicating, representing and developing an understanding of an algorithm?

Form Focus Groups of Teachers and Teacher Leaders

It has worked well in the past to re-arrange participants into focus groups before viewing the video. If they have a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video I have noticed that the discussions are more coherent.

I have noticed a tendency for people to make general comments in response to the video. When I ask follow up questions both the small and large group discussions are richer.

	Focus Questions (as on previous page):	In Previous Workshops Teachers Have Said:	Follow Up Questions:
SUMMARIZE: Focus Group Discussion after Viewing the Video	• What moments appear to you to be important mathematically? Was this a student interchange? What was the teacher's role?	- *Teri brings out what she wants them to get to.* - *When the teacher asked about why the answer got so small every time she focused students on how the model shows the denominators.*	- *What specifically do you think Teri wants to get to? Can you give us an example of her words or actions?* - *The focus on the denominators was at the end of Day 1. Why do you think Teri did not push further at that time?*
Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• What is the teacher's role when the students are exploring? Are her actions and questions effective in bringing out the mathematics? • What are the students doing during the "explore" phase? Are their conversations effective in addressing the mathematics?	- *Teri asked great questions. She answered every question with another question.* - *I am marveling at the flexibility of the teacher. I don't know if I could do that.* - *We did not see any frustration on the part of the kids.* - *What kinds of backgrounds do these kids come from? How do you get them to work in groups?*	- *You said Teri asked great questions. Can you give us an example? What was the purpose of the question?* - *You said you saw no student frustration. What would you like to ask Teri about this? (See Teacher Questions video)* - *What would you guess was the demographic of the students you saw on the video?*

Focus Questions: (cont'd)	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What is the teacher's role in the Summarize phase? How does she pull the mathematics out of the student work?	- *I liked the part where she asks why and keeps asking until they figure out that there are 6 groups of 10 and that explains why you multiply the denominators.* - *I was impressed that the teacher thought of another question to help kids focus on what's happening to the numerators.* - *It was cool to see kids take feedback from the class with no embarrassment* - *How do you learn great questioning skills* - *Teri kept her focus.*	- *How did Teri's extra question focus kids on the algorithm?* - *Remind us of when kids took feedback without embarrassment.* - *Teri said that she should have waited longer to see if a student would have mentioned multiplication, before she replaced "of" by "times." Agree?* - *What makes a good question?* - *How does the teacher help connect different representations?* - *What was Teri's focus?* - *Should she have had every group share a solution?* - *Would you have selected and sequenced the student work differently? (See Student Work)*[Ω]

[Ω] We have Student Work on the DVD. Teachers might like to discuss possible ways of selecting and sequencing this.

Focus Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
• What is the evidence that the students are learning?	- *I can't tell if the kids got why to multiply the numerators.* - *You could see students using the language of dividing up parts, relating this to the denominator.* - *I loved the high expectations that kids have of each other.*	- *Do you think that all kids got why the algorithm works that day? If you were Teri what would you do about this?* - *How do you suppose the students developed these high expectations?*[Ω]
• What evidence is there that students expect to make sense of their various conjectures and strategies? What is the teacher's role in creating and raising these expectations?	- *There is a great discussion about whether the algorithm will always work. I notice that Teri did not lead them.*	- *What is gained by pushing students to explain why the algorithm works?*[Ω]

[Ω] Teachers and Principals often remark on the atmosphere in Teri's classroom. They notice that students are seriously working on mathematical challenges even when the teacher is not directly supervising them, and that challenging each other and giving explanations is the norm. None of these things are just luck; Teri works hard at getting the class to establish norms for their discussions, and then at modeling interest and enthusiasm for listening to student ideas. She talks about this at length in an interview that is in the appendix.

[Ω] There are two goals in pushing students to explain the algorithm. One is that understanding the algorithm may lead to greater retention of when to use this algorithm. The second goal is to encourage students to develop the habit of mind that mathematics should always make sense, and that "why" is always a good question to ask yourself, and others.

	Focus Questions:	In Prior Workshops Principals Have Said:	Follow Up Questions:
Focus Questions for Principals	* What evidence is there of student engagement? Of students taking responsibility for learning?	- *Students were engaged in making sense.* - *Students expected to explain their reasoning.*	- *Can you give and example of student engagement?*
	* How is this way of teaching different from what teachers are currently doing? * How prepared are teachers in my school to teach like this? * What is my role in supporting this program after implementation?	- *Teri guided, facilitated.* - *Teri knows what she wants the kids to understand, and her actions in the Launch and Explore phases are directed towards the math goals that come out in the Summary.* - *I need to find time for teachers to plan together.*	- *How will you help your teachers prepare for the roles you describe?* - *What skills or practices will teachers need to acquire to fill these new teacher roles?*[Ω] - *How will you support teachers?* - *How will teacher evaluations be affected?*

[Ω] When I think about the seamless way that Teri goes about asking questions, choosing student work and sequencing it in a way that allows students to make sense of the mathematical goal, I sometimes despair of being able to help other teachers feel comfortable doing the same. There are so many things to attend to simultaneously. In professional development sessions we do have the luxury of attending to different issues separately, in the hope that, with practice, teachers will assimilate several good practices into their daily routines. For example, we have the student work for the problems shown on this video. Should teachers show all student work? If they do, they show respect for all students, but they may also use up precious time and lose the focus on the mathematical goal. So which work should they select and how? How can they plan ahead to make the selection process both quick and effective? What should they be looking for? Why might one sequence work better than another? Where do errors fit into the sequence of the Summary presentation? To which errors should class time be devoted and which are better handled individually? There are no easy answers to these questions. The goal is to have teachers feel empowered by learning how to orchestrate student work so that the mathematical goal is the focus, and so that students feel ownership in the mathematical outcomes of a lesson. We can begin the process at a workshop, but Principals will have to support the continuation of the learning process.

SUMMARY CONT'D Large Group Discussion	After participants have had an opportunity to talk in their small groups we should have a large group discussion.
After Viewing the Video	Participants often have questions they would like to ask Teri. We may view Teri's Reflections. See the Journal for "Teacher Reflections" for more details.