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"Developing an Algorithm for Multiplying Fractions: *Bits and Pieces II*, Problem 3.3"

LAUNCH:
Before viewing the video of students doing *Bits and Pieces II*, 3.3.

(See Journal for "Teacher Reflections" for an alternative use of this video.)

Before viewing the video of students doing 3.3, participants need to do the Problem. I will use the Getting Ready on page 36 as a Launch. We should use estimation strategies; these tend to imply computation strategies, so it will be my job to help make these connections. In the Explore phase participants are likely to have trouble coming up with convincing models without recourse to their old algorithms. If they want to "change to improper fractions" I might ask, "Suppose students do not think of this strategy. What else could they do?" In the Summary we should share strategies as if participants were students, and I will try to make as explicit as possible the mathematics I saw during their Explore phase, and how I selected "student" work to guide the Summary phase. After sharing solution strategies we can have a discussion about the L-E-S model, as it played out here, and about different mathematical strategies. This foreshadows the video of students doing 3.3, so it *prepares participants to look for and analyze the L-E-S model and the mathematical strategies as they appear on the video*. Below are some guiding questions.

Getting Ready to View the Video: After participants have done 3.3	**Possible "Teacher" Discussion Questions:**	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
	• What strategies for estimating do you think students will suggest in the Launch phase?	- *Students' strategies for estimating include rounding one or both numbers to whole numbers and then using "groups of" language.* - *Another student strategy is using the Commutative Property. For example,* $2\frac{1}{2} \times \frac{4}{7}$ *becomes about* $\frac{1}{2}$ of $2\frac{1}{2}$.	- *Do estimation strategies help students do the actual multiplication computation?* - *What mathematical idea is being used correctly in* $\frac{1}{2} \times 2\frac{1}{2} = 1\frac{1}{4}$, *and is misapplied in the common error you predicted?*

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Would the Launch I used get your kids well started? Was there something to cut? Something that should be included?	- *Launches have to be 5 minutes only.* - *You have to do an example in the Launch because kids don't know what we know.* - *You should not do an example in a Launch.* - *You have to set expectations.*	- *What expectations need to be established in this Launch so students can proceed? Is a time limit appropriate?* - *How much is too much in a Launch? When should we stop this particular Launch so students still have something demanding to Explore?* - *When is an example appropriate?*
• Did we all come up with the same strategy for multiplying mixed numbers?	- *Some of us used repeated addition and some of us used the standard algorithm. Some used drawings.*	- *Could you build on what you already know about fractions?* - *Why does the standard algorithm work?*
• What questions did you need to ask during the Explore phase? Should these have been included in the Launch?	- *I had to ask for help to make a drawing to represent* $\frac{1}{3}$ of $2\frac{1}{2}$ *because it was not just a part of a part.* - *I could see adding* $2\frac{1}{3}$ *repeatedly but I could not account for the* $\frac{1}{2}$ *day.*	- *What are the pros and cons of pushing the Launch to the point where a correct strategy is played out?*

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
• Can you tell from the Summary phase what I was looking for in the Explore phase? Did I choose an effective sequence of "student" strategies to share?	- *You seemed to be looking for different "student" strategies to share; some used drawings and some used only symbols.*	- *Let's look at the "student" work. Is there a reason why one order might work better than another? Do the different "student" strategies connect to each other?*
• What common error(s) might you predict?	- *Students often think that to multiply* $1\frac{1}{2} \times 2\frac{9}{10}$ *we can multiply the whole numbers and multiply the fractions, to get* $(1 \times 2) + (\frac{1}{2} \times \frac{9}{10}) = 2\frac{9}{20}$.	- *Which should be greater:* $1\frac{1}{2} \times 2\frac{9}{10}$ *or* $1\frac{1}{2} \times 2$*?* - *How does the algorithm for "long" multiplication (whole numbers) relate to a strategy for mixed number multiplication? (See video clip of 3.3)*

Note: *If the workshop I am planning is typical then teachers are likely to think that students will have problems with this lesson. My job is to convince participants that out of the struggle will come some good mathematics, but that they will have to plan ahead to be prepared to spot good mathematical ideas and draw them out. Fortunately I have a video of a teacher doing just that. And I have that teacher's reflections on video so participants can think about how she has planned and executed this lesson, and how concerned she was that she would not get everything she needed from the student work --- but she did.*

VIDEO: "Developing an Algorithm for Multiplying Fractions"
(*Bits and Pieces II*, Investigation 3.3, 8 chapters. 22 minutes)

Note: This video has been edited to focus on the learning trajectory from understanding an algorithm for multiplying two fractions less than one, to developing algorithms for multiplying mixed numbers. Real time is 2 class periods.

EXPLORE: While watching the video:
(See "Teacher Reflections" for an alternative use of this video)

Principals benefit from doing the mathematics in 3.3, and having this in front of them as they watch the video. When they are viewing the video they could be asking the same questions as they did for 3.1 and 3.2.

Focus Questions for Principals

Additional questions might be:

- Where does this mathematics fit into your curriculum?
- Is the time spent on students inventing ways to solve problems time well spent?
- Are the student algorithms you see on this video efficient?
- Can the student algorithms play a role in explaining the standard algorithm?
- What is gained and what is lost by the CMP approach to "algorithm?"

Focus Questions for Teachers and Teacher Leaders

As before I should have a copy of the transcript available. And, of course, participants should have copies of 3.3. To Launch into viewing the video I should ask participants to choose from some focal questions and congregate in groups according to interest. As before, it helps if they think they are looking at this video through Teri's eyes, looking for what works and ways to improve. I may need to rephrase the questions below depending on the audience.

- What did the teacher seem to be looking for in the Launch? Was it successful/too long/too short?
- What did the teacher seem to be looking for in the Explore? Was it successful?
- What did the teacher seem to be looking for in the Summary? Was it successful?
- What student interchanges seemed mathematically significant? Was the teacher present?
- What student-teacher interchanges seemed mathematically significant? What was the teacher's role?
- Is there evidence of skill development? Of conceptual development? What is the relationship?
- Did the teacher select and sequence the student work effectively in the summary?
- What do students understand at the end of this lesson?

Form Focus Groups of Teachers and Teacher Leaders

It has worked well in the past to re-arrange participants into focus groups before viewing the video. If they have a few minutes to talk about the focus question *before* the video and then time to debrief in small groups *after* the video I have noticed that the discussions are more coherent.

When I ask follow up questions during the small group discussions the conversation is richer. I need to think more about Follow Up Questions and how to use them during both small and large group discussions.

	Focus Questions:	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions:**
SUMMARIZE: Focus Group Discussion after Viewing the Video	• What did the teacher seem to be looking for in the Launch? Was it successful/too long/too short?	- *I was not sure what the teacher was hoping would come out of the GR/Launch.*	- *Can anyone help identify the mathematical ideas that came up in the Launch phase?*
Alternative ways to conduct discussions: It can be unnecessarily repetitive if the same discuss/ view/discuss format is followed in every pd session. I have tried different formats. Some of these are described in the appendix.	• What did the teacher seem to be looking for in the Explore? Was it successful?	- *Suppose my students don't come up with all the strategies that these kids did. What will I do in the Summary?* - *Teri seemed to be looking for different strategies in the Explore, and she saw a common error.*	- *Do you suppose that Teri always gets all these strategies from students?* [Ω]

[Ω] We have Teri's reflection on the DVD so we can listen to her talk about how lacking in confidence she was at some points, but, by trusting her students and following their lead, the investigation, and particularly the Summary of 3.3 was a great success.

Possible "Teacher" Discussion Questions (cont'd):	**In Previous Workshops Teachers Have Said:**	**Follow Up Questions**
What did the teacher seem to be looking for in the Summary? Was it successful?	- *Teri used different student strategies to get two different and valid algorithms.* - *She used one correct strategy to help explain a common error.*	- *What order did she choose for her student strategies? Would another order have been any better? Explain why or why not.* - *Was it effective to make the common error explicit?*[Ω]
• What student interchanges seemed mathematically significant? Was the teacher present?	- *Students used the Commutative Property, the Distributive Property and rewriting the mixed numbers in the Launch, and again in the Summary.*	- *What do you mean the Distributive Property came up? We did not hear these words. Can you remind us what student(s) said?*
• What student-teacher interchanges seemed mathematically significant? What was the teacher's role?	- *The teacher carefully scripted each student strategies more formally. We want students to take this over.*	- *Can you give an example of Teri scripting a student strategy? Why might she have chosen to do this?*
• Is there evidence of skill development? Of conceptual development? What is the relationship?	- *Students need to practice the strategies they found.*	- *What do you mean "practice?" Where do we find "practice" in CMP?*[Ω]

[Ω] Sometimes it works to take care of a common error right away, so students will immediately know they have an incorrect solution and will be attentive to other solutions. On the video we see Teri wait until she has 2 correct solutions on the board so she can use these to show WHY the common error is incorrect.

[Ω] Teri talks about the place of practice in CMP in one of her reflections.

	Possible "Teacher" Discussion Questions (cont'd):	In Previous Workshops Teachers Have Said:	Follow Up Questions
	• Did the teacher select and sequence the student work effectively in the summary?		- *Might you have sequenced the student work differently?*
	• What do you think students understand at the end of this lesson? •	- *That you can multiply mixed numbers by using either of 2 strategies: rewriting as improper fractions and then applying the algorithm, or "picking it apart."*	
	Possible Focus Questions for Principals:	**In Previous Workshops Principals have said:**	
Focus Questions for Principals	• Where does this topic fit in your curriculum? • Is the time spent on students inventing ways to solve problems well spent?[Ω] • What is gained and what is lost by the CMP approach to "algorithm?		

SUMMARY After participants have had an opportunity to talk in their small groups

[Ω] I need one short article for Principals, to reassure them that this Problem centered approach is based on research. "Mathematical Understanding" (Fuson, 2005) is good. See appendix.

CONT'D we should have a **large group discussion**.