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Transcript for October 23- 25, 2006

"Developing an Algorithm for Multiplying Fractions:
Bits and Pieces II, Using Fractions Operations
Investigation 3"

The class is seen working on Investigation 3.1,
"How Much of a Pan Have We Sold," and 3.2, "Finding a Part of a Part."

The video was shot in real time and edited from 3 days,
approximately 3 hours, to 24 minutes.

Bits & Pieces II, Investigation 3
Class: 6th Grade
Date: October 23-27, 2006

Chapter 1: Launching 3.1.

Time: Approximately 00:00 – 5:48 (Times from start of video)

Slide:

In her reflection at the end of day 1, Teri explains why she extended the launch for 3.1 to include students thinking back to their knowledge about multiplying whole numbers.

Line 1, 00:06 T: I have taught this a few years, and I know from experience that getting kids to think about multiplying with fractions is very difficult. They're used to whole number multiplication where you have seven groups of nine and when you get all, you get more everything multiplies and you get bigger. So this whole notion of taking a part of a part is very difficult, and I know that from past experience. Things I thought were going to be easy were - are not easy for them. So I was trying to pull on what

Line 10, 00:34 they know about multiplication and get them to think harder about what might happen if I multiplied two numbers and the answer didn't get bigger. So I have this big goal. By the end of the week I want them to understand finding a part of a part or a part of a whole and that's difficult to think about when up 'til now in their education they've thought about multiplication as getting bigger.

Slide:

After 15 minutes of students talking about multiplying whole numbers, and about having to redefine the whole when you think about a part of a part, Teri starts the last part of the launch for 3.1.

T: Today, we're going to take a look at some kids are making big brownie pans, and they're gonna sell parts of it at a fundraiser. So, they're having a big brownie pans

Line 20, 1:20 like this, and it's twelve dollars for the whole pan, but people can come up and they can buy just parts of the pan if they want to. Okay?

Class: Um, huh.

T: Let's say that Mrs. Klotz came up and she bought half of this pan. This - hold on, hold on. She bought all of this. So this part over here is what's left in the pan. Okay? Now let's say that all of us walk up and we say "Oh, there's only a half left." We kinda want a lot of it, though. Let's buy three-fourths of that half.

Line 30, 1:55 Is it possible to do that?

Class: [In unison] Yeah.

T: Okay. I used red to show how much is left in the pan. I'm gonna use a different color to show what we're gonna buy of it. Violet, could you come up here on this, could you show us what three-fourths of that would look like?

S1 [Violet]: If I split this this way, it would be, um, fourths, because if you also split this piece which would be there you would have four pieces, and then if you split it this way you, you would have fourths on this pan and

Line 40, 2:32 then you would take these pieces and that would be three fourths of the pan, three fourths.

[Unintelligible comment or question from class.]

S1 [Violet]: Yeah. It would also be three fourths of the

half but also it would be three eighths because you would have this piece too.

Off-camera student: And three eights of the pan, right --

T: Talk to them. They're the ones you have to convince.

S2 [Nikki]: Okay. You said you'd split the other one that you don't have in half, like you'd four it, but you can't,

Line 50, 3:00 then it would be eighths, not fourths.

S1 [Violet]: No, 'cause before I split it this way I had, um, fourths, one, two, three and four, and then I split this one in half, and then I would also do it like that so I would have eight pieces so, um, on this half it would be three fourths but on the whole pan it would be three eighths.

T: Does everybody agree with her about that?

Class [in unison]: Yes.

T: Okay. Thanks, Violet. So let's think about what we

Line 60, 3:33 just did here. She's showing right here three fourths of what?

Class: Of a half.

T: Of a half. Okay. She's showing three fourths of a half. My question is, what fraction of the whole pan did we buy then? What fraction of the whole pan did we buy?

S3 (Drew): Three eighths?

T: Three eighths. What do you guys think?

Class [in unison]: Yes, yeah. Um huh.

T: So if I bought three fourths of this half, then I

Line 70, 4:06 bought three eighths. Does everybody agree with that?

Class [in unison]: Yes.

T: Okay. Nikki?

S2 [Nikki]: I see uh, kind, the pattern. You just change the denominator. You times it by two or three or four. The numerator does not change at all.

T: Okay.

Hayden: I see that too.

T: But do you think based on one situation that's going to happen every time?

S2 (Nikki): No, just with halves.

Line 80, 4:28 T: Okay. Okay. Alright. Well, think about that. But this is just one brownie situation, okay?

Class: Uh, huh.

T: Now we're gonna do a problem in your book that looks really similar to this.

T: Now, let's look at the top of page 33, 'cause that's where the problem starts. Okay? This says all the pans of brownies are square. A pan of brownies costs twelve bucks. You can buy any fractional part of a pan of brownies and pay that fraction of twelve dollars. For example, half a

Line 90, 4:59 pan costs half of twelve dollars. Are you following so far?

Class: Yes, yeah.

T: Okay. So will you look at letter A for me? It says: "Mr. Williams asks to buy one half of a pan that is two

thirds full." And then letter A says to draw a picture of what the brownie pan looks like at first, and then two says use a different color to figure out what you bought. So what we're gonna do for one and two is exactly what we did up here. We're gonna use one brownie pan, two different

Line 100, 5:33 colors. One color to show what's in the pan, and another color to show what we actually did. Okay? And then you're gonna show what fraction they bought and you're gonna get a cost. You're gonna do four things for each brownie problem. Okay.

•Chapter 2 Exploring 3.1

Time approximately 05:48 – 6:50 (Times from start of video)

Slide:

Teri assigns only 3.1A and B and has an extra sheet of problems prepared for students who finish 3.1A and B.

The class explores 3.1A and B for 27 minutes.

Line 1, 5:55 T: Okay. So, what are you, how are you going about doing B? What does it say about answering them, what do we have to do?

Hannah: She's gonna buy three fourths of a pan that's half full.

T: So what's that pan look like before Aunt Serina buys something from it?

(Students are working on the EXTRA PROBLEM)

Ellie: Hey, don't we have make the denominator the same, though?

Line 10, 6:16 Kaylie: Yeah, that's what I'm saying. See, these both go into forty.

Ellie: Okay, okay. So, three fifths . . .

Kaylie: Three fifths times eight equals twenty four fortieths.

T: In about two minutes, we're gonna come together and look at your pans of brownies. So you have two minutes to finish up what you're right in the middle of. And then we're going to stop working and we're all going to come together. Just so you know what's going to happen here in a minute.

Chapter 3: Summarizing 3.1

Time approximately 6:50 – 11:56 (Times from start of video)

Line 1, 6:50 T: I saw very similar things for B.1, 2 and 3, but I saw three different things when I walked around for A. So I asked different individuals would they put their drawings up so we can try to figure them out.

[3.1 A, 1/2 of 2/3]

S7 [Hayden]: It says that he buys half of two thirds, so, they don't, this is sold out right here, and they have two thirds, and he buys half of it, so half of two thirds was one third. So, and it says, um, what fraction of, he bought one third was the fraction of how much he bought.

Line 10, 7:33 He paid, well what would be, he paid, um, the whole thing was twelve dollars. He paid four dollars because you divide four by the denominator. You divide, or you divide the, your denominator by how much the whole pan costs, and I got four dollars. Nikki?

S2 [Nikki]: Didn't he have one half first and then he bought two thirds?

S7 [Hayden]: No, Oh, no. They only, this, they only had two thirds. They only had two thirds.

T: How did you know that, Hayden? Tell Nikki how you knew

Line 20, 8:11 that.

S7 [Hayden]: Because it says Mr. Williams asked to buy one half of the two thirds full.

S2 [Nikki]: Okay.

T: Any other questions for Hayden? I'm just going to write with numbers what I heard you say, Hayden, and tell me if this is right. I heard you say you bought one half of two thirds, and you said it was one third?

S7(unidentified): Yeah, because.

T: Okay.

[A new group at the board]

Line 30, 8:36 T: Let's look up here and listen while this group is talking, okay?

S8 [girl at board, pink OC shirt]: We drew two thirds and then we cut the two thirds in half to see what he would buy. And then, he would buy two sixths, because if you cut it in half he would get three, and then three which is six. And that's equivalent to one third. So he would either, he would get two sixths or one third.

S9 [Becca]: Okay *[hands book to S8]* - unintelligible.

Line 40, 9:03 T: Hold on just a second. Did you have a question for this? Okay, go ahead.

S10 [Ellie]: Well, I think you could, why didn't you just keep it in two, um, thirds, because it never talked about sixes.

S8: Because he buys a half of two thirds, I guess that's -

Becca: We just thought it was easier, it helped us understand it more.

S8: It helped us to find how much money he would have to

pay.

Line 50, 9:30 S__ [Ellie]: Oh, okay.

T: So you just kinda, you cut, you cut it again this way. So what was your answer?

S8, S9: Two sixths.

T: And how did you decide it was two sixths?

Becca: Because if you use like the whole bar, you end up having six pieces, and then you just have, you, he ends up not buying two pieces.

T: Sharing. So I'm just gonna write what I heard you saying and look what it looks like with numbers. You

Line 60, 9:54 bought one half of two thirds and you guys got two sixths. But you said that's the same as the one third that they got. Okay.

Slide

Teri ends the discussion on Day 1 with and example to which she adds a foreshadowing comment.

[New group at the board. 2/7 of 1/3]

T: Could you talk about what you guys did?

S11 (Kristen): Well, um, he had one, there was one third of the pan left over, so we had to divide it into seven pieces, and he bought two sevenths, two sevenths of the one third, and if you divide all the rest into seven pieces, it would be twenty one pieces, so all together he bought two twenty-oneths of the whole pan.

Line 70, 10:42 T: Do you understand what they just said?

Class: Yes.

T: Okay. I'm sorry. I'm just - can you - thanks, you guys. So, you, she bought what, two sevenths of what?

S11(Kristen): Of a third.

T: Of one third. And then you said something. You said if I split these guys into what?

S11(Kristen): Seven pieces.

T: So, is it fair to say that these two girls took each one of these thirds and split them into seven pieces? So

Line 80, 11:06 could I do this, and actually show what you just said?

S11(Kristen): Yeah.

T: Do you see what they did?

Class: Yeah.

T: They split every one of these three pieces, they split into seven pieces. So, how many pieces do I have in my whole thing now?

Class: Twenty one.

T: Twenty one. Now, in this one third, how many pieces did they take?

Line 90, 11:28 Class: Two.

T: Two. Hmm. So, two sevenths of one third. Look how little it got.

Class. Um, hum.

T: You know what. I'm noticing that every time. I'm

noticing, that, ooh, I got a little two sixths, or a third. I got three eighths, I got two twenty oneths. My answer got so little. Why? Why am I getting a little answer? Yeah?

S9 (Becca): Because the denominator is getting larger and that

Line 100, 11:55 makes the pieces smaller?

•Chapter 4: Summarizing 3.1. Proposing an algorithm.
Approximate time 11:56 – 15:50 (Times from start of video)

Slide

Day 2 continues the discussion of solutions of part of a part problems from Day 1.

Hayden spots a pattern.

[Group at the board discussing 9/10 of 1/6]

Line 1, 12:03 S12 [Brooklyn]: What we did is, it was in one sixths so we split it up into six pieces, and then they had just this part and then all this part was gone, and they had, we split this up into ten pieces and then they had nine of them. They bought nine of them and then there was this one and then we figured, to figure out how many, um, we would have for the denominator we had this in ten pieces so we split each of the other six into ten pieces so we got sixty, and then we had nine in

Line 10, 12:44 here and then all the rest wasn't there so we got nine sixtieths.

T: Okay. So can I read it like this? Nine tenths of one sixth, and she's proved up here, nine sixtieths.

Class: Um, hum.

T: We're going to come back and look at all these, 'cause you know what I'm going to do? I'm gonna change that word "of" to a mathematic symbol. What you guys have done here, I love the way that you're all looking at me, I can tell you're listening. Thank you very much. When I take a part

Line 20, 13:17 of another part, I can write that as a multiplication problem.

T: Instead of saying nine tenths of a sixth, I can also say nine tenths times a sixth. Go ahead, Hayden.

S7 (Hayden): I think its times the numerator and numerator and then times the denominator and denominator.

Class: Yeah. Nine times one is nine and ten times six is sixty.

S7(Hayden): Yeah. See, you go like this. Two times one equals two and seven times three equals twenty one, and so that's

Line 30, 13:46 how I get this nine and --

S__ [unidentified]: That's how I did number four.

T: So why, why does that work?

S__ [Taylor]: It helps you figure it out easier.

T: Okay, but why?

S [Taylor]: Because you just know that denominators are equal to what the answer is going to be.

T: Why, though?

S14 [Nikki]: [Unintelligible, like groups

Line 40, 14:22 of that number so you had for number three ten groups, and six in each group.

T: Do I have ten groups of six? Where do I have ten groups of six?

S__: [Unintelligible response]

T: Oh, you're thinking, you're thinking why, why

S15 [Allie]: When she says ten groups of six, she means, she says like, um, like, like you're, you're adding six ten times basically.

S__ [unidentified]: It's like --

Line 50, 14:51 T: Are we doing that in this picture, adding six ten times in any way? Kristen?

S11(Kristen): Like in one of the sixths, you have ten pieces, and then you have six sixths, so you have tenths and . . .

S11 and class: Yeah, yes.

T: Is there six groups of ten instead of ten groups of six?

Class: Yes.

T: So I took every sixth and I split it into ten pieces. Let's check and see if we did that. In every one of these

Line 60, 15:18 thirds, did I split them into seven pieces?

Class: Yes.

T: One, two, three, four, five, six, seven. Hmmm. In each half, did I split it into three pieces?

Class: Yes.

T: Huh. How about over here? In this half, did I split it into four pieces?

Class: Yes.

T: Okay. So that sort of explains the denominator. So why do I multiply the numerator. Is that what you said. Who said - Hayden, was it you who said you multiply these

Line 70, 15:48 two and then you multiply those two? So why, why?

Chapter 5: Summarizing 3.1, Analyzing the Algorithm.

Approximate time 15:50 – 19:50 (Times from start of video)

Slide

Teri tries a new problem to see if that helps students think about the proposed algorithm.

Line 1, 15:53 T: What if I have - let's try something different. What if I had two thirds of, maybe, two fifths? Could we figure that out? Um, remember, I can cross, make it multiplication. Hayden said, I think we're gonna multiply these guys and multiply our numerators. So, if we'd use that strategy, we're gonna end up with about four fifteenths. So, I tell you what. Could you guys help me to do a drawing and see, and see what's happening here? Let's see here. Let's get a big pan of brownies. If I have two

Line 20, 16:33 thirds of two fifths, what does the pan look like to start with?

S1 (Violet): First I would divide it into fifths and then, if I have this and then we would split these two pieces into thirds, and then two thirds of that would be, like, um, these parts.

S1(Violet): I messed up.

T: Where's your one third? Can you show us, okay.

S1: My one third --

T: 'Cause it looked like you split it into three pieces.

Line 30, 17:22 Why don't you do what you did again? 'Cause I, is she taking that green spot? So, how big is this piece and this piece and this piece?

S1(Violet): They're thirds.

T: Are they? So, how could you fill in one of those thirds?

S1(Violet): Um, you could do, wait a second, one third would also be two sixths, which is --

T: Okay.

S1(Violet): Where I split it --

Line 40, 17:46 T: Okay. So can we get rid of that other blue that we have there and that next piece. Okay, okay. Are we okay, is she showing what you think we should show? Thank you, Violet, very much.

•Class: [Unintelligible comments, multiple raised hands.]

T: Okay, hold on, hold on for just a second. She started out with a two fifths, right? Then she's shading in two thirds of that green piece. You're saying no.

S1(Violet): I did two sixths and two sixths is equal to one third, so we would need to do that --

Line 50, 18:18 Class: Yeah.

T: Now is she showing two thirds?

Class: Yup, yes.

T: Thanks.

S__ (Hayden): Maybe you should cut it off - cut - take the thirds and go all the way across so you can see how much of the whole pan it is.

S__ (unidentified): Yeah, like do dotted lines -- yeah, like that --

T: So, if the brownies were there, they would be cut up

Line 60, 18:36 like that?

S__ (Hayden): I think they're all in sixths because you had them into fifths and then you cut them into thirds so now they're all in sixths so you have, let's, er, fifteenths --

Class: Fifteenths --

T: Are we at fifteenths now in the whole pan?

Class: Yeah.

T: Thanks, Violet.

Class: And now we have four of them.

Line 70, 18:56 T: Okay, so let's take a look at this. Is that what we said we were gonna get?

Class: Yeah.

T: Okay. So what did we do in every one of these original fifths? How many pieces are in them?

Class: Three.

T: So we took every fifth and we split it into three pieces. Could that be how we got this?

Class: Yeah.

T: Now, let's look at each of those two fifths. How many

Line 80, 19:16 pieces did we color in in each one of those fifths?

Class: Four.

T: Four? Here's a fifth. How many pieces did we color in? Two here, and how many here?

Class: Two.

T: Could that explain the two groups of two to get this?

S14 (Nikki): For every fifth or every, like, third, because there's fifths and thirds there, you cut them in two.

T: Yup. Because for each of those fifths we want two, two

Line 90, 19:43 thirds in each fifth, two pieces here and two pieces here. What do you think about that?.

Chapter 6, Exploring 3.1C and D.

Approximate time 19:49 – 20:37 (Times from start of video)

Line 1, 19:49 T: What I'm going to have you do is that we're gonna go back to our book. Go back to you book on page 33 --

Slide:

The class continues to explore 3.1C and D for 20 minutes, and to tie their solutions to the proposed algorithm.

T: What is your drawing showing? Six pieces out of how many in the whole thing?

S16 (Jesse): Six pieces -

T: Out of -

S17(Dalton): There's only two full pieces (unintelligible - multiple speakers).

Line 10, 20:19 T: Okay. So now let's go back to what you were doing over there, Dalton.. Does your work over here match this drawing? What were you doing over here? You said, I'm just going to try to solve it first?

S17 (Dalton): Yeah, doing what Hayden said -

T: Okay.

S17(Dalton): Timings it out. Three times two equals six --

T: So six twentieths, is that what you get?

Chapter 7: Summarizing 3.1C and D.

Approximate time 20:36- 22:04 (Times from start of video)

Line 1, 20:36 T: Alright. This is what we're gonna do. We're gonna take a look at all four of these. C.3 and C.4 I asked two groups to do it. C.2, they had a disagreement at their table so they're going to show both ways. Okay. So let's go on and start over there with C.1.

[C.1, 1/3 x 1/4]

S9 (Becca): What we did is we knew that one fourth meant there was four pieces so we divided it up into four pieces. And then it says that we had one out of the four pieces, so we had that, and then we had to divide that up into three

Line 10, 21:11 pieces, so we divided that up into three pieces, and we had one of those three pieces, which is there. And then we divided the rest of them up like if they were there, we divided those up into three, and we ended up get - that gave us a total of twelve, and so we ended up getting one twelfth, which we also figured out like if you did it Hayden's way, too.

[Clapping from class].

T: Okay. So, you guys, how many pieces did she split each fourth into? How many pieces are each fourth split into?••

Class: Three.

Line 20, 21:49 T: Three. Would that explain the four times three? Excuse me, up here?

Class: Yes.

T: And then in this one fourth, how many of them did we take?

Class: One.

T: Would that help explain the one?

T: So what are you thinking at this point about Hayden's idea?

Class: It works.

Chapter 8: Summarizing 3.2, Writing a Class Algorithm.
Approximate time 22:04 – 24:35 end of part 1) (Times from start of video)

Slide:

On Day 3 Teri introduces the class to a new model in 3.2. Students explore 3.2A and B, finding a part of a part and comparing their results from their drawings with the algorithm proposed by Hayden.

Line 1, 22:15 T: Page 35. 3.2. We're still in 3.2. When you finish, if you finish, I would like for you to consider C. Look at C for a second. What observations can you make from Questions A and B that help you write an algorithm? So if you finish B.1, 2 and 3, I've got it started on this sheet. You can grab a marker, come right over here, and start writing. Remember the algorithms we wrote for adding and subtracting? We're gonna try to write an algorithm, algorithm for multiplying, but we're going to kinda do this

Line 20, 22:56 one as a big class.

T: You're already done, so you're good. Right. So now, two, four, how many, how many pieces do you have colored in out of that whole - but well there's nine right here -

S__(unidentified): Nine.

T: But there's also nine --

S__(unidentified): Eighteen --

T: Eighteen out of how many in that whole thing?

S__(unidentified): Thirty?

T: Yup, yup.

Line 30, 23:24 T: So let's look at what some of your classmates put up here for an algorithm and tell me what you think. Do you think that these are the steps that we should tape, take? Multiply the numerators and multiply the denominators and then you put them together as a fraction?

Class: Yes, yeah.

T: Nikki, talk to the folks writing this. What do you think?

S19 [Nikki]: Should you [unintelligible]?

T: What do you think? Do you think you need a picture in

Line 40, 23:47 your algorithm or no?

S19(Nikki): Because, it should, because some times, I don't know, maybe one of these times it might not work. It might, probably not going to, but one of these times it might not --

S__ [unidentified]: But that's how you do it --

S__ (Becca): It's going to work because if it wasn't - it probably wouldn't have worked for all of them.

T: Okay, I'll tell you what, guys.

S__ (unidentified): What if you had one more of like something, if you had one more?

Line 50, 24:10 T: Oh, I just set you up. It's like I paid you to say that. That's exactly what we're going to figure out tomorrow. What happens if I don't just have a fraction of a fraction, if I don't have a part of a part? What if I have a fraction of a whole number? What if I have a fraction of a mixed number? What if I have a mixed number times a mixed

number?
S__ (unidentified): Sweet.
T: Sweet?